Criteria for constructing ice platforms in relation to meteorological variables

Nakawo, M.

Publisher’s version / Version de l'éditeur:

Cold Regions Science and Technology, 6, 3, pp. 231-240, 1983-02

National Research Council Canada — Conseil national de recherches Canada

CRITERIA FOR CONSTRUCTING ICE PLATFORMS IN RELATION TO METEOROLOGICAL VARIABLES

by M. Nakawo

Reprinted from
Cold Regions Science and Technology,
Vol. 6 (1983)
p. 231 - 240

DBR Paper No. 1126
Division of Building Research

Price $1.00 OTTAWA NRCC 22585

Canada

RÉSUMÉ

On a proposé, comme critère de contrôle pour la construction d'une plateforme glaciaire, la mesure de la perte de chaleur constante que l'on peut considérer comme acceptable à la surface de la glace en construction. Sur la base de ce critère et de la relation existant entre les variables météorologiques et le flux de chaleur libéré par la surface dans l'atmosphère, il a été possible d'évaluer une période associée à un cycle d'arrosage et de refroidissement donné. On a établi un graphique qui indique la période en fonction des variables météorologiques.

Cold Regions Science and Technology, 6 (1983) 231–240
Elsevier Scientific Publishing Company, Amsterdam – Printed in The Netherlands

CRITERIA FOR CONSTRUCTING ICE PLATFORMS IN RELATION TO METEOROLOGICAL VARIABLES

Masayoshi Nakawo*
Division of Building Research, National Research Council of Canada, Ottawa, K1A 0R6 (Canada)

(Received August 31, 1981; accepted in revised form September 17, 1982)

ABSTRACT

A criterion for controlling the construction of an ice platform has been proposed in terms of constant heat deficit at a position near the surface of the built-up ice. Based on this criterion for construction and the relation between meteorological variables and the heat flux released from the surface towards the atmosphere, a time period for a single flooding-and-freezing cycle has been estimated. A chart has been constructed for the time period in relation to meteorological variables.

NOTATION

a_c — cloud amount in tenths
$c_i(\theta)$ — "specific heat" of sea ice, $dF/d\theta$ (J/g °C)
c_P — specific heat of air at constant pressure (1.0 J/g °C)
d — thickness of natural sea ice underneath built-up ice (1 m)
E — evaporation heat flux, dQ_E/dt (J/m^2 h)
$F(\theta)$ — heat of freezing of sea ice; total heat released by bringing sea water from θ_m to θ and by freezing it at θ until equilibrium is reached (J/g). 335 J/g is taken for $\theta = -15$°C and salinity = 20%.
Δh — thickness of each flooded layer (0.015 m)
H — sensible heat flux, dQ_H/dt (J/m^2 h)
h_N — thickness of ice platform after a particular layer is built up; height of the current surface (m)
h_P — thickness of ice platform before a particular layer is built up, $h_N - \Delta h$ (m)
K_i — thermal conductivity of sea ice, 7.5×10^3 (J/m h °C)
L_e — latent heat of evaporation, 2.8×10^3 (J/g)
q_a — specific humidity in air (g H_2O/g air)
q_s — specific humidity at the surface; both q_a and q_s are estimated assuming saturation condition; the values of q_a and/or q_s are, for example, 1.60×10^{-3}, 0.64×10^{-3}, 0.23×10^{-3} and 0.008×10^{-3} for -10°C, -20°C, -30°C and -40°C respectively
Q_C — conductive heat loss across bottom surface of a flooded layer during a single cycle (J/m^2)
Q_E — evaporative heat loss towards the atmosphere during a single cycle (J/m^2)
Q_F — total heat released through freezing and cooling of a flooded water layer during a single cycle (J/m^2)
Q_H — sensible heat loss towards the atmosphere during a single cycle (J/m^2)
Q_R — radiative heat loss towards the atmosphere during a single cycle (J/m^2)
Q_T — total heat loss towards the atmosphere across the top surface of a flooded layer during a single cycle, $Q_R + Q_E + Q_H$ (J/m^2)
r — average build-up rate of ice platform, dh_N/dt, 4×10^{-3} m/h
R — radiative heat flux, dQ_R/dt (J/m^2 h)
ΔS — variation in heat deficit in the ice during a single cycle (J/m^2)
t — time from beginning of a cycle or from beginning of construction, whichever applies

*Present address: Department of Applied Physics, Faculty of Engineering, Hokkaido University, Sapporo, Japan 060.

	(h)
T	length of single flooding and freezing cycle (h)
u_a	wind speed (m/s)
x	height from the bottom of the ice sheet (m)
x_c	a height above which temperature is subjected to change during a single cycle; it is considered to be constant during a single cycle but to keep increasing for a whole construction period
y	distance from current surface to level of x_c, $h_N - x_c$ (m)
α	coefficient of surface cooling (3.5×10^{-5} m^{-1})
β	coefficient of heat and water vapour transport, 4.5×10^{-3} (dimensionless)
γ	$(\theta_s - \theta_m)/(\theta_c - \theta_m)$ when construction is started; $\theta_s = -30°C$, i.e., $\gamma = 2.13$ for example calculation in Appendix 1
θ	ice temperature (°C)
θ_a	air temperature (°C)
θ_c	criterion temperature (−15°C)
θ_m	liquidus temperature (−1.7°C)
$\theta_N(x)$	temperature profile at the end of a particular cycle (°C)
$\theta_P(x)$	temperature profile immediately before a particular cycle (°C)
θ_s	surface temperature (°C)
κ	thermal diffusivity of sea ice (4×10^{-3} m^2/h)
ρ_a	density of air (0.0013 Mg/m^3)
ρ_i	density of sea ice (0.91 Mg/m^3)

INTRODUCTION

In the land-fast ice of the High Arctic, floating ice platforms are constructed to support offshore drilling operations. The method has been used successfully by Panarctic Oils Ltd. in sheltered locations of the Canadian Arctic where the natural sea ice is relatively stable (Baudais et al., 1974; Masterson et al., 1979).

