Structure-ground interaction in earthquakes

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STRUCTURE-GROUND INTERACTION IN EARTHQUAKES

by
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L'INTERACTION DU SOL ET DES BATIMENTS LORS DES SEISMES

SOMMAIRE

Un modèle équivalent de simple degré de liberté (S.D.F.) est utilisé afin de déterminer les déplacements relatifs des édifices à un étage sous l'action de séismes en terrain élastique. La méthode présentée évite les désavantages d'employer individuellement les perturbations au hasard ou celles à l'état soutenu. Enfin, on présente aussi une étude détaillée de paramètres.

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STRUCTURE-GROUND INTERACTION IN EARTHQUAKES

By Johann H. Rainer,[1] A. M. ASCE

INTRODUCTION

Analysis of the earthquake response of structures founded on a flexible foundation has recently received considerable attention. Although the idealized structural models in such studies varied substantially in detail, they can generally be grouped into two main categories: (1) Those that employ comparisons of response due to arbitrary base disturbances such as recorded earthquakes (3,7) or artificially generated earthquakes (10,11,12) and (2) those that used steady-state ground disturbances (5,9).

The response comparison between the interaction structure and the fixed-base structure suffers from the fact that the results may be more sensitive to the arbitrary characteristics of the disturbance than to the foundation effects being studied. As the natural frequencies of the structure change with the introduction of foundation flexibility, such a response comparison may show either an increase or a decrease in response, simply because of the characteristics of the particular exciting force used (3,7,11). Whether a reduction or an increase in response is obtained depends on the location of the spectral peaks of the disturbance in relation to the natural frequencies of the fixed-base and interaction systems. Certain general aspects of the interaction phenomenon can, therefore, be masked by the characteristics of the particular ground disturbance chosen. This complicating effect introduced by the frequency shift was overcome by Perelman, et al. (12) by comparing the seismic interaction response with the response of a single degree-of-freedom (SDF) system having the same natural frequency and subjected to the same disturbance.

On the other hand, interaction studies that employ steady-state distur-

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bances are limited by the fact that the results cannot readily be applied to earthquake-type disturbances.

The method of analysis now presented helps to overcome the shortcomings of studies where steady-state or random-type disturbances are used by themselves; in fact, it bridges the gap between the two techniques. To achieve this goal, an equivalent SDF model is derived, representing relative displacement for single-story interaction structures, that implicitly incorporates the effects of the flexible foundation parameters. With this equivalent SDF the properties of the structure are separated from the influences of specific random-type disturbances. Thus the significant parameters of the interaction process can be readily identified and evaluated. When the interaction response to a given ground disturbance is desired, however, well established procedures for SDF systems, such as numerical integration and response spectrum techniques, can be employed.

EQUIVALENT SDF MODEL FOR RELATIVE DISPLACEMENT

For purposes of dynamic analysis many single-story as well as more complex structures can be idealized by a SDF system, i.e., a mass supported by a deformational spring, in turn mounted on a rigid base. If the base is permitted to move horizontally as well as rotationally relative to the undisturbed ground, however, a ground-structure interaction system is obtained. Such a system is shown in Fig. 1, where the ground deforms under the dynamic loads that are applied by the base to the half-space.

With the introduction of rocking and relative horizontal motion of the base, the original SDF system becomes a three degree-of-freedom system. Three modal shapes can, therefore, be expected. For single-story structures the lowest mode will remain dominant, and under most earthquake disturbances the contribution of the second and third modes to the total response of single-story structures can be assumed negligible. This study considers only the effect of the lowest mode of the single-story interaction system.

With this simplification it becomes possible to transform the interaction system into an equivalent SDF model whose properties reflect the effects of the foundation interaction. Established techniques for SDF systems may then be utilized. Briefly, the procedure is to determine the SDF system that matches the interaction system with respect to its frequency response.

The technique is developed in terms of relative displacement, although it is similarly applicable to other structural parameters such as overturning moment. The process will be illustrated by two specific interaction structures, described in Table 1.

