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Evaluation of Moment Redistribution in a Two-Span Continuous Prestressed Concrete Beam


by Venkatesh Kumar R. Kodur and T. I. Campbell

An approach is proposed for determining the percentage of redistribution of moment occurring at failure of a continuous prestressed concrete beam. The approach, which is based on two parameters—the percentage of redistribution x *and the moment ratio* MR*—was developed using results from nonlinear finite element analyses of a large number of continuous prestressed concrete beams. In the proposed approach, the available redistribution of moment is based on the overall structural behavior rather than on the cross-sectional behavior.*

Keywords: continuous beam; nonlinear analysis; prestressed concrete; redistribution of moment; secondary moment.

The bending moments in a continuous beam can be predicted using a linear-elastic analysis, provided the load level is such that the elastic limit is not exceeded in any of the constituent materials. When the elastic limit is exceeded, at any particular load level, the bending moments in the beam will likely differ from those predicted by a linear-elastic analysis. The difference for a particular load level between the actual moment at a section and that determined by a linear-elastic analysis is referred to as redistribution of moment.

The failure load of a continuous prestressed concrete beam depends on the extent of redistribution of moment that occurs prior to failure. The extent of redistribution of moment can be full, partial, or nil, depending on a number of factors.[1,2] To determine the actual amount of redistribution of moment occurring, a nonlinear analysis has to be carried out.

Design codes for concrete structures usually recommend the use of a linear-elastic analysis and either ignore the nonlinear effect or recognize it by applying a somewhat arbitrary adjustment to the design elastic moments. Even to date, there is debate on the extent of redistribution permitted by different codes of practice.[1-3]

The extent of redistribution of moment in a continuous prestressed concrete beam depends on a number of factors. Parametric studies conducted by Kodur[2] have demonstrated that the stiffness of the span and the presence of secondary moments influence the extent of redistribution of moment, and it has been recommended that overall structural ductility be considered in determining the amount of redistribution. The majority of current codes of practice[4-6] base the allowable amount of redistribution of moment on cross-sectional ductility at a critical section, and do not take secondary moments into account in determining the permitted amount of redistribution of moment. Although some codes of practice[7] recommend the use of a detailed nonlinear analysis for determining the extent of redistribution of moment, the applicability of such an approach is limited in many design situations due to the complexity and the effort involved. Some studies[8,9] have called for development of simple approaches for determining redistribution of moment, and an attempt is made to develop such an approach in the current investigation.

A theoretical expression to determine the extent of redistribution of moment is derived and a reasonably simple method for predicting the failure load of a continuous prestressed concrete beam is proposed in this paper. The applicability of the method of design is demonstrated through a numerical example.

INVESTIGATION

The majority of previous studies on continuous prestressed concrete beams considered only some of the parameters that have been shown to affect redistribution of moment. Only a few investigators, such as Santamaria,[10] Moucessian and Campbell,[8] and Scholz,[11] concentrated on developing a simple approach to determine the redistribution of moment for application in a design situation. The studies of Santamaria and Moucessian and Campbell are important since they were

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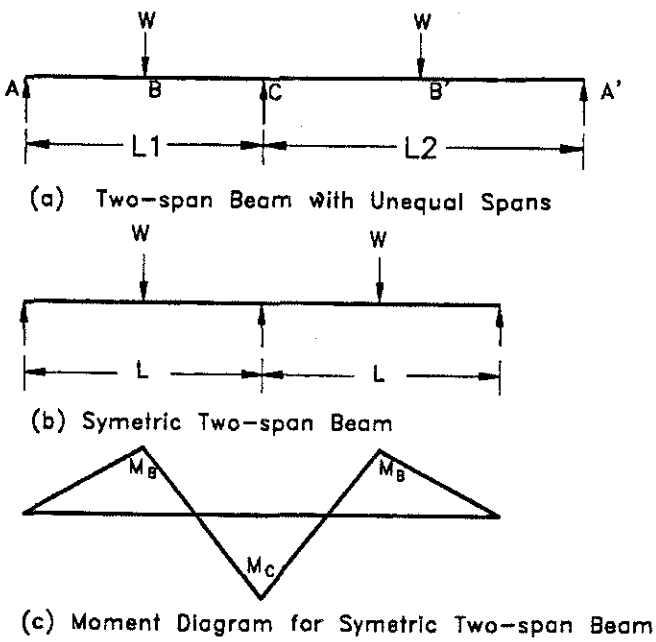

Fig. 1—Two-span beam with concentrated loading

based on the concept of structural ductility, unlike Scholz's study, which was based on cross-sectional ductility. Santamaria did not reach any conclusion since no definite trend was observed in his proposed *PAR-c/d* relationships, where *PAR* (plastic adaptation ratio) was defined in terms of load factors, and *c/d* as the ratio of neutral axis depth- to-effective depth of a section at ultimate.

Moucessian and Campbell proposed an approach based on *PAR*1-*MR* relationships, where *PAR*1 was defined in terms of three failure loads; as the ratio of the extent of redistribution that occurred to the maximum redistribution possible in a beam, and *MR* was arbitrarily defined as the ratio of the difference of the ultimate moment and secondary moment at the support section to that at the critical section in the span. However, the applicability of this approach is limited since the data on which it is based were obtained from the nonlinear analysis of beams in which only a limited number of parameters were varied. Factors such as cross-sectional shape, concrete strength, and span-depth ratio were not considered. The present study attempts to overcome some of the limitations of the *PAR*1-*MR* approach, and to develop a rational approach for determining the extent of redistribution of moment in a continuous prestressed concrete beam.

