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PROBABILITY CHARTS FOR PREDICTING ICE THICKNESS

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IN RECENT YEARS there has been an increase in available information on the thickness of ice covers at various sites in North America. In 1953 Currie[1] made a study of ice thickness and snowfall for the Prairie Provinces and the Northwest Territories; in 1954 Ryder[2] compiled ice thickness records in the Northern Hemisphere for the American Geographical Society; and during the winter of 1958-59 the Meteorological Branch of the Department of Transport established a continuing program of weekly ice thickness measurements on bodies of water adjacent to selected meteorological stations in Canada[3].

There are a number of reasons for this recent increase in ice thickness observations. Empirical and theoretical studies have shown that the bearing capacity of an ice cover is proportional to the square of the ice thickness[4]. Before the bearing capacity of an ice sheet can be estimated, therefore, the thickness must be measured or estimated. Ice thickness records are also required by engineers for predicting possible ice pressure against dams and for consideration in designing water supply intakes where the source of supply is a frozen lake, river or harbour. A record of expected ice thickness is useful also to hydrologists concerned with river run-off or the prediction of the amount of ice that will be melted during the spring break-up of lakes and rivers.

The most satisfactory method of obtaining information on ice thickness is by routine measurement of ice thickness for a number of years at a number of sites. In Canada, with its thousands of lakes, rivers and harbours frozen for many months of the year, however, it will always be difficult to have an adequate number of these ice observation stations. Often it will be necessary to estimate ice thickness at sites several hundred miles away from the nearest ice observation station.

Under these circumstances it is usual to estimate ice thickness using formulae that relate ice growth to meteorological factors. In this paper an alternative method of predicting ice thickness and rate of growth is proposed. Probability charts prepared from ice thickness observations now available in North America are presented. These charts provide information on the probable rate of increase of ice thickness during the ice growth period and the probable maximum thickness that will be attained under a wide variety of site conditions in North America.

Formation of Natural Ice Covers

To appreciate the limitations of probability charts or more elaborate methods for predicting ice thickness it is necessary to understand the various ways an ice cover can form. As several authors have described the formation and development of natural ice covers in detail[5,6], only a brief outline will be given in this paper.

Under normal conditions there is a temperature gradient in the ice cover. Heat flow along this gradient leads to an increase in the thickness of the cover by the formation of ice at the interface between the ice and water. If the ice is thin and the air temperature much below freezing, the temperature gradient and associated heat flow will be relatively large and a rapid increase in ice thickness will result. As the ice cover increases in thickness the temperature gradient usually decreases and the ice grows at a progressively slower rate. Frequently, the average air temperature decreases at the same time and partly offsets the effect of increasing ice thickness on the temperature gradient.

If there is an insulating layer of snow on the ice cover it will reduce the temperature gradient through the ice and the rate of growth will decrease accordingly. If the snow cover is deep enough, the temperature at the ice-snow interface can be very close to 32°F and ice growth will be negligible. As the insulating value of snow can be as much as ten times that of ice, depending on its density, snow cover on the ice has an important influence on the rate at which an ice cover thickens.

Ice does not always grow downward from its lower surface. If the snow cover is heavy enough the upper surface of the ice can be depressed beneath the hydrostatic water level so that water is forced up through holes and cracks and floods the sur-