The platform is built by successive flooding and freezing of layers of sea water on an existing natural sea-ice cover. This procedure is called free flooding (Dykins and Funai, 1962; Dykins, 1963): sea water is transferred to the surface of natural/built-up sea ice and allowed to disperse in all directions from the point of discharge. No constraint is provided to the flow of water.

Because of the construction technique it is difficult to control the thickness of individual layers. The build-up rate depends on the time required for each flooding-and-freezing cycle for a given layer thickness established with a certain rate of discharge. For a rapid build-up rate the period of the cycle has to be shortened.

To produce built-up ice with certain strength characteristics, however, the period of a cycle for a given layer thickness has to be long enough to achieve the required degree of solidification of the flooded layer. If flooding is too frequent and the layer thickness the same, the resultant ice will be of poor quality. Optimizing the construction rate, therefore, can be achieved by minimizing the duration of the flooding-and-freezing cycle while still meeting the time requirement of the strength criterion. As the rate of solidification is a function of meteorological conditions, the optimum length of time will consequently be a function of these parameters.

Observations on the heat budget during construction of an ice platform have provided a relation between rate of heat loss for the various components of the total heat loss from a flooded layer and meteorological variables (Nakawo, 1980). This relation permits an estimate to be made of the criteria for the time period to complete a flooding-and-freezing cycle for given meteorological components when the strength criterion is provided.

One of the problems is that the strength criterion has not been established quantitatively. In actual construction the duration of a cycle depends on the "feeling" the operator has concerning the degree of solidification of the resultant ice. As a consequence, the layers of an ice platform may not be of uniform strength. This could take place, for example, when an operator is changed. In addition, assessment of strength tends towards the weak side over the long term. It is necessary, therefore, to quantify the strength criterion (the degree of solidification).

An attempt has been made to correlate the strength criterion of "assessment" with thermal processes and to develop a thermal criterion for the quality of the built-up ice. Based on this proposal a time period for each flooding-and-freezing cycle is discussed in relation to meteorological variables.

The symbols used are defined in Notation.

THERMAL CONDITIONS FOR FLOODING AND FREEZING CYCLE

Temperature profiles during a single flooding-and-freezing cycle are shown schematically in Fig. 1 (based on the author's observations). The profile at the end of the previous freezing period, immediately prior to the flooding under consideration, is given by a solid line designated "previous profile". Temperature increases with depth, showing an inverse S shape: the temperature gradient decreases at first, then increases, with a minimum temperature gradient at the middle of the ice cover.

When a water layer is applied at the surface the temperature profile changes drastically (shown by a dotted line in Fig. 1). The water freezes after flooding has ceased and the released heat is removed by radiative, sensible, and latent heat fluxes across the top surface towards the air and by conductive heat flux across the bottom surface of the layer towards the ice beneath (Adams et al., 1960). A temperature profile during this solidification period is shown by the broken line in Fig. 1. The profile at the end of the freezing period is shown by a solid line designated "new profile".

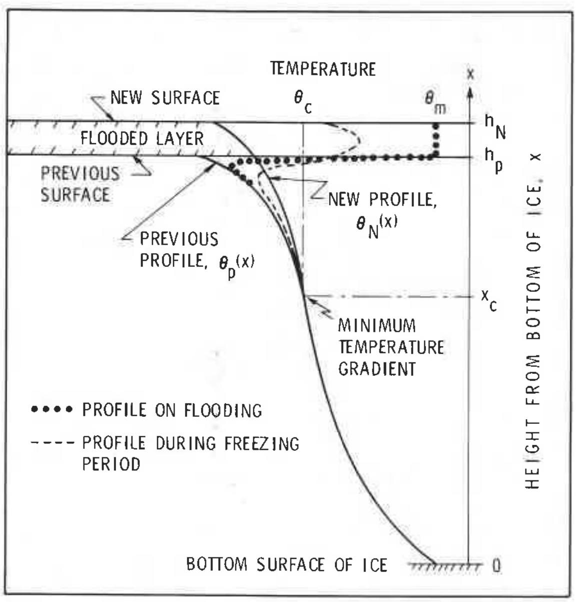

Fig. 1. Temperature profiles during flooding and subsequent freezing period.