THEORETICAL EXPRESSION FOR REDISTRIBUTION OF MOMENT

By equating the total available rotation at the central support region with the inelastic rotation required to achieve a percentage of redistribution x in a symmetric two-span prestressed concrete beam (Fig. 1), the following expression can be derived (see Appendix A*)

$$\left(\frac{x}{100-x}\right) = \frac{3}{2}EI\left(\frac{1}{EI_c} - \frac{1}{EI_{cy}}\right)\left(\frac{d}{L} + 0.1m_1\right) + \frac{M_{sec}}{M_c} = y_{ine} + y_{sec} \tag{1}$$

where EI is a measure of the flexural stiffness of the span, EI_C is the flexural stiffness at failure of the support critical section, EI_{Cy} is the flexural stiffness of the support section at first yield of the reinforcement, d is the distance from the extreme compression fiber to the center of the tension force at the support section, L is the span length, M_{sec} is the secondary moment at the support section, M_C is the moment capacity of the support section, and m_1 is a factor to account for the type of loading (see Appendix A*). The flexural stiffness, at any load level, is defined as the ratio of the moment to the curvature. The terms y_{ine} and y_{sec} in Eq. (1) represent the extent of redistribution resulting from inelastic action and secondary moments, respectively. Eq. (1) was derived in a manner similar to that used by Mattock.[12]

The parameter $PAR1$ may be defined as

$$PAR1 = \frac{W_{col} - W_{le}}{W_{pl} - W_{le}} \tag{2}$$

where W_{col}, W_{le}, and W_{pl} are the failure loads based on nonlinear, elastic, and plastic analyses, respectively. By introducing the relevant expressions for W_{col}, W_{le}, and W_{pl} in terms of the moment capacities and secondary moments at the span and support critical sections (see Appendix A*), the percentage of redistribution of moment x can be related to the parameter $PAR1$ as follows

$$PAR1 = \frac{\frac{x}{100-x}}{MR} \tag{3}$$

where

$$MR = \left(\frac{M_c + M_{sec}}{M_c}\right)\left(\frac{T_f(M_B + aM_c)}{a(1-a)s_1(M_c + M_{sec})} - 1\right) \tag{4}$$

and M_B and M_C are the ultimate moment capacities of the span and support critical sections, respectively, a is the ratio of the distance of the span critical section from the end support to the span length, when the span and center support section ultimate strengths are developed simultaneously, s_1 is a factor used in defining the bending moment at the central

*The Appendixes are available in xerographic or similar form from ACI headquarters, where they will be kept permanently on file, at a charge equal to the cost of reproduction plus handling at time of request.

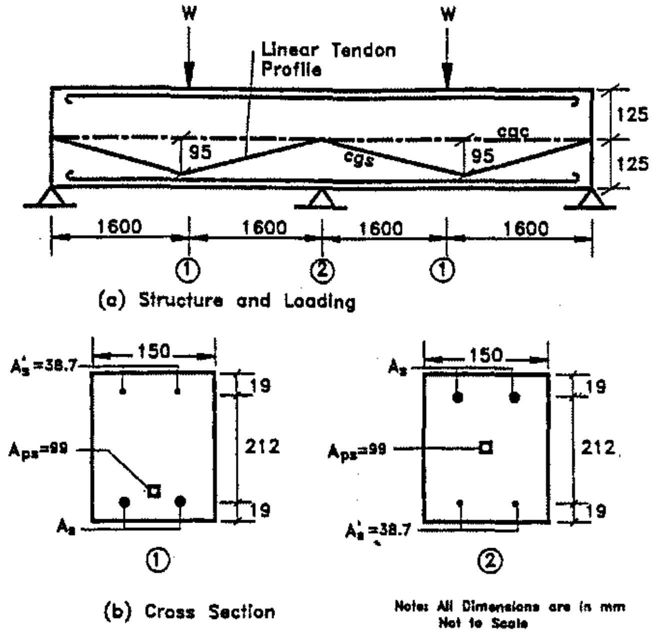

Fig. 2—Layout of Group A beams (1 in. = 25.4 mm)

support (see Appendix A*), and $T_f = 1.0$ for a concentrated load and 2.0 for a uniformly distributed load in each span.

Substituting for $x/(100 - x)$ from Eq. (1) in Eq. (3) gives

$$PAR1 = \frac{\frac{3}{2}EI\left(\frac{1}{EI_c} - \frac{1}{EI_{cy}}\right)\left(\frac{d}{L} + 0.1m_1\right) + \frac{M_{sec}}{M_c}}{MR} \qquad (5)$$

It can be seen from Eq. (4) that MR accounts not only for the moment capacities and secondary moments at the critical sections, but also for the location of the span critical section and the type of loading.