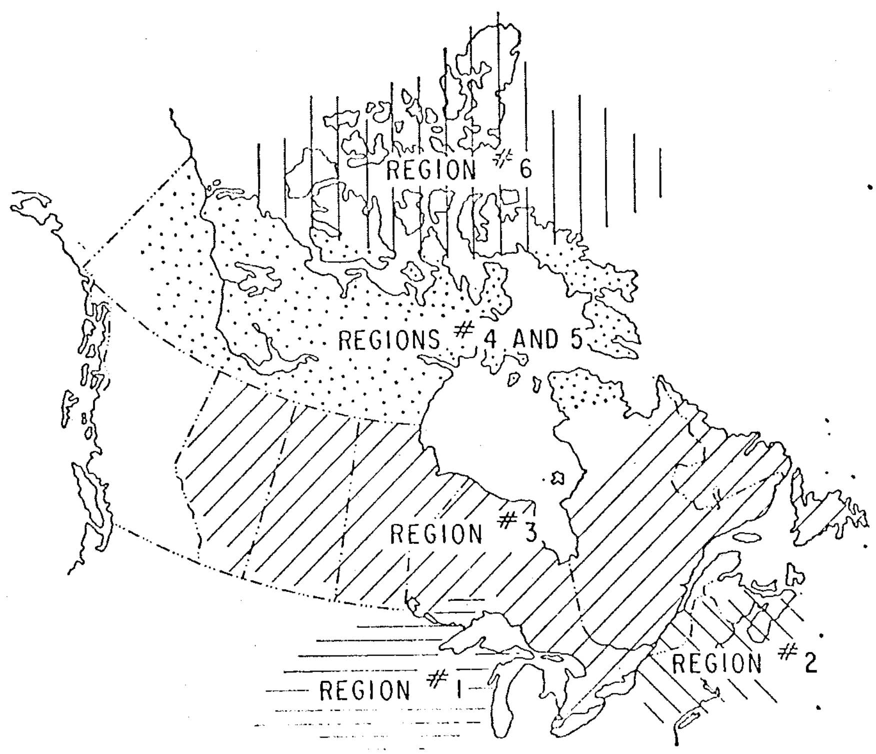

Fig. 1—Regions used for ice thickness probability studies

face. The slush that forms and freezes on the top is sometimes called white ice because of its colour. Its growth depends on the amount of snow that falls on the ice cover and on the meteorological conditions that prevail after the surface has been flooded. If the weather is cold enough, the freezing slush on the surface can produce a rapid increase in total ice thickness. If mild weather prevails, especially if the snow is not wetted to its full depth, a layer of slush can remain unfrozen. Sandwiched between the snow and the ice, or between two layers of ice, this slush can produce poor conditions for vehicle traffic. Such water or slush layers can also be produced by melting and run-off during mild periods.

From a consideration of the process of ice cover formation, it is clear that the thickness of ice and the depth of snow on the cover are major factors in determining the rate at which ice will thicken. The weather conditions that occur during the period of ice formation are also important in determining the rate of heat loss from the surface. The main meteorological variables affecting heat loss at the surface are the air temperature, total net radiation, wind velocity and relative humidity. Of these air temperature is the most important for ice thickness predictions normally required by engineers.

If theoretical or empirical formulae are used to predict ice thickness, quite detailed observations on weather, ice and snow cover conditions are required. The accuracy of the prediction will depend not only on the validity of the formulae used but also on the accuracy with which weather conditions and snow and ice cover properties required for the calculation, can be predicted. Because of the difficulties at many sites in measuring and predicting the required variables, the use of these formulae is often unsatisfactory. Probability charts based on past ice thickness observations offer a useful alternative for predicting ice growth rates and ice thickness, particularly for sites where details of weather, snow and ice conditions are not available.

Analysis Procedure

Two sources of ice thickness records were used in the analysis. These were Ryder's study of ice thickness on lakes and rivers in the Northern Hemisphere[2], and a recent compilation of ice thickness measurements taken by the Meteorological Branch of the Department of Transport[3]. The records compiled by Ryder were somewhat sporadic, usually with only two or three years of continuous observation at a site. Details such as depth and density of snow or quality of ice were often not recorded. Nevertheless, these records were useful as they were from many different geographical locations in the Northern Hemisphere and probably covered a wide range of conditions. Most of the ice thickness observations taken by the Meteorological Branch were obtained in the Arctic and sub-Arctic at weekly intervals.

As the available records were limited and covered a large geographical area, no attempt was made to make a detailed analysis of individual stations. Rather, the total area was broken down into six general regions (Fig. 1), which are listed in Table I with the number of ice observations used in the analysis.

From these observations the average rate of ice growth was calculated for two-week periods from the time ice first started to form until maximum ice thickness was attained. On the average, this ice growth period varies from about six to seven months in the Far North (October-April) to about two to three months in the south (December-February). Growth rates were grouped according to initial ice thickness in the following manner:

(1) Initial ice thickness at the beginning of two-week period less than 10 in. for all regions (for a comparison of ice-growth rates when the ice is thin).