Consider the heat exchange of the flooded layer during the whole cycle (flooding plus freezing). Consider Q_R, Q_H and Q_E (the heat losses towards the atmosphere across the top surface by radiation, convection and evaporation, respectively) and Q_C the conductive heat loss across the bottom surface of the flooded layer towards the ice underneath. All the terms are considered positive when the heat is taken away from the layer. They are balanced with the heat Q_F released by the freezing and cooling of the flooded water. The heat budget equation, therefore, is given by

$$Q_F = Q_R + Q_H + Q_E + Q_C \tag{1}$$

As it is impossible to separate the latent heat and specific heat effect for sea ice, the concept of heat of freezing, F, is introduced. This is the total heat released by bringing the sea water from the liquidus temperature θ_m to a certain temperature θ and freezing it at θ until equilibrium is reached. F is therefore temperature dependent. (This F is slightly different from the heat of melting defined by Anderson (1960) and the heat of fusion defined by Ono (1967), but essentially the same for practical use.) Considering that a temperature profile θ_N is established at the end of a cycle in the flooded layer under consideration, Q_F is given by

$$Q_F = \int_{hp}^{h_N} \rho_i F(\theta_N) \mathrm{d}x \tag{2}$$

where h_N and hp are the height of the top and bottom surfaces of the layer.

As shown in Fig. 1, the temperature does not change significantly with time during a cycle at a level lower than the height x_c, which corresponds approximately to the minimum temperature gradient during the cycle. It is only above this level that the temperature changes with time owing to flooding with sea water. This level was maintained about 0.5 m below the surface (Nakawo, 1980); x_c is considered to be constant during a single cycle, although it increased for a whole construction period as the surface built up.

Consider next the heat budget of the ice portion, $x_c < x < hp$, which is subjected to temperature change during one flooding-and-freezing cycle. The change in the stored heat in this portion of ice is the difference between the conductive heat across the plane of $x = h_P$ and that across the plane of $x = x_c$. The former is Q_C and the latter is expressed by $K_i \left[\frac{\partial\theta}{\partial x}\right]_{x_c} \cdot T$, in which T is the duration of the flooding-and-freezing cycle since $\left[\frac{\partial\theta}{\partial x}\right]_{x_c}$ does not change appreciably during a cycle (Nakawo, 1980). The heat budget equation, therefore, is given by

$$\int_{x_c}^{h_P}\int_{\theta_P(x)}^{\theta_N(x)} \rho_i c_i(\theta)\,d\theta dx = Q_C - K_i\left[\frac{\partial\theta}{\partial x}\right]_{x_c} \cdot T \quad (3)$$

By rearranging eqns. (1), (2) and (3), assuming a constant density, and considering $c_i(\theta) = \frac{dF(\theta)}{d\theta}$, the following equation is obtained:

$$Q_R + Q_H + Q_E = F(\theta_c)\rho_i\Delta h - K_i\left[\frac{\partial\theta}{\partial x}\right]_{x_c} \cdot T + \Delta S \quad (4)$$

where

$$\Delta S = \rho_i\left[\int_{x_c}^{h_N}\int_{\theta_N(x)}^{\theta_c} c_i(\theta)d\theta dx - \int_{x_c}^{h_P}\int_{\theta_P(x)}^{\theta_c} c_i(\theta)d\theta dx\right] \quad (5)$$

In a physical sense, the second term on the right-hand side of eqn. (5) is the heat deficit in the ice layer $x_c < x < h_P$ immediately prior to flooding, assuming that the heat stored in the layer is taken to be zero when $\theta = \theta_c$ throughout the layer above x_c. The first term is the heat deficit in the layer $x_c < x < h_N$ at the end of the freezing period. The term ΔS means, therefore, a net change in the heat deficit during a single flooding-and-freezing cycle. The value of ΔS depends on the length of the freezing period; ΔS is positive when the freezing period is sufficiently long, and negative when it is short.

For practical purposes short-wave radiation can be neglected in assessing Q_R since the ice platform is constructed during winter in the high Arctic. Q_R is therefore due only to long-wave radiation. It is, however, dependent on cloud cover, surface temperature, and air stratifications (temperature, vapour pressure, etc.), all very complex to examine analytically for a flooding-and-freezing cycle. The author has shown (Nakawo, 1980) that dQ_R/dt is less sensitive to various meteorological conditions when there is no cloud, and that it can be approximated by a decay-type function corresponding to a cooling trend of the surface temperature with time. When Nakawo's equation is combined with an expression for the effect of cloud cover (Ambach and Hoinkes, 1963),

$$\frac{dQ_R}{dt} = R = [0.15 \times 10^6 + 0.21 \times 10^6 \exp(-0.614t)] \times (1 - 0.060\, a_c^{1.2}) \quad (6)$$

Nakawo (1980) has also proposed the following semi-empirical equations for Q_H and Q_E:

$$dQ_E/dt = E = -\beta L_e \rho_a u_a (q_a - q_s) \times 3600 \quad (7)$$

$$dQ_H/dt = H = -\beta \rho_a c_P u_a (\theta_a - \theta_s) \times 3600 \quad (8)$$

where θ_s is assumed to be given by

$$\ln\frac{\theta_a - \theta_s}{\theta_a - \theta_m} = -\alpha u_a \times 3600\, t \quad (9)$$

since H accounted for more than 50% of the total heat loss towards the atmosphere. The value of 3.5×10^{-5} m^{-1} was obtained for α from one set of data. The β value ranged from 4×10^{-3} to 5×10^{-3}, and 4.5×10^{-3} will be used for the following calculations.