From Eq. (1) or Eq. (5), it can be inferred that:

a. The extent of redistribution of moment increases with stiffness of the span, plastic hinge length (reflected by the term $0.1\,m_1$), and secondary moment M_{sec}.

b. The greater the value of EI_C corresponding to a higher c/d ratio at the support section, the lower the amount of redistribution will be.

c. An increase in the span-depth ratio L/d results in decreased redistribution of moment.

d. The extent of redistribution of moment is influenced by the type of loading as reflected by the factors a and s_1 in the expression for MR [Eq. (4)].

e. Concrete strength and partial prestressing index, whose effects are accounted for indirectly in the computation of MR through the values of M_B and M_C, also influence the extent of redistribution.[2]

In Eq. (1), as a conservative estimate, the stiffness of Span EI can be assumed to be equal to the stiffness of the critical span section at yield of the reinforcement, and d can be assumed to be equal to the total depth of the member. Since the derivation of Eq. (1) is based on a number of simplifying assumptions, its applicability may be limited in practice. Nevertheless, this relationship is useful in identifying the various parameters that influence the extent of redistribution of moment in a two-span prestressed concrete beam.

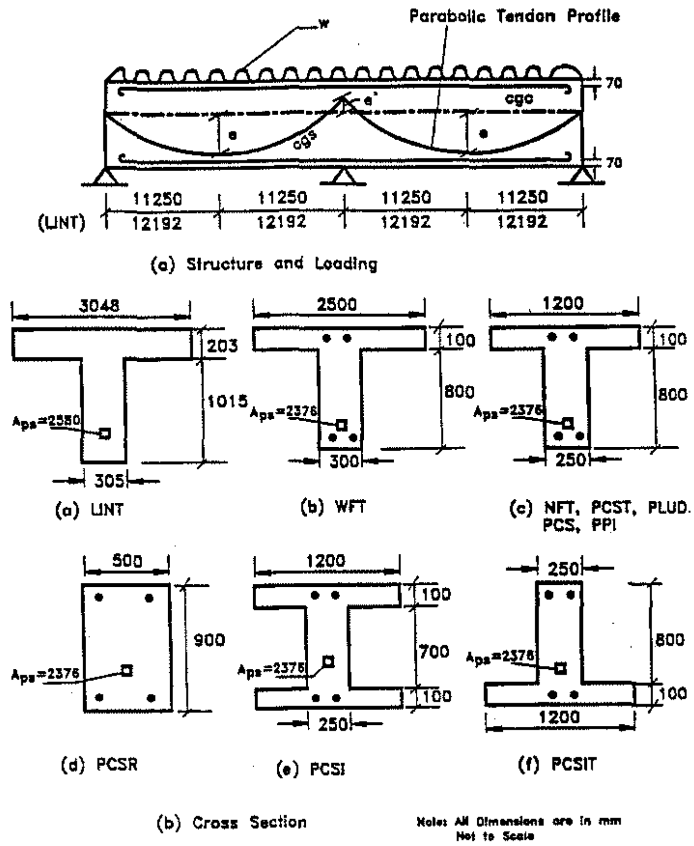

Fig. 3—Layout of Group B beams (1 in. = 25.4 mm)

REDISTRIBUTION OF MOMENT USING NONLINEAR ANALYSIS

To establish the validity of the expressions derived in the previous section, and to examine the influences of the different parameters, a detailed parametric study was carried out. Sixty-six beams, which were divided into two groups (Group A and Group B), were analyzed using a computer program,[2,13,14] and the failure load was evaluated in each case. The parameters varied, and included EI and c/d at the support and span critical sections, type of loading, cross-sectional shape, span-depth ratio, concrete strength, secondary moment, and partial prestressing index.

Twenty-six of the beams studied by Moucessian[15] were selected as Group A beams. All the beams were symmetric over two spans of 3.2 m (10.5 ft) each, had a rectangular cross section 150-mm-(5.9-in.)-wide and 250-mm-(9.8-in.)-deep, and were subjected to a concentrated load at the center of each span (Fig. 2).

The 40 Group B beams, which were selected from the two-span beams used in the parametric studies of Kodur[2], were loaded symmetrically with uniformly distributed load and had wide-ranging characteristics, such as cross-sectional shape (rectangular, T, I, and inverted T), secondary moment, concrete strength, and c/d values (Fig. 3). The location of the critical section in the span for a beam loaded with uniformly

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Table 1—Details and results of analyses for Group A beams

