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INVESTIGATION OF UNCERTAINTY IN EGRESS MODELS AND DATA*

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ABSTRACT

The use of computational analysis to predict building egress during emergency situations has been steadily increasing in recent years. However, there is a general lack of data for use in computational egress models, and there are no benchmarks against which to test the predictive capability of the computational egress models. As a result, how well these models are able to predict a priori the time to egress a building is generally unknown, as are those variables that have the most significant impact on the predicted outcomes. To begin addressing the issues of evaluating the predictive capability of egress models, and the uncertainty and variability associated with the models and the available data, a three-year research effort is underway. The study methodology and preliminary results are presented.

INTRODUCTION

Accurate predictions of the time required to egress a building under various conditions are crucial to decisions regarding the acceptability of a building design. Accurate predictions of total time to egress require both robust models, i.e., models that capture the major factors of egress, as well as a large amount of good, reliable and readily available input data for the model, such as body sizes and walking speeds. Accurate predictions also require a methodology for expressing variability in factors such as occupant health, mobility, and location at the time of a fire event – parameters generally unknown prior to an event.

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To help provide more accurate predictions of egress time, assessment of computational egress models to determine model structure, dominant variables, and sources, types, magnitudes and importance of uncertainty and variability is needed. Collection and analysis of data sets to support this computational modeling, especially for assessing emergency egress scenarios where data are extremely sparse and little is known about the validity of the data, is likewise essential. Ultimately, a methodology to quantify uncertainty and variability in both the input data and the predictions of the models must be developed and standardized.

These needs are well documented in the literature. Shields and Proulx,[1] for example, note shortcomings in the availability of data, in the current state of modeling, and in the application of current models. In their paper on creating a database for evacuation modeling, Fahy and Proulx[2] provide an extensive list of data needs, and conclude that "it is essential that engineers, designers and building officials have available to them accurate information upon which to base any assumptions of occupant time to start and movement speed that will be used in the evaluation of an engineered building design." Fahy[3] reiterated this concern at the *National Research Council Workshop to Identify Innovative Research Needs to Foster Improved Fire Safety in the United States*.

Previous work on uncertainty in fire-safety calculations has focused on the other side of the safety equation, i.e., prediction of the time to development of untenable conditions. Notarianni[4,5] and others[6,7,8] have discussed the importance of identifying and addressing uncertainty in data, models, and fire safety engineering analysis, as the failure to do so can lead to inappropriate use of data, misinterpretation of model output, and ultimately to misinformed decisions for life safety system design. This work focuses on accurate predictions of time to egress so that complete information is available to the decision-makers.

OVERALL SCOPE OF RESEARCH PROGRAM

To begin addressing the issue of uncertainty and variability in egress data and in computational models for egress analysis, a three-year research began in September 2002 under a grant from the National Institute for Standards and Technology, Building and Fire Research Laboratory (Grant 60NANB2D0138), *Uncertainty in Egress Models and Data: Investigation of Dominant Parameters and Extent of Their Impact on Predicted Outcomes*. The primary goals of this work are to:

- Understand sources of uncertainty and variability in egress models.
- Apply and refine a method of uncertainty analysis to computer egress modeling
- Identify "cross-over" variables that may have an impact on the results of the egress model that is significant enough to cause a change in an engineer's design of a building.
- Provide building engineers with guidance in the appropriate use of computer egress models.

In meeting the above goals, the following tasks have been outlined:

- Collection of data sets for use in and for verification of the predictive capabilities of egress models,
- Documentation of the actual range of input variables used in egress models, and valid statistical representations of these variables
- Identification of sources, type, magnitude and importance of uncertainty and variability in data (input) for egress models, and
- Comparison of predictions of time to egress based on two egress models using a "blind" evacuation scenario (using a data set not used for the uncertainty analysis identified above).

The research plan follows generally the methodology proposed by Notarianni[4,5] for use with any fire-safety calculation. The approach involves the following. Sensitivity analyses are conducted to identify the dominant input variables, i.e., those with the strongest correlation to, and thus the most potential to, change the outcome prediction. Collections of data sets are used to develop statistical representation of values of these dominant input variables to account for uncertainty and in some cases variability. In cases where sufficient data is not available, variables may be assigned broader ranges of potential values, which would account for their uncertainty, but would necessitate a greater number of simulations to complete the analysis. It is expected that some variables that show low statistical correlation may be set to best-guess or average values. After bounding the applicable variables, a Monte Carlo analysis will be applied to conduct a large number of predictions using the selected models. These simulations will be run, and their results will be analyzed to determine the level of uncertainty associated with each variable.