Equations (7) and (8) indicate that $E = H = 0$ when $u_a = 0$. As the buoyancy term is significant (Nakawo, 1980), however, neither E nor H would be zero even if $u_a = 0$. It would be more appropriate to express E and H as

$$E = -(A + Bu_a)(q_a - q_s) \quad (10)$$

$$H = -(A' + B'u_a)(\theta_a - \theta_s) \quad (11)$$

where A, B, A' and B' are constants, rather than use eqns. (7) and (8) in which A and A' are taken to be zero. Equations (7) and (8) were obtained for wind speed >1 m/s. Thus, if eqns. (7) and (8) are used for a low wind speed <1 m/s, each flux could be underestimated. Nevertheless, the equations would be useful, since a calm day is very rare during winter in the Arctic.

CRITERION FOR DEGREE OF SOLIDIFICATION

Dykins (1963) mentioned that sound ice can be produced only if each flooded layer is allowed to solidify completely before the next lift is applied. Sea ice, however, does not solidify completely in the strict sense until its temperature is low enough for all the salts involved to be in the solid phase, and this temperature is lower than −50°C under normal atmospheric pressure. Sea ice is generally at temperatures higher than −50°C, even in the high Arctic. In the normal temperature range its strength increases with a decrease in brine volume as the temperature is lowered. Temperature, therefore, could be introduced as a parameter for the criterion.

There is a problem, however, in using ice temperature as a criterion because it varies with depth into the ice. An alternative could be the heat deficit in the ice, estimated by integrating the difference between the actual temperature profile at the end of a cycle and a given standard temperature profile. (A proposed criterion for obtaining the same degree of solidification for every layer is to allocate a certain value of heat deficit with respect to the standard temperature profile for each flooding-and-freezing cycle.)

Suppose this criterion were to be applied in construction, taking as the standard profile the constant temperature, θ_c. Take as a criterion the condition that the heat deficit should have the same value at the end of every cycle, i.e., ΔS in eqn. (5) is equal to zero. Equation (4) becomes

$$Q_T = F(\theta_c)\rho_i \Delta h - K_i \left[\frac{\partial\theta}{\partial x}\right]_{x_c} \cdot T \qquad (12)$$

where

$$Q_T = Q_R + Q_E + Q_H$$

and T is the period of the flooding-and-freezing cycle.

As shown in Appendix 1, the second term on the right-hand side of eqn. (12) $K_i[\partial\theta/\partial x]_{x_c} \cdot T$ can be neglected during all but the early construction period. Equation (12) is thus reduced to

$$Q_T = F(\theta_c)\rho_i \Delta h \qquad (13)$$

for a series of flooding cycles.

To obtain a consistent degree of solidification for all layers, therefore, eqn. (13) should be satisfied for a specific value of θ_c. This temperature will be called the "criterion temperature".

The value of θ_c has to be compatible with the strength criterion. As mentioned in the Introduction, however, the criterion has not been well defined. The operator's estimate of the degree of solidification of the ice near the surface at the end of each cycle is the only criterion available at present. Degree of solidification is considered to be a function only of temperature near the current surface since salinity does not vary from layer to layer in built-up ice (Nakawo and Frederking, 1981). When the duration of each cycle is adjusted to produce a similar degree of solidification (based on the operator's best estimate), the temperature near the surface would also be the same for all cycles. The operators, therefore, are essentially trying to keep ΔS close to zero, i.e., θ_c constant.

The author observed that θ_c fluctuates around −15°C, with a deviation of about ± 3°C during construction of an ice platform. The strength criterion now being used, therefore, can be expressed as θ_c = −15°C in terms of criterion temperature. This value for θ_c will be used for the calculations shown in the following sections.

OPTIMUM CONDITION

Equations (6) to (9) provide for prediction of the heat fluxes towards the atmosphere, with time, if the three meteorological variables θ_a, u_a and a_c are available. An example of this estimate is shown in Fig. 2(a), with values −30°C, 2.0 m/s and 5, in tenths, for θ_a, u_a and a_c, respectively. Each flux, and hence the total heat flux (thick solid line), decreases appreciably with time as the surface temperature is lowered.

The total heat loss Q_T in a single cycle may be obtained by integrating the total heat flux over time as follows:

$$Q_T = \int_0^T (R + E + H)\,\mathrm{d}t \qquad (14)$$

in which T is the period of the flooding-and-freezing cycle. The dependence of Q_T on T is shown in Fig. 2(b), calculated with the same values of meteorolog-