Beam	Span A_s, mm²	Support A_s, mm²	Span (c/d)	Support (c/d)	M_B, kN.m	M_C, kN.m	M_{sec}, kN.m	MR	W_{le}, kN	W_{col}, kN	W_{pl}, kN	$PAR1$	x, percent
SP50-A1	1632	1778	0.607	0.650	112.2	101.5	7.8	0.13	182.22	191.86	203.69	0.531	5.39
SP50-A5	1260	1778	0.509	0.650	99.1	101.4	8.5	0.03	183.10	184.70	187.29	0.768	0.94
SP50-A9	1073	1778	0.458	0.650	91.6	101.4	8.8	-0.03	183.68	177.06	177.83	0.741	-4.08
SP50-A13	894	1778	0.410	0.650	83.8	101.4	9.2	-0.10	184.27	168.15	168.09	0.988	-10.50
SP50-A18	715	1778	0.351	0.650	75.3	101.4	9.5	-0.16	184.90	157.78	157.44	0.978	-19.10
SP50-A14	894	1378	0.410	0.561	83.8	86.8	8.7	0.00	159.12	158.15	158.92	0.858	-0.67
SP50-A2	1632	1171	0.607	0.520	112.2	78.4	7.2	0.36	142.57	171.60	189.13	0.604	18.19
SP50-A6	1260	1171	0.509	0.520	99.2	78.4	6.9	0.24	142.25	164.30	173.03	0.706	14.43
SP50-A10	1073	1171	0.458	0.520	91.6	78.4	8.1	0.15	144.15	159.20	163.43	0.799	10.33
SP50-A15	894	1171	0.410	0.520	83.8	78.4	8.5	0.07	144.73	152.20	153.69	0.876	5.41
SP50-A19	715	1171	0.351	0.520	75.3	78.4	8.8	-0.02	145.35	142.70	143.05	0.868	-2.07
SP50-A3	1632	557	0.607	0.390	112.1	50.3	6.3	0.92	94.35	150.20	171.57	0.697	39.99
SP50-A7	1260	557	0.509	0.390	99.1	50.3	6.9	0.72	95.33	142.40	155.34	0.767	35.96
SP50-A11	1073	557	0.458	0.390	91.6	50.3	7.2	0.60	95.88	137.60	145.88	0.821	33.23
SP50-A16	894	557	0.410	0.390	83.8	50.3	7.6	0.47	96.45	131.40	136.14	0.874	29.43
SP50-A20	715	557	0.351	0.390	75.3	50.3	7.9	0.34	97.05	123.40	125.49	0.921	23.92
SP50-A4	1632	184	0.607	0.310	112.1	29.9	5.7	2.00	59.30	139.10	158.84	0.787	61.55
SP50-A8	1260	184	0.509	0.310	99.1	29.9	6.3	1.65	60.27	131.30	142.60	0.852	58.76
SP50-A12	1073	184	0.458	0.310	91.6	29.9	6.6	1.45	60.80	126.20	133.14	0.897	56.75
SP50-A17	894	184	0.410	0.310	83.8	29.9	6.9	1.25	61.35	119.90	123.40	0.939	54.01
SP50-A21	715	184	0.351	0.310	75.3	29.9	7.3	1.02	61.95	111.00	112.76	0.963	49.60
SP50-A22	1400	36	0.548	0.285	104.4	21.1	5.8	2.82	44.75	132.62	143.68	0.882	71.46
SP50-C1	1171	1778	0.517	0.652	78.7	101.7	0.5	-0.05	170.32	162.38	162.00	1.000	-4.91
SP50-C2	1171	1171	0.517	0.517	78.7	78.7	0.9	0.11	132.70	145.74	147.64	0.897	9.04
SP50-C3	1171	557	0.517	0.391	78.7	50.7	1.3	0.51	86.73	124.11	130.12	0.863	30.67
SP50-C4	1171	184	0.517	0.307	78.7	30.3	1.7	1.27	53.33	112.59	117.37	0.927	53.98

Note: 1 in. = 25.4 mm; 1 kip.ft = 1.356 kN.m; 1 kip = 4.448 kN.

distributed load was assumed to be at approximately 0.4 L from the end support.

Prestressing in all beams was by means of bonded post-tensioning, and varying amounts of nonprestressed reinforcement were provided. Details on the amounts of nonprestressed reinforcement are given in Tables 1 and 2 for Group A and Group B beams, respectively, where the designations (as used by Moucessian[15] and Kodur[2]) for identifying the beams are adopted. Additional information is given by Moucessian and Campbell[8] and Kodur[2] for Group A and Group B beams, respectively. The material properties were kept constant for Group A beams, while some of the properties were varied for Group B beams. The properties of the concrete, nonprestressed, and prestressed reinforcement, as well as the amounts of prestressed reinforcement, are given in Table 3. The maximum compressive strain in the concrete at failure was assumed to be 0.004 in all cases.

Results from the computer analysis, namely the failure loads, secondary moment, (c/d) ratios, and the moment capacities of the critical sections, are given in Tables 1 and 2 for Group A and Group B beams, respectively. The three loads given are the failure loads based on plastic analysis (W_{pl} or w_{pl}), a computer analysis (W_{col} or w_{col}), and an elastic analysis (W_{le} or w_{le}). The moments M_B and M_C correspond to the moment capacities of the span and the support critical sections, respectively, while M_{sec}, which is the secondary moment at the central support section, is based on an elastic analysis.

The parameters $PAR1$, MR, and x were evaluated for each beam. The values of $PAR1$ and MR were computed using the relationships in Eq. (2) and (4), respectively, while the value of x was computed according to the following relationship [see Eq. (A.14) in Appendix A*]

$$\left(\frac{W_{col}L}{s_1} - M_{sec}\right)\left(1 - \frac{x}{100}\right) = M_c \qquad (6)$$

The term W_{col} in Eq. (6) represents the total load on a span at failure as predicted by the computer analysis. It should be noted in Tables 1 and 2 that the value of MR, and consequently x, is less than zero for some beams. These beams had a high c/d value at the support section and a low c/d value at the critical span section. As a result, the more critical section is the span section, as opposed to the central support section, and consequently, redistribution of moment is from the span to the support section in these beams.

The $PAR1$ values close to unity in Table 1 indicate that almost complete redistribution of moment occurred in the majority of the Group A beams, while partial redistribution occurred in the Group B beams, as indicated by the lower $PAR1$ values in Table 2.

EVALUATION OF THEORETICAL EXPRESSION FOR PERCENTAGE OF REDISTRIBUTION

To evaluate the validity of Eq. (1), the value of x (designated x_{Eq}) for each beam was computed from this equation and compared with the actual value of percentage of redistribution at failure obtained from the computer analysis. The beam properties required in the computation of x_{Eq} were readily available from the results of the nonlinear analysis. The onset of yielding was assumed to occur at a section when either the nonprestressed reinforcement reached a strain level of 0.002 or the prestressed reinforcement reached a strain level of 0.01.