Initial Efforts

The first year of the research program, which has been completed, involved data gathering and preliminary assessment efforts. Efforts included:

- Selecting egress models for evaluation,
- Collecting and analyzing data sets from previous evacuation and research work,
- Assessing the egress models to determine dominant variables, sources, types, magnitudes and importance of uncertainty and variability, and how the models differ,
- Assessing the above models against a data set and comparison of model results,
- Identifying sources and type of uncertainty and variability in data (model inputs), and
- Initial efforts to identify appropriate treatment of uncertainty and variability based on model specifics.

For this effort, the egress models selected for analysis are STEPS (Simulation of Transient Evacuation and Pedestrian movementS)[9] and EXIT89.[10]

<u>STEPS</u>

The STEPS computer program implements an optimization evacuation model, including queuing, and has been used over the past five years on a wide range of projects, from mass transit stations,[11] to complex assembly spaces,[12] to high-rise buildings.[13] The model supports travel through a variety of egress routes as generated within the model for simulation by the modeler. With the ability to establish the various egress paths, different egress scenarios can be readily simulated to quantitatively differentiate between egress scenarios. The model begins with the establishment of people types and people groups. The people types contain the information about the travel speed of the person under various conditions. These conditions would typically be the travel speed on horizontal surfaces, while traveling up stairs, and while traveling down stairs. People groups are used so that different characteristics for certain groups of people can be used. For example if the population had a significant number of elderly or mobility impaired people, they could be represented with a slower travel speed.

Typically developed through the importing of CAD drawings, the geometry of the modeled space is added onto a plane over which egress will occur. The plane is broken into square grid cells typically ranging from 0.4 to 0.5 meters on a side. The geometric configurations for walls, partitions, columns and furnishings found on the drawing are interpreted by the STEPS model as blockages. Where blockages occur, the grid is marked as impassable and occupants cannot traverse those cells. For multi-level models, the addition of planes spanning from one level to another provides for the development of egress paths using stairs. From the grid cells generated as the interpretation of the CAD drawings, available grid cells that can be occupied by people are determined. Only one person can occupy a grid cell at a time. At each time step, the model calculates a "score" for each grid cell in relation to the exits for a particular plane. The occupant will move to a grid cell with a score lower than the presently occupied grid cell provided that the target grid cell is not already occupied. Scoring for the model is based on an algorithm that incorporates the time needed to reach a target, the time needed to queue at a target, and the patience of the building occupants.[14] A further discussion of the variables required for this algorithm is provided in the results section of this paper.

Along with geometric characteristics, people groups and exits are placed onto a plane to complete the modeling. The people groups placed on planes can be configured to randomly distribute people in the plane. The exits provide "goals" or "targets" for the people on the plane. The scoring, described above, is calculated in relation to the various exits available to the evacuee. The exits either provide a way for evacuees to leave the model (exit the building) or to move onto another plane (another floor). Exit routes that allow evacuees to leave the building would, for example, model an occupant leaving a floor, traversing the stairs, passing through a lobby, and leaving the building.

EXIT89

The EXIT89 program developed by Fahy was intended to model evacuation of a large building with the ability to track the path of each occupant.[10] The model uses traffic speeds dependent on flow densities as measured by Predtechenskii and Miliinskii. The model can be used in conjunction with a zone model such as CFAST to incorporate the effects of fire and smoke spread on the occupants' evacuation behavior.