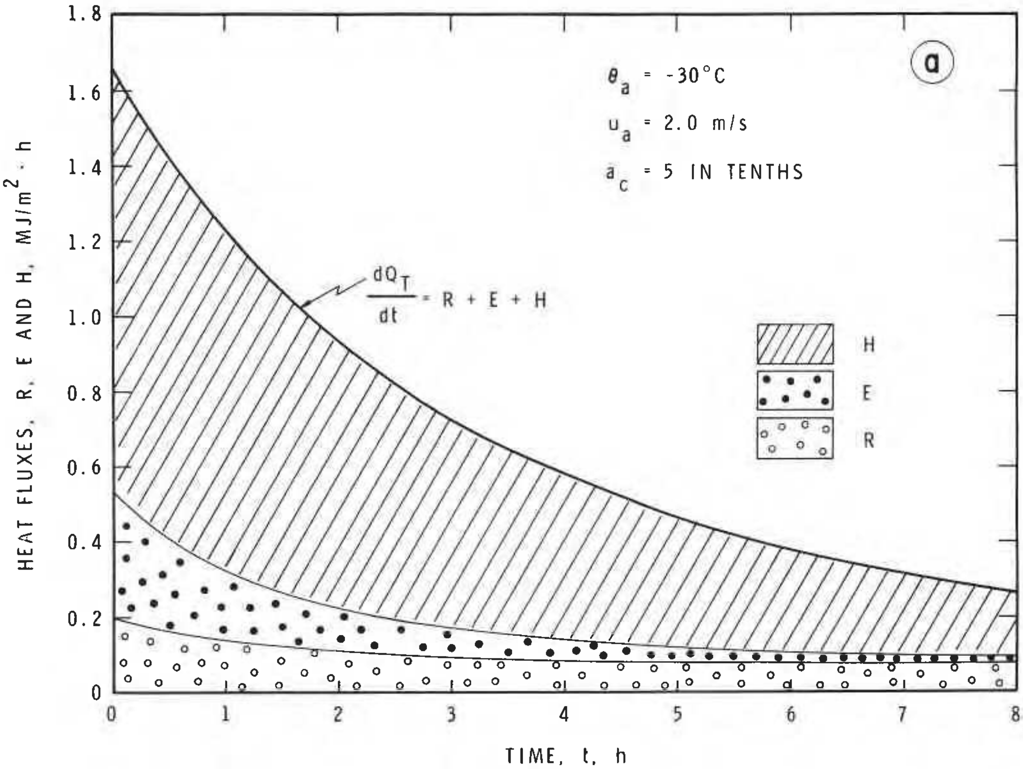

Fig. 2(a). Heat fluxes towards air across top surface of flooded layer versus time.

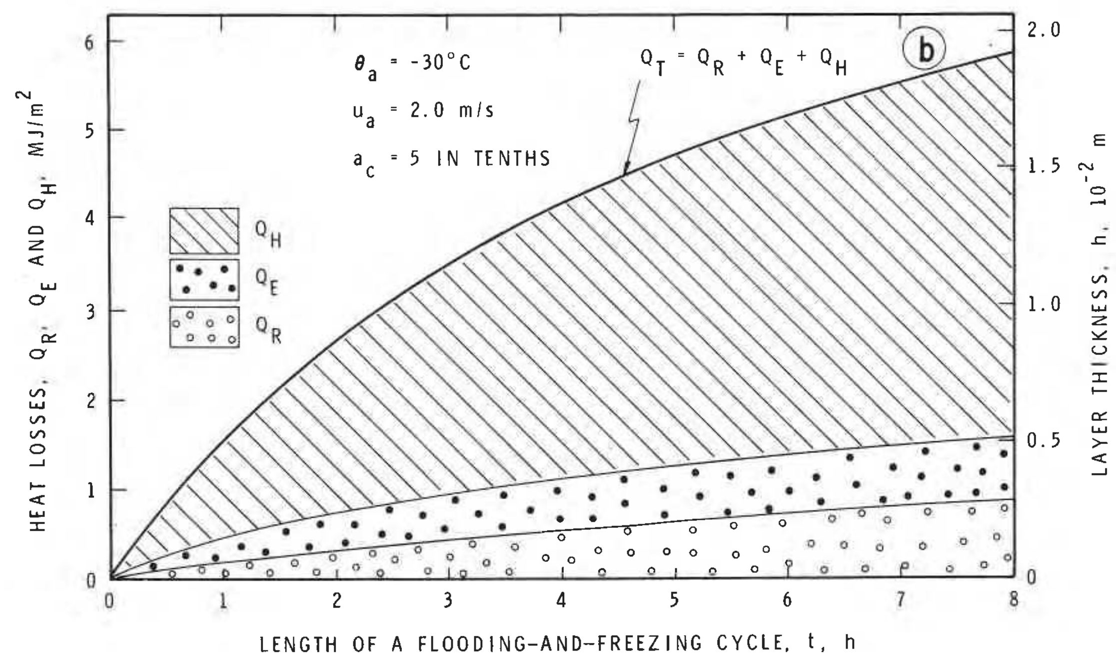

Fig. 2(b). Total heat loss and various heat losses across top surface of flooded layer against time of flooding-and-freezing cycle. The right-hand ordinate corresponds to the layer thickness for a given total heat loss.

ical variables that are used for Fig. 2(a). Each component of heat loss, Q_R, Q_E and Q_H, is also shown in Fig. 2(b).

As mentioned in the previous section, θ_c is assumed to be −15°C. Taking the values of $F(-15°C)$, ρ_i and Δh as 335 J/g, 0.91 Mg/m^3 and 0.015 m, respectively, eqn. (13) (which is the criterion for the construction) requires Q_T to be 4.57 MJ/m^2. This Q_T value, as indicated in Fig. 2(b), corresponds to T = 4.7 h. The cycle is therefore to end at t = 4.7 h and the next flooding to be applied. The optimum condition is thus obtained in terms of the duration of a cycle for the particular meteorological conditions.

The optimum duration of a cycle is presented in Fig. 3 for various meteorological conditions; some of the features are summarized below:

(1) The optimum duration, T, increases rapidly with increase in air temperature, decrease in wind speed, and increase in cloud.

