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Brown, W. G.

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BY

W. G. BROWN

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ABSTRACT

Stress corrosion rate theory, probability theory and stress distribution theory were combined and used to correlate failure test results for large plates of soda-lime glass from different manufacturers. Examples demonstrate the design implications of the theory.

UNE THEORIE DE LA DUREE DE LA CHARGE POUR LA CONCEPTION DU VERRE

SOMMAIRE

La théorie du taux de corrosion de la contrainte, la théorie de la probabilité et la théorie de la distribution de la contrainte ont été réunies afin de mettre en corrélation les résultats obtenus dans des essais de ruine effectués sur de grandes lames de verre soude-chaux provenant de différents fabricants. Des exemples montrent l'importance de la théorie dans la conception du verre.

A Load Duration Theory for Glass Design*

W. G. Brown
National Research Council, Ottawa, Canada.

Introduction

Probably every designer of glass has been faced, at one time or another, with the frustrating experience of trying to find a value for the stress at which glass fails.

The search had to be abandoned, of course, with the realization that glass failure is as much dependent on the stress raising properties of surface flaws and scratches as on the intrinsic material strength of glass. As if this were not enough, it has been known for some time now that the strength of glass depends also on the duration of loading and on the temperature and relative humidity of the environment. The purpose of this paper is to gather together what is already known of these various phenomena into a practicable formulation for glass strength that can be used for glass design. The order of presentation is to discuss stress-corrosion, followed by statistical implications and probability theory and then to show the applicability of the theory for large glass plates.

Stress corrosion

The upper sketch in Figure 1 shows the micro-appearance of a glass surface, pitted with minute flaws and scratches as a result of manufacture and handling processes. For a specific flaw as indicated in the lower sketch, we have Inglis' equation[1] to show the approximate relationship between flaw tip stress σ_m, applied tensile stress σ, flaw depth x, and flaw tip radius ρ:

$$\sigma_m \approx 2\sigma \, (x/\rho)^{1/2} \qquad (1)$$

Because of the sharpness of glass surface flaws, very high stresses can occur at flaw tips. (It should be kept in mind that in Figure 1 and equation (1) no real physical interpretation need be given to the tip radius ρ.)

Since water vapour induces chemical corrosion in glass[2], albeit slowly, the geometry of a flaw can be expected to change with time. Furthermore, the corrosion rate depends, as well, on the stress magnitude. This stress corrosion phenomenon is described by the following equation:

$$dz_\perp/dt \approx K \cdot RH \cdot e^{-(\gamma_0 - \gamma_1 \sigma_{11})/RT} \qquad (2)$$

Here, $dz_\perp/dt$ is the time rate-of-progress perpendicular to the glass-corrosion interface. It is approximately proportional to the relative humidity RH[4], and to an exponential term[2] dependent on temperature T and stress σ_{11} parallel to the interface. (K, γ_0, γ_1 and R are constants.) Development and understanding for equation (2) comes mainly as a result of the work of Charles[2,3] and Weiderhoen[4].

Changes in dimensions x and ρ will depend overwhelmingly on the tip stress and it is not difficult to show that the rate of change of (x/ρ) will also be approximately of the form of equation (2). Furthermore, the stress term in equation (2) can always be approximated as a power term as in equation (3), that is, as the ratio of tip stress to temperature raised to power n, i.e.:

$$d(x/\rho)/dt \approx K_1 \cdot RH \cdot e^{-\gamma_0/RT} \cdot (\sigma_m/T)^n \qquad (3)$$

With equation (3) as base, we can then go on directly, after substitution from equation (1) and integration, to develop the following:

$$\int_0^{t_f} RH \cdot e^{-\gamma_0/RT} (\sigma/T)^n \, dt = \text{constant}$$

$$= \frac{RH\, e^{-\gamma_0/RT}}{(n+1)\,T^n} \cdot \sigma_\epsilon^{\,n+1}/\beta \qquad (4)$$

Equation (4) is a statement that the cumulative effect of arbitrary time-dependent stress σ applied to a specimen until failure at time t_f, is constant. Charles[2,3] finds experimentally that $n = 16$ for soda-lime glass, hence, for example, with constant rate of stress increase β, integration shows that the failure stress σ_ϵ is proportional to the stress rate raised to the 1/17 power. In other words, we

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expect that tests will show the measured failure stress to decrease with decreasing loading rate. It is also clear from equation (4) that the failure stress $\sigma\epsilon$, obtained at a standard rate of stress increase, gives a complete and characteristic description of the strength of each specimen.