The model requires a network description of the structure, which includes the geometry of the compartments, openings between rooms, and the number of people located at each node of the building. The user must decide whether the occupants will evacuate at the first sign of fire or if they will delay some period of time. The decision process to evacuate or delay is implicitly handled as a time delay before the program begins to calculate the evacuation for a given occupant. The delay times are a user option and can be assigned to all occupants of a node or can be randomly assigned to individuals. Also considered in the model is whether the occupant will use the shortest route or nearest exit, or use a familiar route. To achieve this, the user must specify the appropriate route for an occupant to follow. Population and travel densities are based on body sizes, which are also user specified. Disabled occupants can be added, but are done by simply modifying the travel speed for that particular occupant and does not account for assistance from other occupants. By the developer's admission, the model is not capable of simulating behavioral considerations explicitly. Rather, certain considerations can be modeled implicitly by incorporating delays and modified occupant parameters. Essentially, the model calculates a flow of people from one node to the next (modifying the speed of movement according to population density and environmental conditions) until all occupants have exited the building. This assessment will build upon previous validation efforts,[10,15] resulting in more explicit understanding of sources of uncertainty and appropriate methods for treatment thereof.

STUDY METHODOLOGY AND RESULTS TO DATE

As noted above, the research plan follows generally the methodology proposed by Notarianni.[4,5] As used in this research program, a five-phase process is being used to carry out the study:

Phase 1: Construction of Base Models
Phase 2: Identification of Variables and Possible Values
Phase 3: Monte Carlo Analysis
Phase 4: Statistical Analysis of Computer Model Results
Phase 5: Identification of Significant Variables

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Phase 1: Construction of Base Models

STEPS and EXIT89 were used to model a 6-story (plus basement) office building in London, Ontario (the London building) and a 14-story apartment building in Calgary (the Calgary building), for which data was available from actual evacuations. The modelers were provided with information regarding the building geometry, the number of occupants on each floor of the building, and the pre-movement times for many of the building occupants.

Preliminary results indicate that STEPS and EXIT89 both provide good approximations of the travel time from the London building, given that the modeler knows the number of people that were present on each floor and their pre-movement times. The actual total evacuation time of the London building was 226 seconds. The three initial STEPS simulations predicted total evacuation times of between 222 and 226 seconds; the three initial EXIT89 simulations predicted total evacuation times of between 199 and 203 seconds.

STEPS also provided a good approximation of the travel time from the Calgary building. The actual evacuation time from the building was 790 seconds. Four initial STEPS simulations predicted total evacuation times of between 726 and 745 seconds. Simulation of the Calgary building with EXIT89 is ongoing.

Phase 2: Identification of Variables and Possible Values

A number of variables were identified that could be modified within STEPS and EXIT89. Included in these variables are building geometry (number of floors, furniture layout, number of exits, stair width, etc.), occupant characteristics (number of people, age distribution, walking speeds, pre-movement times, patience, etc.), and model-specific variables (grid spacing, time step, etc.). Building geometry was not modified in this initial study. All simulations assumed that the geometry was the same, as observed during the actual evacuation of the London and Calgary buildings. Future work may investigate geometry changes, such as varying door and stair widths, or blocking exits. Distribution curves of values were formed for many variables representing occupant characteristics or model-specific variables. When available, experimental observations reported in available literature were used to form these distribution curves. A sample distribution curve for pre-movement times is provided in Figure 1.

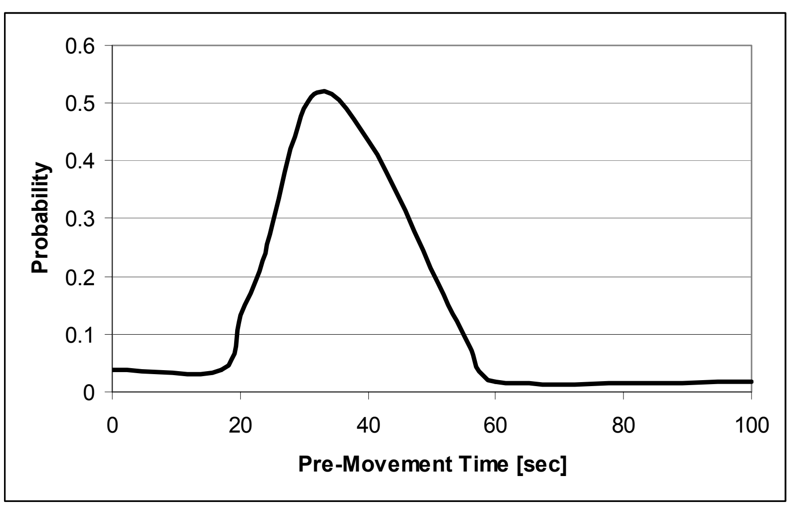

Figure 1: Sample Distribution Curve of Values for Pre-Movement Time (in seconds) for Occupants of Office Buildings. Formed from NRC "London Building" Data and Purser[16]

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One of the factors limiting the field of egress modeling is the relative scarcity of data for some occupant characteristics. Future work is required to expand upon the literature review begun in this study. In addition, techniques should be developed to combine statistical information that is already available from different studies, but in varying formats. Some research has been done in the fields of risk management[17] and weather forcasting[18] investigating the combination of data from multiple sources.