(2) Initial ice thickness at beginning of two-week period from 0 to 20, 20 to 40 and over 40 in. in Regions 3, 4, 5 and 6. (As ice thicknesses are normally less than 20 in. in Regions 1 and 2, the records available were not complete enough to warrant subdividing these areas further.)

The above classification of ice-growth rates by region and ice thickness was selected after several trial classifications. These preliminary studies indicated that the break-down is adequate for the purposes of this paper. A more detailed classification was not justified because of limitations on the accuracy and length of ice thickness records available for analysis.

The growth rates were divided into class intervals covering the observed range of ice growth on lakes, rivers and harbours. Frequency distributions were then made of ice-growth rates for each region and range of initial ice thickness. These distributions were then plotted as cumulative frequency curves on arithmetic probability paper. Several of these frequency curves were then plotted on one diagram to make up what is termed, in this paper, a probability chart.

Probability Charts Obtained From Analysis

Figure 2 shows the frequency curves plotted on probability paper for all the observed ice-growth rates, without regard to snow depth, for the different regions and ranges of initial ice thickness.

Figure 2a gives the distribution curves for the six regions when ice thickness at the beginning of the two-week averaging period was less than 10 in. Region 4, fresh water lakes and rivers from latitude 60 to 70°N, was not included because there were insufficient observations to warrant analysis. The distribution curve for salt water indicates that sea-ice thickens more rapidly than fresh-water ice, probably because the heat of fusion of salt-water ice is less than that for fresh-water ice and the ice-growth rate is larger for the same rate of heat loss. The fresh-water curves indicate a slightly higher rate of growth for more northerly regions, although not as much as might be expected; the difference is not great because the weather conditions that prevail during freeze-up are approximately the same for all regions. A map giving average freeze-up dates and months when average air temperature is 32°F was prepared by Burbidge and Lauder[7]. It shows that the average date of freeze-up almost coincides with the date when the average air temperature is 32°F. In the Far North of Canada freeze-up occurs during September, whereas in the south it is early in December.

Figure 2b gives distribution curves for Regions 3 and 4 for different ranges of ice thickness. There is comparatively little displacement in the frequency curves for different thickness ranges, and it is apparent that as the winter season progresses the decrease in rate of heat loss due to increasing ice thickness and snow depth is largely offset by more severe weather conditions. Hence the range of ice-growth rates is only slightly higher for ice thickness under 20 in. than for ice thickness in the range 20 to 40 in. or 40 in. and greater. The reason the probability curve for ice thicknesses of 40 in. or more is apparently slightly higher than that for the 20- to 40-in. range may be due to insufficient observations.

Figure 2c, showing the distribution curves for salt water in Regions 5 and 6, indicates that the rate of ice growth decreases with increasing ice thickness. This decrease may be due in part to the dependence of the specific heat of sea ice on temperature.