Fig. 4 shows a comparison of the x_{Eq} values and the percentage of redistribution as obtained from the computer analysis for each of the Group A beams. It can be seen that x_{Eq} compares well with x from the computer analysis at higher

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Table 2—Details and results of analyses for Group B beams

Beam	Span			Support			Span (c/d)	Support (c/d)	M_B, kN.m	M_C, kN.m	M_{sec}, kN.m	MR	w_{le}, kN/m	w_{col}, kN/m	w_{pl}, kN/m	$PAR1$	x, percent
	e, mm	A_s, mm^2	A_s', mm^2	e', mm	A_s, mm^2	A_s', mm^2											
LINT1	794	0	0	180	0	0	0.049	0.455	4765	3646	1897	0.26	74.58	76.79	87.12	0.176	4.31
LINT2	786	0	0	84	0	0	0.049	0.494	4727	3226	2025	0.32	70.65	73.17	84.21	0.186	5.48
LINT3	777	0	0	-19	0	0	0.049	0.541	4687	2765	2165	0.40	66.33	69.76	81.04	0.233	8.44
LINT4	773	0	0	-78	0	0	0.049	0.571	4665	2506	2244	0.46	63.91	67.75	79.27	0.250	10.22
WFT1	450	400	400	95	400	6000	0.082	0.288	3149	2714	1175	0.19	61.46	63.59	69.59	0.262	4.74
WFT2	450	400	400	95	400	4000	0.082	0.401	3149	2612	1165	0.23	59.69	61.93	68.90	0.244	5.16
WFT3	450	400	400	95	400	3000	0.082	0.454	3149	2569	1164	0.24	58.99	60.92	68.21	0.209	4.54
WFT4	450	400	400	95	400	2000	0.082	0.506	3149	2457	1157	0.28	57.11	59.25	67.86	0.199	5.22
NFT1	389	3500	400	155	400	5900	0.333	0.345	3779	2710	756	0.59	54.77	66.53	79.84	0.469	21.54
NFT2	389	3500	400	155	400	4700	0.333	0.426	3779	2635	756	0.62	53.59	64.74	79.34	0.433	21.13
NFT3	389	3500	400	155	400	3500	0.333	0.500	3779	2515	756	0.68	51.69	62.29	78.53	0.395	21.06
PCST1	388	1500	400	155	400	10,000	0.197	0.229	3291	2786	845	0.34	57.38	65.01	72.39	0.508	14.77
PCST2	388	1500	400	155	400	6500	0.197	0.306	3291	2739	841	0.36	56.57	64.19	72.08	0.491	14.96
PCST3	388	1500	400	155	400	4000	0.197	0.468	3291	2567	837	0.43	53.79	60.73	70.92	0.405	14.61
PCST4	388	1500	400	155	400	1600	0.197	0.602	3291	2270	832	0.56	49.02	55.96	68.83	0.350	16.21
PCSR1	338	400	4300	312	400	4300	0.213	0.220	3160	3048	474	0.34	55.65	65.06	72.02	0.575	16.34
PCSR2	338	400	4300	312	400	1500	0.213	0.309	3160	2910	468	0.39	53.38	62.34	71.09	0.506	16.30
PCSR3	338	400	3800	312	4000	400	0.229	0.450	3139	3627	538	0.17	65.82	67.96	75.55	0.220	3.60
PCSR4	338	400	3800	312	9200	400	0.229	0.590	3138	4522	625	0.00	78.75	75.24	81.56	-27.730	-9.53
PCSI1	338	2000	400	312	2000	400	0.211	0.216	3719	3607	468	0.36	64.40	76.09	84.88	0.571	17.02
PCSI2	338	2000	400	312	3000	400	0.211	0.286	3719	3847	485	0.30	68.45	76.66	86.50	0.455	11.89
PCSI3	338	2000	400	312	5300	400	0.211	0.437	3719	4343	520	0.19	76.85	79.77	89.80	0.226	4.09
PCSI4	338	2000	400	312	8000	400	0.211	0.579	3718	4768	555	0.11	84.12	82.75	92.67	-0.160	-1.85
PCSIT1	216	400	10,000	407	1500	400	0.212	0.193	3054	3377	18	0.35	53.64	65.50	72.49	0.629	18.18
PCSIT2	216	400	10,000	407	3000	400	0.212	0.297	3054	3746	73	0.25	60.34	67.52	74.97	0.491	10.81
PCSIT3	216	400	10,000	407	5300	400	0.212	0.446	3054	4243	148	0.13	69.38	70.87	78.30	0.167	2.17
PCSIT4	216	400	10,000	407	8000	400	0.212	0.589	3054	4673	223	0.05	77.38	73.98	81.18	-0.890	-4.82
PLUD1	389	2000	400	0	400	11,500	0.232	0.271	3257	2107	748	0.67	45.12	54.37	66.21	0.439	21.74
PLUD2	389	2000	400	0	400	5900	0.235	0.432	3257	2033	718	0.73	43.48	52.43	66.63	0.387	21.80
PLUD3	389	2000	400	0	400	2600	0.235	0.647	3257	1747	730	0.94	39.14	46.32	64.56	0.283	20.65
PLUD4	389	2000	400	0	2000	400	0.235	0.796	3257	1659	757	1.01	38.17	44.24	63.92	0.236	18.80
PCS5	388	1500	400	155	400	10,000	0.137	0.218	3374	2801	851	0.37	57.71	66.25	73.85	0.529	16.17
PCS6	388	1500	400	155	400	6500	0.137	0.267	3374	2771	846	0.38	57.16	65.68	73.65	0.517	16.28
PCS7	388	1500	400	155	400	4000	0.137	0.380	3374	2669	842	0.42	55.49	63.56	72.96	0.462	16.07
PCS8	388	1500	400	155	400	1600	0.137	0.502	3374	2463	838	0.50	52.16	60.01	71.58	0.404	16.78
PCS9	388	4500	400	155	400	10,000	0.195	0.218	4219	2802	743	0.72	56.02	71.04	87.64	0.475	25.32
PCS10	388	4500	400	155	400	6500	0.195	0.267	4219	2771	744	0.73	55.54	70.98	84.73	0.529	26.07
PCS11	388	4500	400	155	400	4000	0.195	0.380	4219	2669	744	0.78	53.93	67.97	86.75	0.428	24.97
PCS12	388	4500	400	155	400	1600	0.195	0.502	4219	2463	744	0.90	50.68	63.47	85.37	0.369	24.73
PPI1	388	3500	400	155	400	4700	0.333	0.426	3779	2635	760	0.62	53.65	64.74	79.34	0.432	21.03