Many of the STEPS model-specific variables do not represent quantifiable properties that can be obtained experimentally. Among these are people's patience, model randomness, time-step, lock-solver depth, patience coefficient, walking coefficient, and queuing coefficient. Coefficients were randomly assigned values between 0 and 1 for each simulation, while time-step was varied between 0.01 and 5 seconds, and lock-solver depth was varied between 1 (default) and 10. The effect of changing the grid size was tested during the Calgary building simulations, using grid sizes of 0.3m, 0.4m, 0.5m, and 0.6m.

Phase 3: Monte Carlo Analysis

For the efforts to date, Phases 3-5 have focused on STEPS. In Phase 3, 2000 STEPS input files were created and run using the Monte Carlo method for each of the buildings investigated. Variable values were randomly chosen from the distribution curves formed in Phase 2 of the analysis.

Phase 4: Statistical Analysis of Computer Model Results

The output from the STEPS simulations was analyzed using a method described by Notarianni[19]. A cumulative distribution function (CDF) was created to display the probabilities associated with an estimated evacuation time for the London building and for the Calgary building. This was accomplished by graphing each evacuation value against its rank, as displayed in Figure 2.

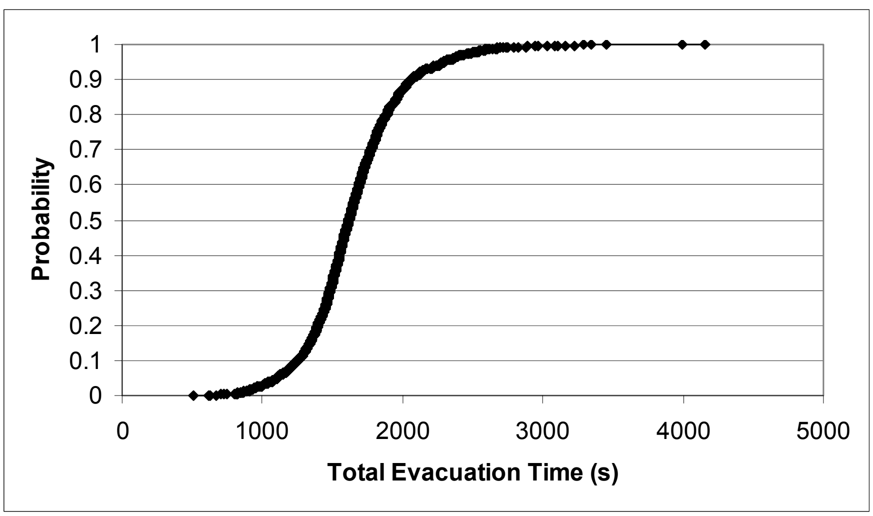

Figure 2: Cumulative Distribution Function of Total Evacuation Time (Calgary Building)

This graph provides the probabilities that are associated with a range of evacuation times. For example, according to this analysis, there is a 90% chance that it will take 2000 seconds or more to evacuate the London building. Therefore, if the acceptable level of uncertainty were 95%, it would be prudent to design fire/life safety systems for the building that could maintain tenable conditions for at least 2300 seconds.

Phase 5: Identification of Significant Variables

Using a method identical to that described above, it is possible to view the simulation data in a way that identifies the significance of the variables that were randomly chosen in the Monte Carlo analysis of Phase 3. A sample of some analysis techniques is provided below, looking at the effect of varying occupant load on the total evacuation time from the Calgary building.

Sample Analysis - Occupant Load of Calgary Building

Figure 3 provides a CDF that reflects the effect of the total building occupant load on the total evacuation time for the Calgary building. The two series represent simulations with a low occupant load (0-25% of the maximum number of people simulated) and a high occupant load (75-100% of the maximum number of people simulated). This figure provides some insight into the effect of occupant load on total evacuation time. The general trend in the Calgary building, as one might expect, is that higher occupant loads result in higher total evacuation times from the building.