The general validity of equation (4) is demonstrated when applied to other, independent test results as in Table I. Here, we see theory and experiment in close agreement for loading rate differences of 800 to 1.

Table I. Failure stress results (Kropschott and Mikesell[5])

	Comparative	Predicted
Loading Rate, β	Failure Stress	Failure Stress ($\beta^{1/17}$)
1 lb/in^2	1.00	1.00
10 lb/in^2	1.11	1.14
800 lb/in^2	1.48	1.49

Probability implications

Considering again the flaw or scratch geometry, we recognize that dimensions x and ρ will be random, hence, each specimen will fail at a different value of $\sigma\epsilon$. If we now make a plot of the percentage of specimens which fail we obtain the cumulative failure probability P_f as in Figure 2. Here, in the upper figure, we simply plot the total fraction of all specimens which have failed at specific values of $\sigma\epsilon$. A few specimens will always fail at very low stresses and a few will not fail except at very high stresses.

Considering the effect now of changing the specimen size, obviously, large areas of glass will fail at lower stresses than small areas because a large area is more likely to contain major flaws. This is readily visualized by considering simple tension tests on areas of different sizes as in the lower sketch of Figure 2. For area size A_o, the probability of withstanding a given stress is $(1 - P_{fo})$, from the diagram. For specimens twice as big, the withstanding probability is $(1 - P_{fo})^2$, that is, the withstanding probability of each half-area is $(1 - P_{fo})$. Similarly, for area A, the withstanding probability is equal to that for area A_o raised to the power A/A_o. Hence:

$$P_f = (1 - P_{f_o})^{A/A_o} \qquad (5)$$

It is easy to see from equation (5) that for constant failure probability, the corresponding failure stress decreases with increasing area. It might be mentioned in passing that glass areas of practical size such as windows are hundreds and thousands of times larger than usual laboratory samples and that under these conditions, average failure stresses for the large areas will be of the order of only 1/4 those of the small areas. Under these circumstances, tests on a limited number of small samples give no useful information for large areas.

Correlation of test results for large plates

We turn now to application of the load duration theory and probability-area implications to the practical problem of window glass. In particular, we have available two independent sets of test results for large, square areas of soda-lime plate glass obtained by two different manufacturers. In one case, the glass area was constant at 10 square feet, and uniform pressure tests were carried out to break the glass in about 30 seconds. In the other set of tests, the glass area varied between 36 and 100 square feet and pressure loading was applied more slowly to cause failure in about 15 minutes.

Both sets of tests were carried out with approximately simple edge support (i.e. very small edge moments). Consequently, although deflections and stresses at failure fall within the large (non-linear) range, the stress distributions over the plate surfaces will be the same as long as the elastic parameter $q/E(a/h)^4$ is the same. Here, q is the applied uniform pressure, E is Young's modulus, a is the plate width and h is its thickness. For a limited range of $q/E(a/h)^4$, it is permissible to write, for any stress:

$$\sigma/E(a/h)^2 \approx B\,[q/E(a/h^4]^{s/n} \qquad (6)$$

and to make the appropriate substitution for σ in equation (4). Here, B and s are constants.

No real distribution of failure stresses σ_ϵ will follow precisely any of the standard mathematical forms. For convenience, however, we can adopt the Weibull distribution[6] in the form:

$$P_A = 1 - \exp\,[-\,k\,(A/A_o)\,\sigma_\epsilon{}^m] \qquad (7)$$

The appropriate substitutions of equations (4) and (6) into equation (7) now give:

$$P_A = 1 - \exp\Big[C(A/A_o) \cdot \Big\{ E^{n\text{-}s}\,(a/h)^{4s\text{-}n} \cdot \int_0^{t_f} (RH/T^n) \cdot \exp\,[-\,\gamma_o/RT\,q^s\,dt] \Big\}^{m/n+1}\Big] \qquad (8)$$

Here, C is a combined constant for square plates.

In both sets of tests referred to, the variation of q with time was determined and the integral in equation (8) evaluated and expressed in terms of mean or average breaking load q. A value for $s = 12.3$ was then determined by plotting q vs h for the tests carried out on the small plates of 10 square feet area. In these tests, the coefficient of variation on q had also been determined, the average value being 21.5%. With this information and Weibull's mathematics[6], it was possible to infer $m \approx 7.3$. From the results of this test series, it was then possible to infer average bursting strengths for the larger more slowly loaded plates of the second, independent series of tests.