Note: 1 in. = 25.4 mm; 1 kip/ft = 14.63 kN/m; 1 kip.ft = 1.356 kN.m.

Table 3—Material properties of Group A and Group B beams

Material	Group A beams	Group B beams*
Concrete	$f_c' = 40$ $E_i = 40{,}000$ $f_t' = 3.79$ $\varepsilon_{cu} = 0.004$	$f_c' = 40$ $E_i = 40{,}000$ $f_t' = 3.79$ $\varepsilon_{cu} = 0.004$
Nonprestressed reinforcement	$f_y = 310$ $\varepsilon_y = 0.002$ $E_y = 200{,}000$ $\varepsilon_{su} = 0.1542$	$f_y = 400$ $\varepsilon_y = 0.002$ $E_y = 200{,}000$ $\varepsilon_{su} = 0.1542$
Prestressed reinforcement	$A_{px} = 99$ $f_{se} = 1209$ $f_{pu} = 1860$ $\varepsilon_{pu} = 0.05$	$A_{px} = 2376$ $f_{se} = 1209$ $f_{pu} = 1860$ $\varepsilon_{pu} = 0.05$

*$f_c' = 34.5$; $f_t' = 2.93$; $E_i = 29{,}923$; $f_{se} = 1037$; and $A_{px} = 2580$ for Beams LINT1, LINT2, LINT3, LINT4. $f_c' = 50$; $f_t' = 4.24$; and $E_i = 50{,}000$ for Beams PCS5, PCS6, PCS7, PCS8, PCS9, PCS10, PCS11, and PCS12.

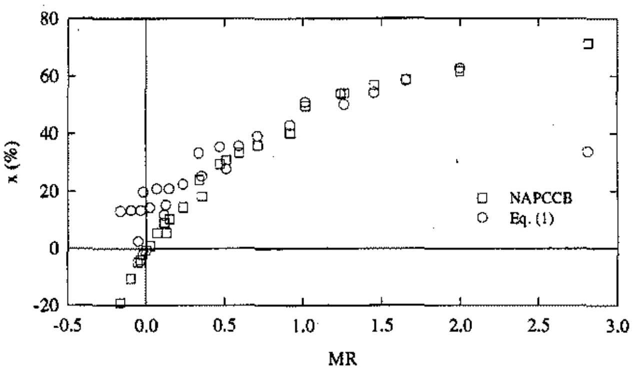

Fig. 4—Comparison of x *from Eq. (1) with* x *from computer analysis for Group A beams*

values of MR, but that for the lower values of MR, the predictions from Eq. (1) overestimate the percentage of redistribution of moment. For Group B beams, as shown in Fig. 5, Eq. (1) overestimates the percentage of redistribution for all the beams. It can also be seen from Fig. 4 and 5 that, for Group A beams having negative MR values, and for Group B beams having MR values less than about 0.13, the x values are negative, indicating that redistribution is taking place from the span critical section to the central support section in these beams.

To examine the previous trends, some of the parameters of Eq. (1), namely the expression for plastic hinge length (as implied by m_1), the stiffness of the span (EI), and the M_{sec}/M_C ratio were varied, and the values of x_{Eq} were computed. Good agreement was found to be possible between the x values from Eq. (1) and from the computer analysis for certain values of EI of the span or for a certain percentage of secondary moments; however, no general agreement was possible for the

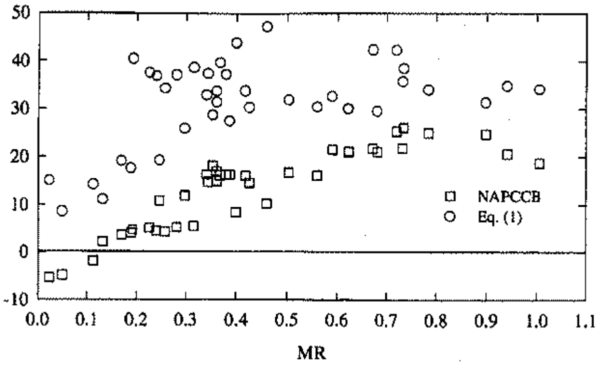

Fig. 5—Comparison of x *from Eq. (1) with* x *from computer analysis for Group B beams*