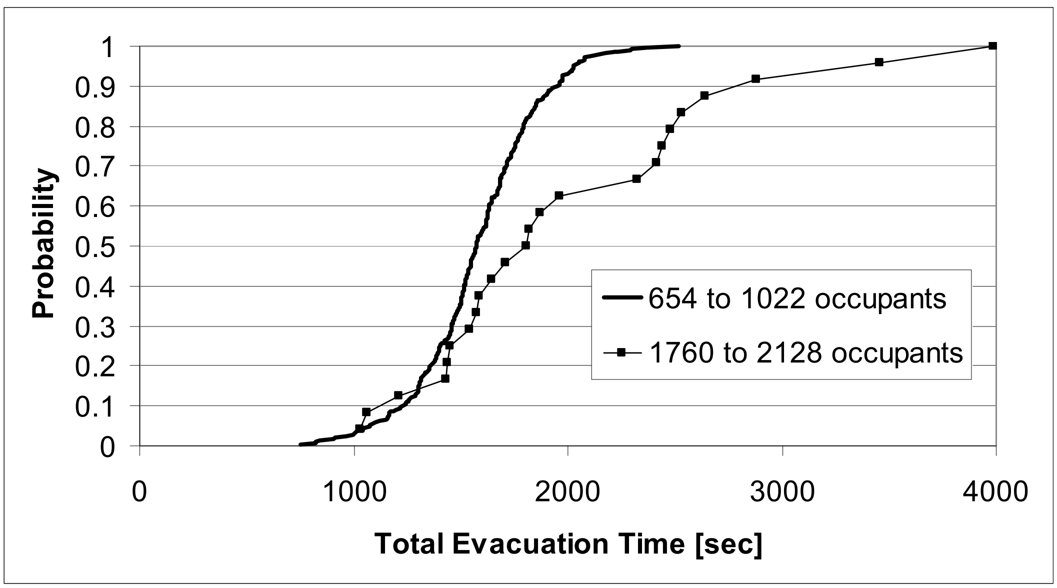

Figure 3: Cumulative Distribution Functions of Total Evacuation Time for Two Ranges of Total Building Occupant Loads (Calgary Building)

In addition, the graph in Figure 3 demonstrates that as we approach 90% to 100% certainty the effect of the occupant load on the total evacuation time for the Calgary building increases dramatically. An alternate method of displaying the data in Figure 3 is to graph the difference in the two series.

Figure 4 represents the difference in total evacuation times, subtracting the 0-25% of peak load (654 – 1022 occupants) series from the 75-100% of peak load (1760 to 2128 occupants) series. Figure 5 indicates that as the probability threshold increases, the effect of the occupant load on the total evacuation time increases rapidly. At the 95% certainty level, the difference between the low occupant load and the high occupant load in the Calgary building varied by almost 1500 seconds (25 minutes).

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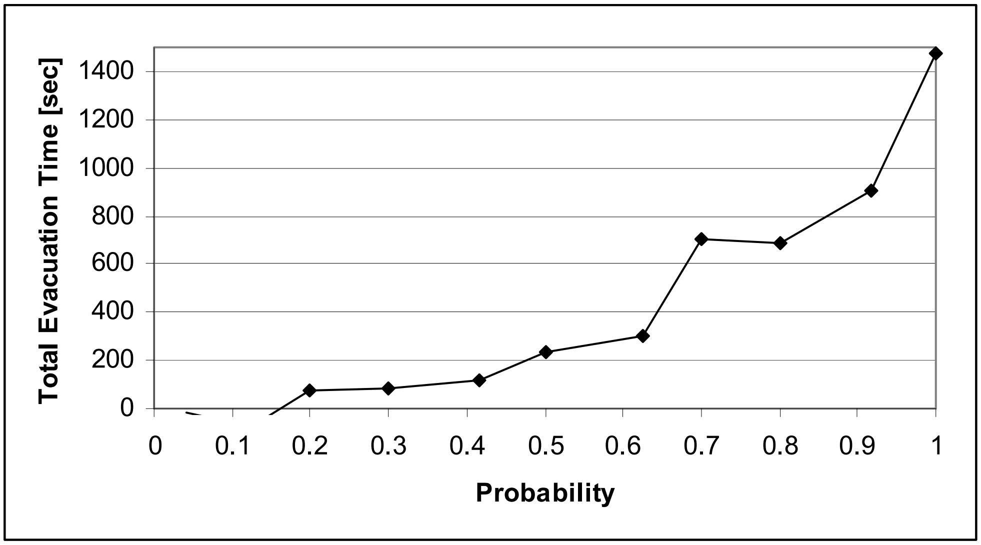