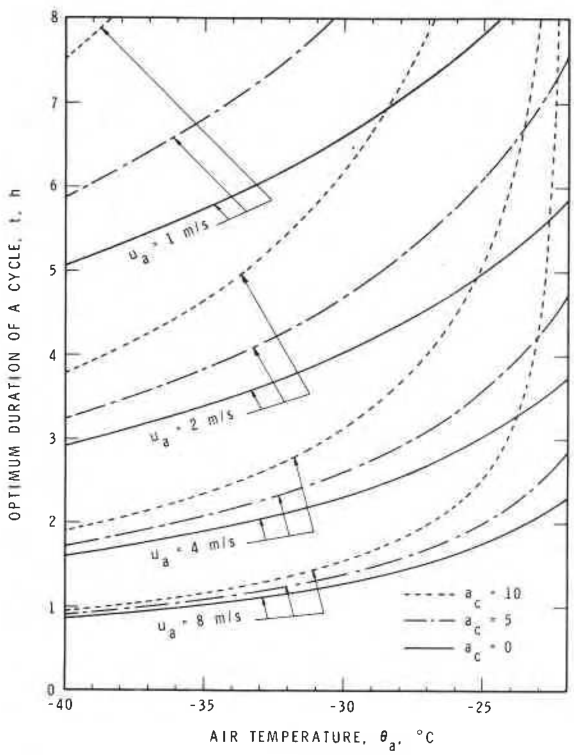

Fig. 3. Compiled chart of optimum length of time of flooding-and-freezing cycle for various meteorological conditions. Q_T is taken to be 4.57 MJ/m² h, which corresponds to Δh = 0.015 m.

(2) For a given cloud cover, the effect of the air temperature increases with decrease in wind speed.

(3) When the air temperature is low, the wind speed is more important than cloud, the effect of which is almost negligible for high wind speeds.

(4) As temperature increases, the effect of cloud cover becomes increasingly significant.

(5) The effect of cloud cover for a given temperature increases rapidly as wind speed decreases.

DISCUSSION

Effect of layer thickness

An estimate of the optimum condition was made in the previous sections for a constant layer thickness, Δh = 0.015 m, an average value encountered in the field. Thickness, however, fluctuates, usually from flooding to flooding but also from location to location within a flooded area. The Δh value is, so long as the current procedure for flooding is followed (Baudais et al., 1974), within a range of about 0.01 to 0.02 m during most of the construction period. The layer thickness Δh corresponding to Q_T is plotted in Fig. 2(b) as a reference axis, indicating the relation between Δh and T. Estimation of optimum length of a cycle can also be obtained for a different layer thickness by means of the same procedure.

The rate of heat loss or rate of increase in Q_T is larger in the earlier part of a cycle, as shown in Fig. 2(a and b). If a thinner water layer is applied, the growth rate of the ice platform is faster under the same meteorological conditions and the same criterion. If Δh = 0.005 m, for example, Q_T being 1.52 MJ/m^2, T is about 1 h (see Fig. 2(b)). It would take only about 3 h, therefore, to build up 0.015 m of ice, whereas it would take 4.7 h if a layer of 0.015 m were applied in one cycle. This comparison may be exaggerated because eqns. (6–9) were obtained with layers of 0.01 to 0.02 m (Nakawo, 1980); it may have to be modified for layers as thin as 0.005 m. Taking this limitation into account it is still considered effective to reduce the layer thickness to establish a faster growth rate for a platform under given meteorological conditions and the proposed criterion.

OPTIMIZATION OF CRITERION TEMPERATURE

The criterion temperature was taken to be −15°C in the calculations shown in the previous section since the temperature at $x = x_c$ was observed to be about this value during an actual construction. Each flooding-and-freezing cycle was adjusted through the operator's estimate of the degree of solidification of the built-up ice (strength criterion). That no accidents have occurred seems to justify this procedure. In other words, −15°C may be an adequate value for θ_c.

The criterion, however, may be too conservative with regard to the strength of the ice, and a higher value could be taken for θ_c. If −10°C is taken for θ_c, for example, the same thickness of built-up ice can be achieved more rapidly, or thicker ice can be made in the same period.

What should be the appropriate criterion temperature? Only by optimizing it in relation to both the total thickness of the platform and the mechanical properties of the resultant ice can this question be answered. The total thickness that can be obtained in the period available for constructing the platform can be predicted as a function of θ_c by the use of eqn. (13) and the data on general ambient conditions. Future investigations are needed to establish the relation between ice quality and the criterion temperature for construction.

ACKNOWLEDGEMENTS

The author is grateful to R. Frederking and L.E. Goodrich for their kind help in the preparation of this paper. It is a contribution from the Division of Building Research, National Research Council of Canada, and is published with the approval of the Director of the Division.

APPENDIX 1

Upward heat conduction from the bottom of an ice sheet

The criterion used to determine the duration of the flooding cycle is that the heat deficit of the layer $x_c < x < h_N$ at the end of a flooding cycle is a given constant value. Equation (13) requires Q_T to be equal to the total amount of heat released in freezing a flooded layer and bringing its temperature from θ_m to θ_c. When ice is built up according to eqn. (13), therefore, it is considered that layers with a temperature of θ_c are piled up successively just below the height corresponding to the current value of x_c. The heat conduction equation was used to examine how the temperature profile, and hence the temperature gradient, vary with time for $x < x_c$.