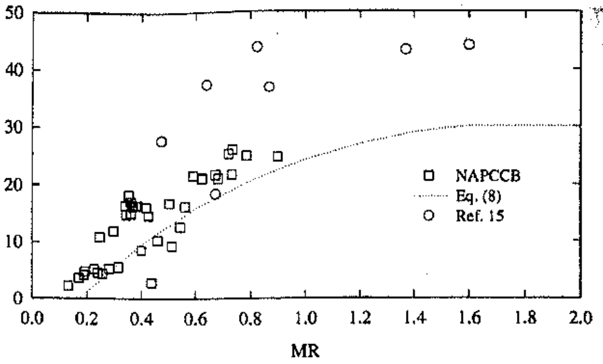

Fig. 7—Proposed x-MR *relationships for beams with uniformly distributed loading*

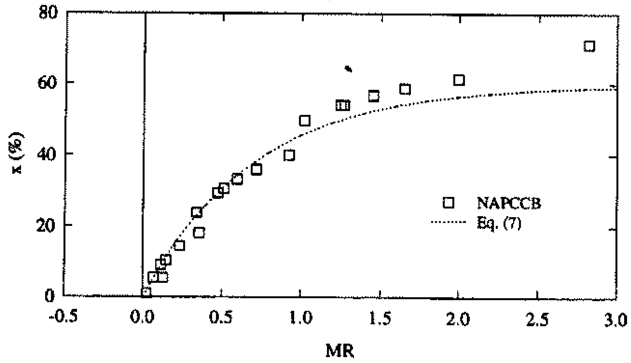

Fig. 6—Proposed x-MR *relationship for beams with concentrated loading*

whole range of MR. It was found that the plastic hinge length did not significantly alter the value of x_{Eq}, but that secondary moment had a significant influence on the extent of redistribution of moment for the Group B beams. The fact that reasonable agreement was observed for the Group A beams, as can be seen in Fig. 4, may be attributed to the relatively small magnitude of the secondary moments present in these beams (see Table 1), with the result that the second part of Eq. (1), y_{sec4}, does not influence x_{Eq} to a great extent. However, in the Group B beams, as can be seen from Table 2, the secondary moments are of considerable magnitude when compared to the moment capacities of the critical sections, and thus, influence the behavior to a greater extent.

Further, as noted in References 2, 3, 16, and 17, there are wide-ranging opinions as to whether secondary moments decrease, increase, disappear, or remain constant in the inelastic range. It should be noted that the expression for x_{Eq} was derived based on the assumption that the secondary moments remain invariant throughout the loading range.[17] However, in the computer analysis, the secondary moment is treated as part of the overall moments, and so the variation of x_{Eq}, as compared to x from the computer analysis, may be attributed to the role played by secondary moments in determining the extent of redistribution of moment.

Various attempts[2] were made to improve the agreement between x_{Eq} and x from the computer analysis, for both groups of beams, by applying modification factors to $(y_{ine} + y_{sec})$ in Eq. (1). It was found that while reasonably good agreement could be obtained in some cases, the agreement between x_{Eq} and x from the computer analysis varied depending on the magnitude of the secondary moments and the value of MR.

While the theoretical expression for x-MR in Eq. (1) is very useful in identifying the different parameters that may influence redistribution of moment, its usefulness in the design process is limited. The effort involved in determining the required parameters is significant, and Eq. (1) also requires the separation of available redistribution into two components—the inelastic component and the secondary moment component. This necessitates an answer to the question as to what happens to secondary moments in the inelastic range. As an alternative to this theoretical expression, further effort was concentrated on finding a simple empirical relationship for x based on data from the nonlinear analyses.

PROPOSED RELATIONSHIPS FOR PERCENTAGE OF REDISTRIBUTION

The variation of x with MR is shown in Fig. 6 and 7 for Group A and Group B beams, respectively. It can be seen from Fig. 6 that the value of x increases with MR, with the rate of increase being higher at the lower range of MR values. It can also be seen that the percentage of redistribution can be as high as 70 percent for some beams. Fig. 7 shows that the variation of x with MR for Group B beams follows a trend similar to that for Group A beams, but that the data points are more scattered. This can be attributed to the wide-ranging characteristics of Group B beams and to the value of MR, which is dependent on the location and the moment capacity of the critical section. In the case of a beam subjected to a concentrated load, the location of the critical section in the span region is well-defined, but in the case of a beam loaded with uniform dead load (UDL), as in the case of Group B beams, the location of the span critical section is not clearly defined and is dependent on the relative moment capacities of the critical sections.

The plots of x against MR in Fig. 6 and 7 appear to follow a definite trend, and, as a result, an attempt was made to define a relationship between x and MR in each case. The proposed relationships between x and MR are shown in Fig. 6 and 7, and are defined as follows:

For beams with concentrated load

$$x = 60\left[1 - e^{-\left(\frac{MR}{0.7}\right)}\right] \qquad 0 \le x \le 60 \qquad (7)$$

For beams with uniform dead load (UDL)

$$x = 45\left[1 - e - \left(\frac{MR}{0.7}\right)\right] - 10 \qquad 0 \le x \le 30 \qquad (8)$$

These proposed relationships indicate that the extent of redistribution of moment is dependent on loading type. A beam with a certain value of *MR*, subjected to concentrated load redistributes moment to a higher degree than when loaded with uniform dead load (UDL). The maximum permitted percentage of redistribution is 60 percent for beams with concentrated loading, while it is 30 percent for beams with uniform dead load (UDL). It should be noted that the value of x computed from the relationship in Eq. (7) is probably conservative for some cases of loading, such as loading at one-third the span from the central support. This is because Eq. (7) was arrived at by varying the position of the concentrated load along the span length, since the extent of redistribution not only depends on the loading type but also on the position of the concentrated loading.