Properties of Frequency Response Curves.—The dynamic characteristics of a linear system are completely determined by the frequency response curves. For a particular response parameter, e.g., relative displacement, the frequency response curve is defined as the ratio of response to disturbance under steady-state conditions as a function of frequency.

Frequency response curves are usually presented in terms of a nondimensional response parameter such as the ratio of the relative displacement amplitude to the ground displacement amplitude plotted versus frequency. In deriving an equivalent SDF system, however, it is advantageous to plot the ratio of relative displacement amplitude to ground acceleration amplitude.

TABLE 1.—PARAMETERS FOR SAMPLE CALCULATIONS

Parameter (1)	Structure number 1 (2)	Structure number 2 (3)
(a) Structural Parameters		
m_1, in pounds per second squared per inch	1,000	4,000
m_0, in pounds per second squared per inch	1,000	1,000
h, in feet	40	80
r, in feet	15	15
V_s, in feet per second	300	800
ω_0, in radians per second	10	20
ω_1, in radians per second	7.59	7.60
λ, as a percentage	2.0	2.0
(b) Equivalent SDF Model for Relative Displacement		
ω_1, in radians per second	7.59	7.60
$\left(\frac{\omega_1}{\omega_0}\right)^2$	0.576	0.144
λ_e, as a percentage	1.37	0.192

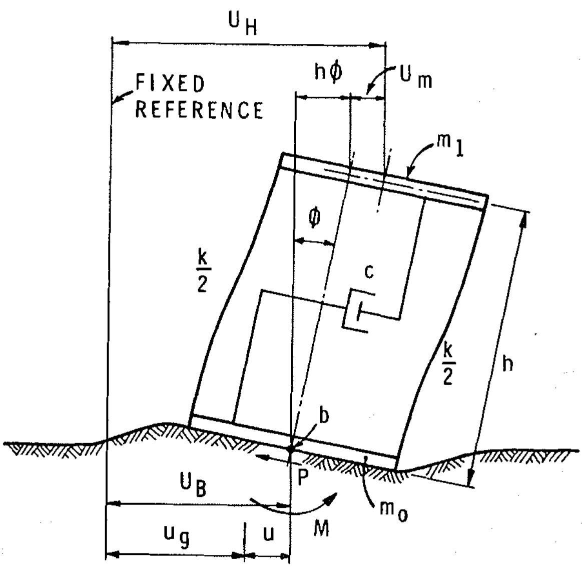

FIG. 1.—INTERACTION SYSTEM

This ratio is termed the displacement response factor. Sample curves of the displacement response factor versus frequency are presented in Fig. 2(*a*) for the particular structure (No. 1, Table 1*a*). Although both a real and an imaginary component are generally present, it is satisfactory with small amounts of damping to consider merely the vectorial sum of the real and imaginary components, i.e., the amplitude frequency response curve. The latter will be used exclusively herein.

Derivation of Equivalent SDF Model.—For the particular structure (No. 1, Table 1*a*) Fig. 2(*a*) presents the displacement response factor for the fixed-base structure with resonance frequency ω_0 and the same parameter for the