It was assumed, for simplicity, that the thermal diffusivity of natural and built-up sea ice is the same, being independent of temperature. The equation of heat conduction is then given by

$$\frac{\partial^2\theta}{\partial x^2} - \frac{1}{\kappa}\frac{\partial\theta}{\partial t} = 0\,. \tag{A1}$$

Neglecting the natural growth of sea ice at the bottom, the boundary condition is

$$\theta = \theta_m \text{ at } x = 0\,. \tag{A2}$$

The initial condition would be expressed by the following equations, assuming a constant temperature θ_c in the hypothetical built-up ice and a linear temperature profile in natural ice before construction is started:

$$\theta = \theta_c \qquad \text{where } x > d \tag{A3}$$

$$\theta = \frac{\gamma(\theta_c - \theta_m)}{d}\,x + \theta_m \quad \text{where } 0 < x < d \tag{A4}$$

$$\text{with } \gamma = \frac{\theta_s - \theta_m}{\theta_c - \theta_m} \tag{A5}$$

These initial conditions are shown with solid lines in Fig. A1, taking the values indicated in the Notation.

The solution of eqns. (A1) to (A4) is given by

$$\theta = \frac{(\theta_c - \theta_m)}{2d}\left[(\gamma x + d)\,\mathrm{erf}\left(\frac{x+d}{2\sqrt{\kappa t}}\right) - (\gamma x - d)\,\mathrm{erf}\left(\frac{x-d}{2\sqrt{\kappa t}}\right) + \left(\frac{2\gamma\sqrt{\kappa t}}{\sqrt{\pi}}\right)\left\{\exp-\left(\frac{x+d}{2\sqrt{\kappa t}}\right)^2 - \exp-\left(\frac{x-d}{2\sqrt{\kappa t}}\right)^2\right\}\right] + \theta_m \tag{A6}$$

which is valid for $x < x_c$.

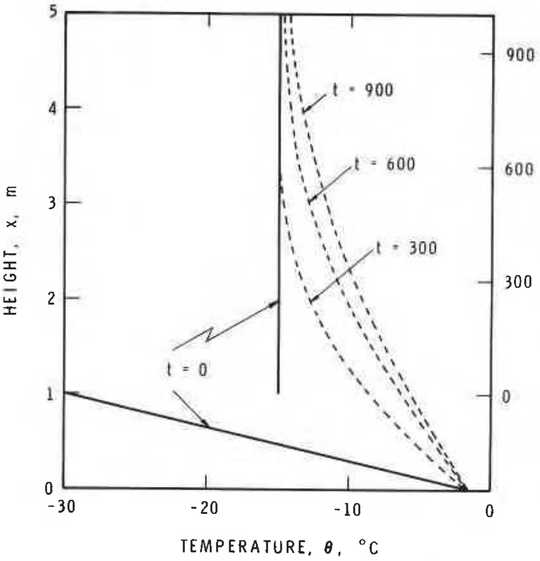

Fig. A1. Temperature profile at different times after construction is started. The initial thickness (natural sea ice) is taken to be 1 m. The right-hand ordinate corresponds to the time when the built-up ice reaches a given height from the bottom of the ice sheet.

Temperature profiles are shown with dotted lines in Fig. A1 for various times after construction is started. The temperature gradient is obtained by differentiating eqn. (A6).

$$\frac{\partial\theta}{\partial x} = \frac{(\theta_c - \theta_m)}{2d}\left[\gamma \operatorname{erf}\left(\frac{x+d}{2\sqrt{\kappa t}}\right) - \gamma \operatorname{erf}\left(\frac{x-d}{2\sqrt{\kappa t}}\right)\right.$$
$$\left. + \frac{(1-\gamma)d}{\sqrt{\pi\kappa t}}\left\{\exp -\left(\frac{x+d}{2\sqrt{\kappa t}}\right)^2 + \exp -\left(\frac{x-d}{2\sqrt{t}}\right)^2\right\}\right] \tag{A7}$$

Assuming x_c increases with time at the same rate as the build-up rate of ice,

$$x_c = d - y + \dot{r}t \tag{A8}$$

where

$$y = h_N - x_c \tag{A9}$$

Variation of the temperature gradient at $x = x_c$ is obtained with the values given in the Notation by eqns. (A7) and (A8) for various values of y (Fig. A2).