Figure 4: Sensitivity of Total Evacuation Time to Total Building Occupant Load

Evaluating Uncertainty Importance

In addition to the creation of CDFs, as shown above, useful information can be gained by evaluating the importance of each variable in relation to the total evacuation time (or any other quantity that is measured). Each variable's importance can be calculated on a scale from –1 to 1, where a correlation coefficient of 1 indicates a very strong direct correlation, 0 indicates no correlation, and –1 indicates a very strong inverse correlation.

The value of the correlation coefficient at which a variable becomes statistically significant can be calculated using the following equation:

$$t = \frac{c}{\sqrt{1-c^2}}\left(\sqrt{n-2}\right)$$

Where t is related to the confidence level, which is typically chosen as 95%[20], c is the statistical significance of the correlation coefficient, and n is the number of samples in the data set. For the London and Calgary buildings, both of which used sample sizes of 2,000, it was determined that correlation coefficients with an absolute value of 0.021 or less were statistically significant.

Table 1 provides the correlation coefficients that were calculated from the results of the London and Calgary building simulations. This table provides a partial list highlighting some of the more statistically significant variables.

In general, the individual variables showed less correlation to the total evacuation time in the London Building than in the Calgary building. Some variables correlated well to total evacuation time in the Calgary building, but not in the London building, suggesting that the importance of the variables in the egress models is tied to the geometry of the building. This hypothesis is somewhat enforced by the fact that the grid size, which was the one geometry-linked variable tested was the variable with the highest correlation coefficient. The effect of varying the grid size has not yet been tested for the London Building; this work will be completed as the study progresses.

Variable	Correlation Coefficient	
	Calgary Building	**London Building**
Grid Size	0.398	Not yet tested
Time Step	0.248	-0.034
Total Occupant Load	0.197	< 0.021 (not significant)
Pre-movement Time (top floor)	0.183	0.042
Walking Speed – Elderly Down Stairs	-0.121	-0.020
Walking Speed – Disabled Down Stairs	-0.099	Was not tested
Door Flow Rate	-0.052	-0.107
Number of People on Level 2	0.067	-0.046
Number of People in Basement	Not applicable	0.044
Patience of Elderly	< 0.021 (not significant)	-0.039
Queuing Coefficient	-0.026	-0.038
Randomness	0.056	-0.036

Table 1: Correlation Coefficients for Variables – Calgary Building and London Building

In some cases, increasing or decreasing the value of a variable might lengthen the total evacuation time in one building while having the opposite effect in the other. For example, the correlation coefficient for the time step was positive (0.248) in the Calgary building, indicating that an increase in time step correlated to an *increase* in total evacuation time; at the same time. The correlation coefficient for the time step in the London building was negative (-0.034) indicating that an increase in the time step would generally *decrease* the total evacuation time.

CONCLUSIONS

A study has been conducted to apply the method of Notarianni[21] to computer egress modeling. This method uses a Monte Carlo technique to evaluate the uncertainty associated with simulation of evacuation times from a building. An analysis has been completed using the computer egress model "STEPS" for a 6-story office building in London, Ontario and a 14-story apartment building in Calgary. A similar analysis is underway using the computer egress model "EXIT89".

Preliminary findings appear to indicate that there is not necessarily a specific group of variables that is important for all building types. The significance of the variables seems to be case-specific and may be closely tied to the geometry of the building.

Future work will involve the variation of grid size in the London building with the STEPS model, and completion of the method of analysis for both the Calgary building and the London building using EXIT89. Additional buildings will also be tested using this type of analysis to provide a large pool of data from which comparisons of variable importance can be made.

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