Figure 3a and 3b are frequency

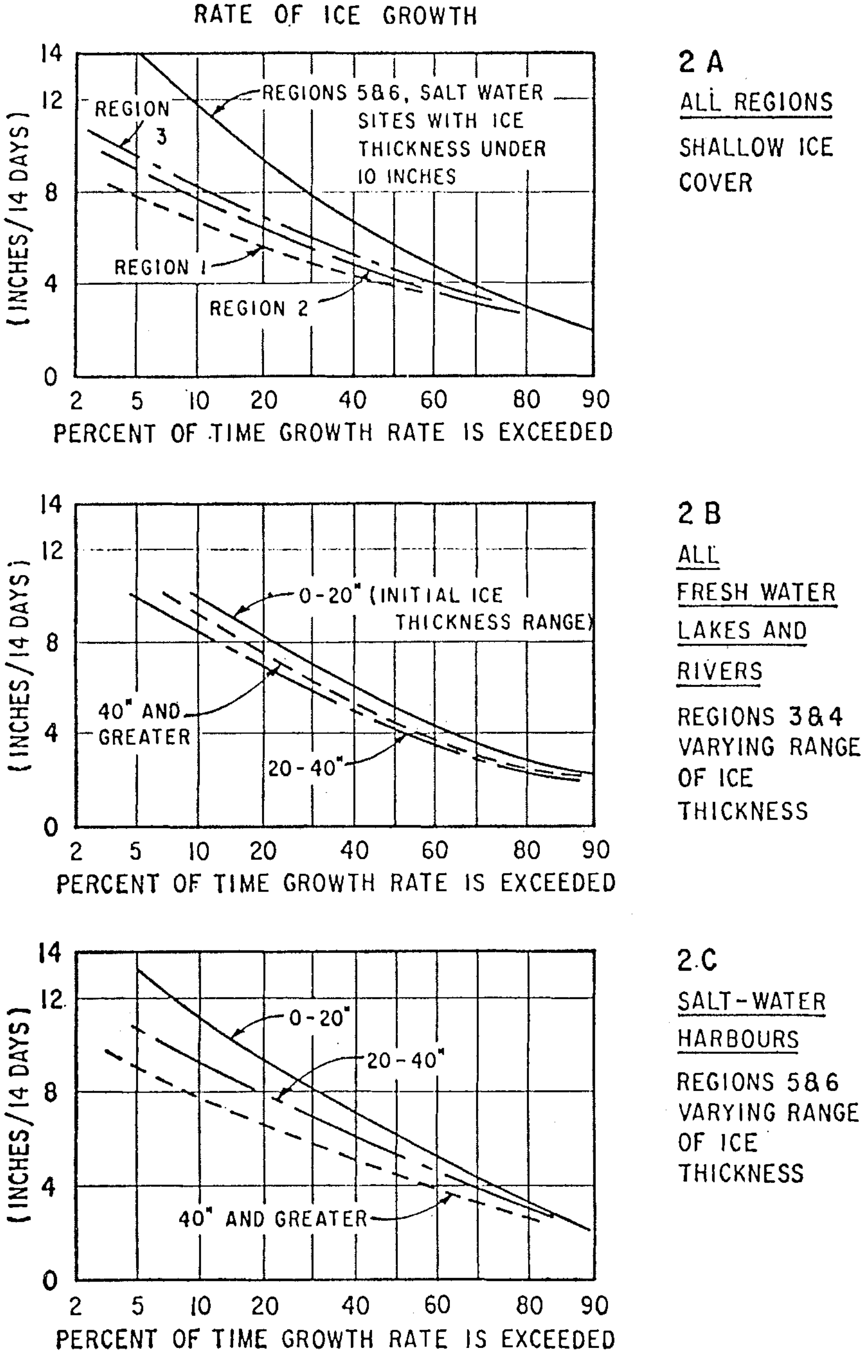

Fig. 2—Ice growth rates plotted as probability curves for different regions in North America

plots showing the effect of snow depth on ice-growth rates. Probability curves have been prepared showing ice-growth rates with snow cover depths less than 2 in. and 6 in. or more for all stations where snow depth was recorded.

Figure 3a shows that the average rate of ice growth with snow cover less than 2 in., for ice thicknesses from 0 to 20 in., is from 2 to 2.5 times as great as that when the depth of snow exceeds 6 in. The limited records available for analysis indicated that the rate of sea-ice growth was higher than that of fresh-water ice for these ranges of ice thickness and snow cover depth. Again, this is probably the result of the lower heat of fusion of sea-ice.

Figure 3b shows that for an ice thickness range of 20 to 40 in. the average rate of ice growth with snow cover less than 2 in. is more than twice the growth rate with a snow cover 6 in. deep or greater.

In this analysis of the effect of snow cover on ice-growth rates at least three cases were noted where the ice-growth rate was extremely high although snow cover was deep. These were not used in preparing Figs. 3a and 3b. The growth rates (in these cases) varied from 8 to 10 in. per week for snow cover over 10 in. deep and initial ice thickness under 20 in. During these periods of rapid growth the reported depth of snow decreased from 10 to 6 in. or less. It would appear that these ice-growth rate anomalies were related to white ice or "snow ice" formation.