Table II is intended to show that the results of these two sets of tests correlate very well on the basis of the

Table II. Effects of pressure load duration and area (large plates)

(1) Plate Size Mfr. No. 2	(2) Average Bursting Pressure	(3) Indicated Pressure by Mfr. No. 1	(4) Ratio, Column (3) ÷ Column (2)	(5) Calculated Effects: Duration	(6) Calculated Effects: Area	(7) Calculated Effects: Combined
8′ × 8′ × 3/4″	1.85 lb/in²	2.9 lb/in²	1.5	1.34	1.38	1.8
6′ × 6′ × 1/2″	1.70 lb/in²	2.4 lb/in²	1.4	1.31	1.24	1.6
9′ × 9′ × 1/2″	0.77 lb/in²	1.4 lb/in²	1.8	1.34	1.44	1.9
10′ × 10′ × 1/2″	0.62 lb/in²	1.2 lb/in²	1.9	1.34	1.51	2.0
6′ × 6′ × 3/8″	1.17 ib/in²	1.6 lb/in²	1.4	1.30	1.24	1.6
8′ × 8′ × 3/8″	0.66 lb/in²	1.1 lb/in²	1.7	1.33	1.38	1.8
10′ × 10′ × 3/8″	0.42 lb/in²	0.82 lb/in²	1.9	1.34	1.51	2.0

present theory even though the apparent, measured load-bearing capacity of the two sets of plates differs by up to nearly 100%. Column (2) of the table gives measured average bursting pressures for the various plate sizes of column (1), while the indicated bursting pressures of column (3) are those which would be observed if the failure mechanism required only that the applied stress at failure should be the same as that observed in the tests on the smaller, more rapidly loaded plates. In other words, column (3) results from the old engineering assumption that failure occurs when the applied stress reaches some critical value. The ratio of indicated and actual bursting pressures in column (4) shows that this simplistic assumption gives errors of between 40 and 90 per cent.

In columns (5) and (6), equation (8) has been used to calculate the portions of the ratios of column (4) which are due to time-duration and glass area effects. The load duration difference of 15 minutes in one test series compared with 30 seconds in the other gives a bursting pressure difference of approximately 33% (column (5)), whereas a 10:1 change in plate area changes the bursting pressure by 51%. The combined (multiplied) effects of columns (5) and (6) are given in column (7); the results are in good agreement with column (4).

The results of Table II represent, in effect, a description of the strength of large, practical areas of soda-lime plate glass. With this in mind, it was possible to infer a value for k in equation (7) with the help of surface stress distribution measurements made by Kaiser[7]. With k determined, the strength characteristics of large areas of soda-lime plate glass are thereby completely (albeit approximately) determined.

Examples

Table III gives some rather arbitrary examples of application in design. Thickness requirements have been calculated for a specific nominal failure probability of 0.001 for square plates of 4 and 100 square feet area respectively, with their edges supported in several different ways. An applied pressure of 0.2 psi for 60 seconds would correspond, for example, to a 100 mph wind load. The same pressure for 3 years might represent the cumulative snow load on a skylight in a 30 year design lifetime. (Aquarium windows can be handled in the same way.)

We first note in Table III that the manner in which the glass edges are held can have a marked effect on thickness requirements. (Simple support here means very light edge restraint and fixed support means rigidly held edges.) The two cases of two-edge support show thickness reductions of up to 50% are possible with rigidly held edges.

Turning to load duration, we see that thickness requirements for three years are roughly twice as great as for 60 second loading. The effect of temperature has also been indicated, where it will be noted that reducing the temperature from 70°F to 32°F reduces the required thicknesses by about 16% — in other words, glass is somewhat stronger at lower temperatures.

Table III. Required plate thickness to withstand a uniform pressure load of 0.2 lb/in² for failure probability of 1 in 1000

Type of Support	2′ × 2′ Plate: 60 sec at 21°C h (in.)	2′ × 2′ Plate: 3 yrs at 21°C h (in.)	2′ × 2′ Plate: 3 yrs at 0°C h (in.)	10′ × 10′ Plate: 60 sec at 21°C h (in.)	10′ × 10′ Plate: 3 yrs at 21°C h (in.)	10′ × 10′ Plate: 3 yrs at 0°C h (in.)
Simple (All Edges)	0.08	0.18	0.15	0.59	1.34	1.15
Simple (2 Opposite Edges)	0.18	0.28	0.25	1.11	1.74	1.60
Fixed (2 Opposite Edges)	0.12	0.21	0.18	0.78	1.22	1.13

Conclusion

As far as is known, past design practice has always neglected load duration effects and has also frequently overlooked statistical implications and even the influence of type of edge support. In any event, it now appears possible to design glass on an improved rational basis. In this connection, it might be mentioned that the total cost of the manufacturers' tests referred to was of the order of one-half million dollars. Consequently, there is considerable justification for deriving as much value as possible from the results.