The majority of the beams in Group B have *MR* values in the range of 0.20 to 0.9 (see Table 2) and such values are representative of beams commonly used in practice. To extend the range of *MR* values, additional data obtained from Reference 15 are plotted in Fig. 7. It can be seen that the values of x are quite high for these additional beams and this can be attributed to the large ratio of M_C/M_B and the low span-depth ratio of 12 for these beams. As a result, it was decided to restrict the proposed upper limit of x for beams with uniform dead load (UDL) to 30 percent.

While the upper limits for x in Eq. (7) and (8) may seem to be high compared to the limits in some current codes of practice,[4-6] it should be noted that the Danish code[18] allows redistribution as high as 66 percent. A high percentage of redistribution is possible in a beam having a low ratio of the ultimate moment capacity at the support section to that at the span section, as can be seen from Tables 1 and 2.

USING *x-MR* RELATIONSHIPS IN DESIGN AND ANALYSIS

The proposed *x-MR* relationships in Eq. (7) and (8) may be used to predict the percentage of redistribution of moment in partially prestressed concrete (PPC) beams in lieu of a detailed nonlinear analysis. Given a two-span continuous PPC beam with certain dimensions and cross-sectional properties, and knowing the type of loading, the ultimate moment capacities of the critical sections can be determined using strain compatibility, while the appropriate values of secondary moments can be established from an elastic analysis. The moment ratio *MR* can be computed from Eq. (4), using appropriate values of a and s_1. Knowing the value of *MR* and the type of loading, the value of x can be determined using either Eq. (7) or (8). The design moments can then be established and the failure load found using the design moments. The calculation of the design moment at a section should be as follows:

a. Determine elastic moments due to dead and live load.

b. Modify by the algebraic addition of the secondary moment.

c. Redistribute the support moment by a percentage x.

d. Adjust the moments in the span accordingly.

The analysis procedure previously outlined can be incorporated into design, since the design process is largely one of analyzing possible structural configurations. Application of the previous procedure is illustrated through a numerical example in Appendix B*. The advantage of the proposed method is that the failure load of a beam is obtained by taking into account the characteristics of the whole beam, including secondary moment.

CONCLUSIONS

1. The theoretical expression [Eq. (1)] for the percentage of redistribution of moment is useful in identifying the various parameters that influence the amount of redistribution occurring at failure in a two-span continuous prestressed concrete beam.

2. The most important parameters affecting the redistribution of moment are the stiffness of the critical cross sections, cross-sectional shape, loading type, concrete strength, span-depth ratio, partial prestressing index, and magnitude and nature of the secondary moments.

3. The proposed approach for determining the redistribution of moment is based on a relationship between the percentage of redistribution x and the moment ratio *MR*. This insures that the extent of redistribution is related to overall structural characteristics of the beam, including the effect of secondary moments and the loading type.

4. The proposed method determines the failure load of a continuous beam making use of the parameters that must be computed in the normal course of design.

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CONVERSION FACTORS

1 in.	=	25.4 mm
1 kip	=	4.448 kN
1 ksi	=	6.895 MPa
1 kip-ft	=	1.356 kN.m

NOTATION

a	= ratio of distance of span critical section from end support to span length when span and center support section ultimate strengths are developed simultaneously
A_{ps}	= area of prestressed reinforcement
c	= distance from extreme compression fiber to neutral axis at ultimate limit state
d	= distance from extreme compression fiber to center of tension force
E_i	= modulus of elasticity of concrete
E_s	= modulus of elasticity of nonprestressed reinforcement
EI	= flexural stiffness
f_c'	= specified compressive strength of concrete
f_{pu}	= ultimate stress in prestressed reinforcement
f_{se}	= effective stress in prestressed reinforcement
f_t'	= modulus of rupture of concrete
f_y	= yield stress in nonprestressed reinforcement
K	= curvature
l_p	= plastic hinge length in each span adjacent to center support
L	= span length
m_1	= factor to account for type of loading
M	= moment

*The Appendixes are available in xerographic or similar form from ACI headquarters, where they will be kept permanently on file, at a charge equal to the cost of reproduction plus handling at time of request.

MR = moment ratio
M_{sec} = secondary moment due to prestress
P = prestressing force
PAR = plastic adaptation ratio (based on load factors)
$PAR1$ = plastic adaptation ratio (based on failure loads)
s_1 = variable factor used in defining bending moment of central support (for concentrated load at midspan $s_1 = 16/3$; for uniformly distributed loading $s_1 = 8$)
T_f = 1.0 for concentrated load; 2.0 for uniformly distributed load in each span
w = uniformly distributed load
W = concentrated load
x = percentage redistribution of moment
y_{ine} = component of redistribution due to inelastic action
y_{sec} = component of redistribution due to secondary moments
ε_{cu} = strain at ultimate in concrete
ε_{pu} = strain at ultimate in prestressed reinforcement
ε_{su} = strain at ultimate in nonprestressed reinforcement
ε_y = strain at yield in nonprestressed reinforcement

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