If meteorological conditions are highly favorable for ice growth, particularly when the ice cover is thin, large rates of ice growth can probably occur during one or two-day periods. It is unlikely that such extreme conditions would continue for two weeks. The probability curves shown on Figs. 2 and 3, therefore, give the average upper and lower limits of ice-growth rates that can be expected for two-week periods. It would be expected that the deviations on either side of the 50% time line should correlate with the deviation of the actual average weather for the period from the climatological average. Unfortunately, the number of ice thickness records available was not sufficient to check this hypothesis.

Use of Probability Charts to Predict Ice-Growth Rates

Probability charts are based on observations taken from the time ice first starts to form until maximum ice thickness is developed and are valid for this period only. With these charts the probable rate of ice growth and ice thickness can be estimated at a site at any time during this ice growth period.

If, for example, the ice cover on a fresh-water lake is 6 in. thick, and 16 in. is required for a given load, how long will it take for the thickness to increase from 6 to the required 16 in.? From Fig. 2b one can say that 50% of the time the ice cover will grow at a rate of 5 in./14 days. Thus on the average it will take four weeks to increase 10 in. in thickness. If, however, there is deep snow on the ice and the weather is relatively mild the growth rate could be as low as 2 in./14 days (90% of time). Conversely, if the air temperature is below normal during this period and there is no snow cover the growth rate could be as high as 10 in./14 days or greater (10% of time), and the required growth would occur in two weeks.

It is evident that the rate of ice growth can vary widely, depending on snow cover and weather conditions. With the aid of past weather records and predicted weather conditions, an observer can put limits on the expected rate of ice growth and ice thickness obtained from the probability charts and make operational decisions based on this information.