It should be noted that the present procedures can be extended to the problem of glass edges, the strength of which is important, particularly as a result of temperature-induced stresses in windows. Sealed, double-glazed window units present another design problem that can now be handled with the help of the present methods. Particularly with small units, the action of temperature and barometer changes causes a bellowing effect with cumulative loading conditions that may cause unusually high rates of failure.

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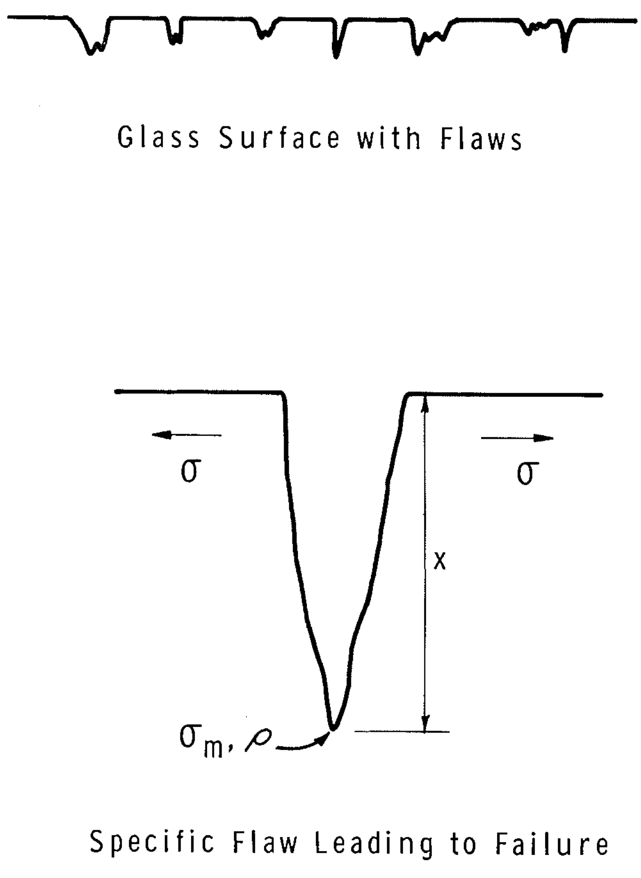

FIGURE 1. Schematic representation of surface flaws

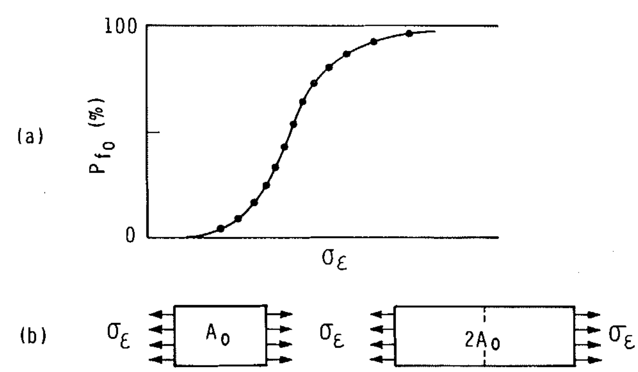

FIGURE 2. (a) Cumulative failure probability for glass area A_0 (b) Illustrating tensile tests on areas A_0 and $2A_0$

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ERRATA LIST

Page 75 — 2nd column, line 11 and reference 4, proper name is Wiederhorn.

Page 76 — Table 1. Loading rate should be lb/in^2—sec.

In paragraph including equation (6) the terms should be

$\frac{q}{E}\left(\frac{a}{h}\right)^4$ and $\frac{\sigma}{E}\left(\frac{a}{h}\right)^2$

In equation (8) the exponent on a/h should be $4s-2n$, the small square brackets should close after RT, and the entire expression enclosed by $\{\ \}$ should be raised to the exponent $\frac{m}{n+1}$ and the equation terminated at this point with the large bracket.

W.G. Brown