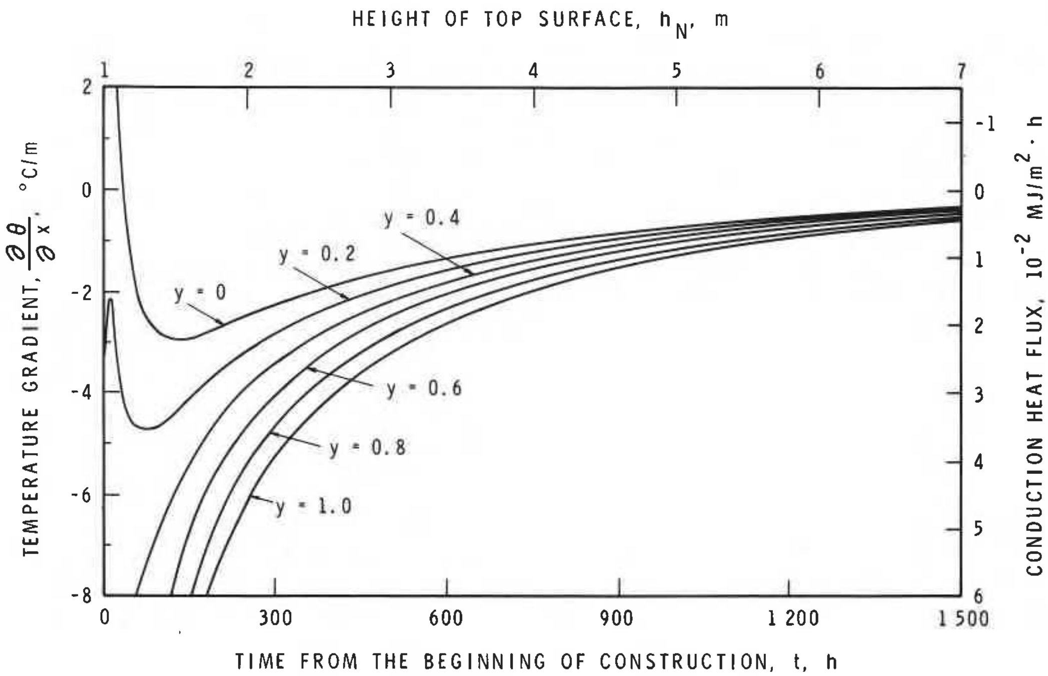

Fig. A2. Variation of temperature gradient at different depths from the top surface. The upper abscissa corresponds to the height of the top surface of the built-up ice established at a given time. The right-hand ordinate shows the corresponding heat flux for a given temperature gradient.

The value of y was observed to be about 0.5 m (Nakawo, 1980). The lines for $y = 0$ and 0.2 in Fig. A2 are therefore meaningless, since eqns. (A6) and (A7) are applicable only for $x < x_c$. For y in the range of 0.4 to 0.6 m, the temperature gradient increases asymptotically to zero; that is, the heat conducted upward at $x = x_c$ decreases with time, approaching zero asymptotically (the scale is shown in the right-hand ordinate).

It decreased from 0.21 MJ/m^2 h, which is the value corresponding to the temperature gradient in the natural ice at $t = 0$, to about 0.03 MJ/m^2 h in the first 300 h. During this time about 1 m of ice was built up. The value of 0.03 MJ/m^2 h at $t = 300$ h is small compared with the rate of heat loss at the surface during a flooding cycle (radiative heat flux is, for example, about 0.1 MJ/m^2 h (Fig. 2(a)). As the rate of heat flow by conduction continues to decrease with time, it can be neglected in estimating the optimum flooding conditions, except for the earlier stage of construction.

During this period conductive heat may have to be taken into account. Equation (12) should then be used for the calculation rather than eqn. (13). Selective deep flooding, however, is usually carried out in this early period in order to smooth the rough surface of natural ice (Baudais et al., 1974). The method of free floodings, as discussed in this paper, is applied afterwards. Early floodings, therefore, are outside the scope of this paper.

REFERENCES

Adams, Jr., C.M., French, D.N. and Kingery, W.D. (1960), Solidification of sea ice, J. Glaciol., 3(28): 745–761.

Ambach, W. and Hoinkes, H. (1963), The heat balance of alpine snow-field (Kesselwandferner, 3240 m, Oetztal Alps, August 11–September 8, 1958), Preliminary communication; Int. Ass. of Scientific Hydrology, Commission of Snow and Ice, General Assembly of Berkeley, IAHS Publication No. 61, pp. 24–26.

Anderson, D.L. (1960), The physical constants of sea ice, Research, 13(3): 310–318.

Baudais, D.J., Masterson, D.M. and Watts, J.S. (1974), A system for offshore drilling in the Arctic Islands, J. Canadian Petroleum Technology, 13(3): 15–26.

Dykins, J.E. (1963), Construction of sea ice platforms, in: Kingery, W.D. (Ed.), Ice and Snow, pp. 289–301.

Dykins, J.E. and Funai, A.I. (1962), Point Barrow trials – FY 1959; Investigations on thickened sea ice, U.S. Naval Civil Engineering Lab., Technical Report R 185.

Masterson, D.M., Anderson, K.G. and Strandberg, A.G. (1979), Strain measurements in floating ice platforms and their application to platform design, Canadian J. of Civil Engineering, 6(3): 394–405.

Nakawo, M. (1980), Heat exchange at a surface of a built-up ice platform during construction, Cold Regions Science and Technology, 3(4): 323–333.

Nakawo, M. and Frederking, R. (1981), The salinity of artificial built-up ice made by successive floodings of sea water, Proc. IAHR Int. Symp. on Ice Problems, Quebec City, 27–31 July, 1981, Vol. II, pp. 516–524.

Ono, N. (1967), Specific heat and heat of fusion of sea ice, in: Oura, H. (Ed.), Physics of Snow and Ice, pp. 599–610.

Publications of the Division may be obtained by mailing the appropriate remittance (a Bank, Express, or Post Office Money Order, or a cheque, made payable to the Receiver General of Canada, credit NRC) to the National Research Council of Canada, Ottawa. K1A 0R6. Stamps are not acceptable.

A list of all publications of the Division is available and may be obtained from the Publications Section, Division of Building Research, National Research Council of Canada, Ottawa. KIA 0R6.