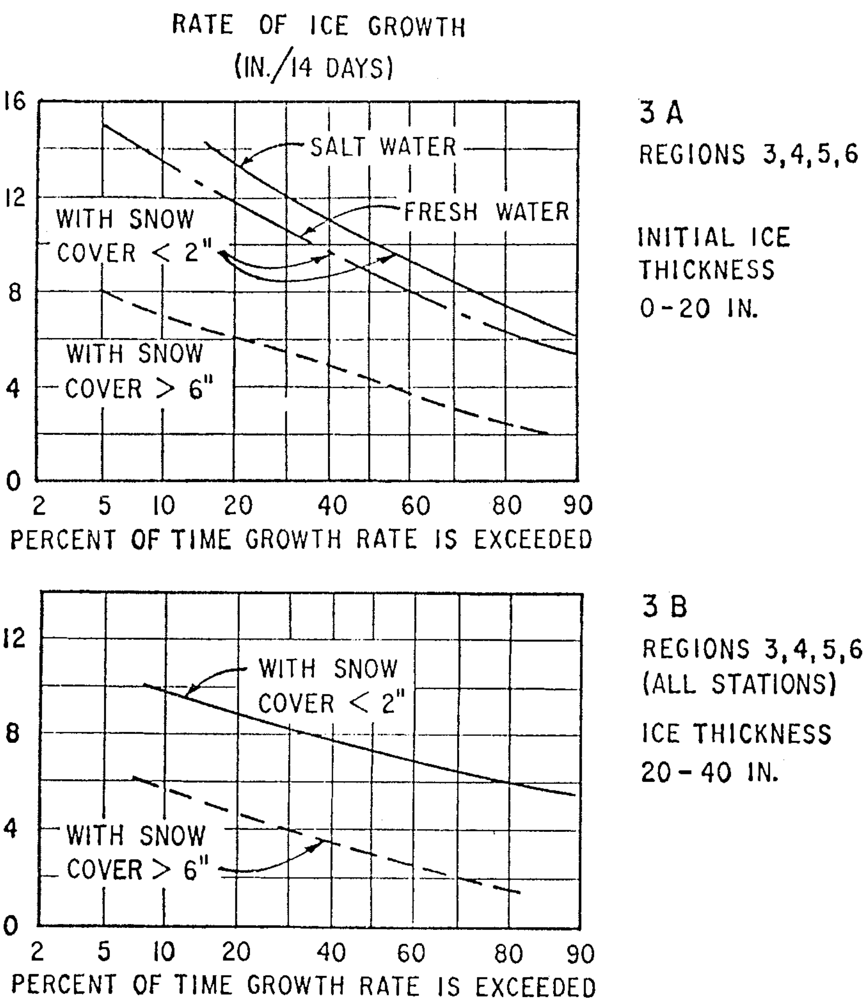

Fig 3—Ice growth probability curves showing effect of snow cover on ice growth rates

The merits of removing the snow cover or of accelerating ice growth by flooding or snow compaction can be more readily assessed with this knowledge of the possible limits of natural ice growth.

It should be kept in mind that the main reason for variation in observed ice-growth rates is the natural variation that occurs in snow depth and weather conditions. This natural variation introduces uncertainty into estimates of ice-growth rates, whether the estimates are made by use of formulae or by probability charts. Prediction of ice-growth rates from probability charts based on actual observations are probably at least as reliable and usually more convenient than predictions based on formulae.

Maximum Ice Thickness

An estimate of maximum ice thickness is often required in such engineering problems as the design of dams for maximum ice pressure or the design of water intakes operating under winter conditions. If ice thickness observations are not available at a site, an estimate of the thickness can be obtained from the information on the rate of ice growth compiled in this study. Using average ice-growth rates it is possible to relate ice thickness to the length of the period of ice growth. The average rates of growth for average weather conditions, snow cover less than 2 in. and snow cover greater than 6 in., have been used to plot the ice-growth lines shown on Fig. 4

Observations on maximum ice thickness for cases where the date of freeze-up was recorded were plotted on Fig. 4 against the number of days from freeze-up to the time of maximum ice thickness. The observed values fall within the limits obtained from the probability curves. It was noted that for areas in the Far North plotted points tend to be near the upper limit of no snow cover. An explanation could be that at these northern sites ice covers are blown clear of snow; a cover greater than 6 in. seldom persists for the entire ice formation period. Similarly, in southern regions plotted points tend to be nearer the lower limit, indicating that significant snow cover occurs more frequently on ice in these areas. It should be noted that there is an upper limit to maximum ice thickness at every site, governed partly by the length of the ice-growth period; this varies from a few weeks in the south to as much as 250 days in the Far North.

As maximum ice thickness at a particular site will vary from season to season, observed maximum ice thicknesses should be used whenever available in preference to other information. In a recent circular the Meteorological Branch compiled information on maximum ice thicknesses on lakes and rivers in Canada[8]. Bilello has prepared preliminary climatological maps of maximum ice thicknesses for Arctic and sub-Arctic regions[9]. For sites where maximum ice thickness observations are not available the limits shown on Fig. 4 can provide useful information that may be satisfactory for many design problems.

Heat Conduction Through Ice Covers

It is possible to estimate the limits of the expected heat loss by conduction through ice covers by converting ice-growth rates given in the probability charts to rate of heat loss. This information might be useful for certain design problems.

Table II gives the probability of occurrence of a given heat loss through freshwater ice, assuming the heat of fusion for ice is 144 Btu/lb.

TABLE I

Location and Number of Observations of Ice Thickness Used in Analysis

	No. of Observations	*Classification*
Region 1 — Mid-western states and Southern Manitoba	107	Fresh-water lakes and rivers
Region 2 — New England States	105	Fresh-water lakes and rivers
Region 3 — Canada east of Rockies and south of latitude 60°N	261	Fresh-water lakes and rivers
Region 4 — Latitude 60°N to 70°N	385	Fresh-water lakes and rivers
Region 5 — Latitude 60°N to 70°N	488	Salt-water harbour
Region 6 — Above latitude 70°N	577	

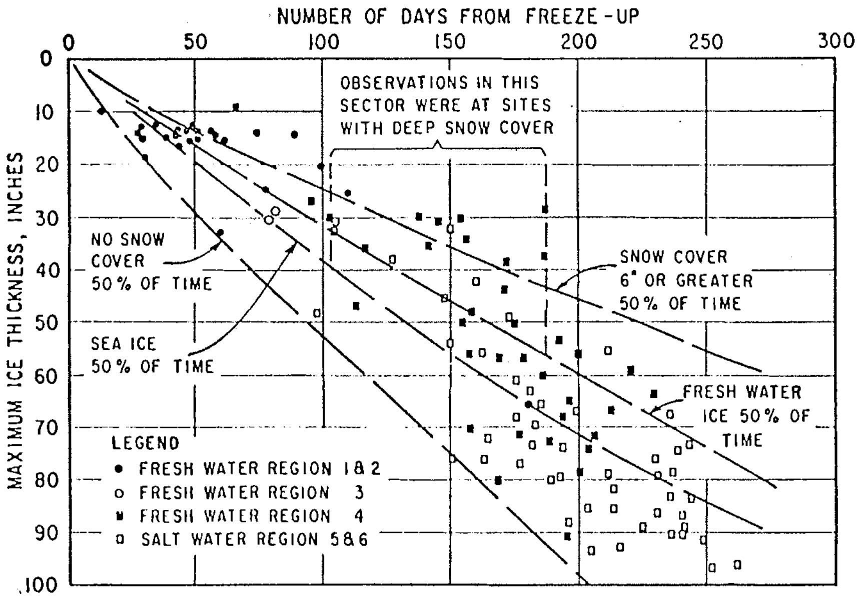

Fig. 4—The relationship between the number of days from freeze-up to maximum ice thickness

The calculation was not carried out for sea-ice; this calculation is not difficult, but it does require a knowledge of the dependence of the heat of fusion of sea water on salinity and of the dependence of the specific heat of sea-ice on temperature and salinity.

For shorter periods heat conducted through the cover can exceed the rates listed in Table II, but for periods of two weeks or more the values given should be quite representative of the average heat loss that will occur.

In the Far North where there is no short-wave solar radiation during the mid-winter months, the heat conducted through the cover equals the heat losses due to long-wave radiation, evaporation and convection. In southern areas the amount of short-wave radiation absorbed by a snow or ice surface should be added to the values in Table II to give the combined heat loss due to evaporation, convection and long-wave radiation. In December at latitude 45°N the average short-wave radiation absorbed by an ice cover would be about 200 to 275 Btu/sq ft/24 hr, or about equal to the 50% value listed in Table II.

CONCLUSIONS

This analysis of ice-thickness observations on a regional basis indicates that probability charts can be used to obtain a reasonable estimate of the probable rate of ice growth and ice thickness at any site in Canda. The average ice growth rates shown on the probability charts can also be used to relate maximum ice thickness to the length of the period of ice growth. The ice growth rates can also be converted into average heat loss from the ice cover to provide information on the rate of heat loss from ice covers in different regions.

Depth of snow cover on ice is shown to have a marked effect on the rate of ice growth. The average rate with a snow cover of 6 in. or more is about one-third to one-half that with a snow cover under 2 in.

Because the rate of ice growth depends on highly variable snow and weather conditions, the only sure way of obtaining satisfactory information on ice conditions at a particular site is to make appropriate measurements of ice thickness for a number of years[10]. As these records improve ice thicknesses can then best be estimated in the same way as air temperature is now estimated from climatological records. When such records are not available at a site, ice probability charts used with discretion can provide a useful method of putting limits on the rate of ice growth and maximum ice thickness to be expected.

Acknowledgements

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TABLE II

Heat Loss Through Fresh-Water Ice Covers (for all regions, all snow cover conditions, and all ranges of initial ice thickness)

Probability of Occurrence	*Ice Growth in./14 days*	*Heat Loss cal/sq cm/24 hr*	*Heat Loss Btu/sq ft/24 hr*
Percentage of time equal to or greater than			
10	8.7	126	465
20	7.0	104	376
30	6.2	90	332
40	5.3	77	281
50	4.8	69	256
60	4.2	61	225
70	3.6	52	192
80	3.0	43	159
90	2.0	29	107

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