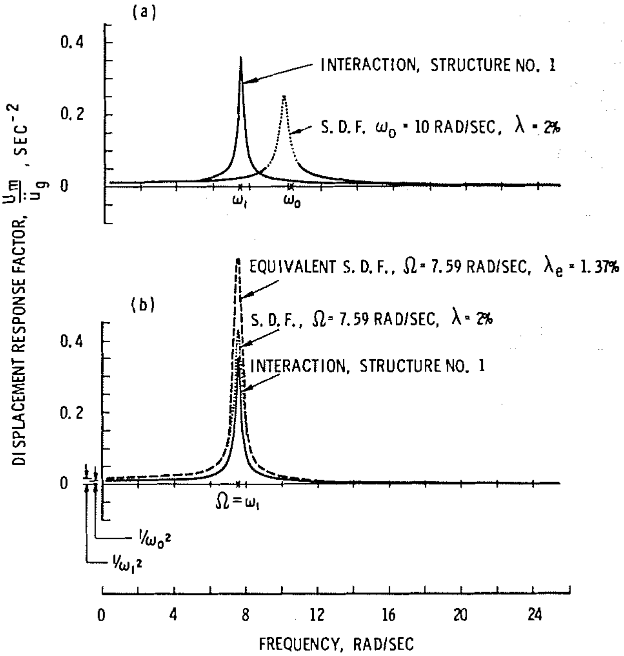

FIG. 2.—FREQUENCY RESPONSE CURVES FOR STRUCTURE NO. 1, RELATIVE DISPLACEMENT

interaction system with fundamental resonance frequency ω_1. Two significant features in these curves are apparent: (1) A decrease of resonance frequency has occurred and (2) the intercept at zero frequency is the same for the fixed-base structure as for the corresponding interaction system.

Fig. 2(*b*) shows the frequency response curves for: (1) The interaction structure in the solid line with fundamental resonance frequency ω_1; (2) the SDF system with resonance frequency $\Omega = \omega_1$ and damping ratio $\lambda = 2\%$; and (3) the equivalent SDF model for relative displacement of the interaction structure. With reference to Fig. 2(*b*), the transformation of the interaction parameter into the equivalent SDF is achieved in three main stages: (1) Determination of proper resonance frequency; (2) determination of a multiplication factor for the entire frequency response curve; and (3) determination of an equivalent damping coefficient to account for the magnitude of the frequency response curve at resonance.

Determination of Fundamental Resonance Frequency.—The fundamental resonance frequency for the interaction system may be computed by determining the eigenvalues of the system once a standard eigenvalue problem has been formulated. This will be described further in the section Interaction Model. Alternatively, a numerical search of the response curves may be employed to detect the peak amplitude located at the resonance frequency, ω_1, of the fundamental mode. This latter method was used to obtain the numerical results presented herein.

Multiplication Factor for Frequency Response Curve.—The multiplication factor for transforming the interaction frequency response curve into that of an SDF system is obtained by comparing the zero frequency intercepts of the respective curves shown in Fig. 2(*b*). The zero frequency intercept corresponds to the case where the interaction structure is subjected to a constant base acceleration $\ddot{u}_g$. For both an interaction system and a fixed-base structure this induces an inertia force in the top mass, m_1, equal to $m_1\ddot{u}_g$, which in turn causes a relative interstory displacement $U_m = m_1\ddot{u}_g/k$ in which k = interstory spring stiffness. For $\ddot{u}_g = 1$

$$U_m = \frac{m_1}{k} = \frac{1}{\omega_0^2} \qquad (1)$$

The SDF system with resonance frequency Ω, however, has a zero frequency intercept of $1/\Omega^2$. The frequency response curve for relative displacement of the interaction structure is thus brought to coincide with that of the equivalent SDF model by multiplying the former by $\omega_0^2/\Omega^2 = \omega_0^2/\omega_1^2$. This results in the dashed curve shown in Fig. 2(*b*).

Equivalent Damping.—The third parameter required for a complete description of the equivalent SDF model is the equivalent damping ratio λ_e. This can be determined simply from the magnitude, M_e, of the resonance peak for the nondimensional frequency response curve for U_m/u_g

$$\lambda_e = \frac{1}{2M_e} \qquad (2)$$

in which $M_e = M_I\,(\omega_0/\omega_1)^2$ and M_I = peak amplitude of the nondimensional frequency response curve for relative displacement U_m/u_g at resonance frequency ω_1.

Because all amplitudes of the frequency response curve have been increased by $(\omega_0/\omega_1)^2$, the response computed with the aforementioned equivalent SDF is too large by the factor $(\omega_0/\omega_1)^2$.

With these three parameters, i.e., fundamental resonance frequency ω_1, multiplication factor $(\omega_0/\omega_1)^2$, and equivalent damping λ_e, the displacement response curve of the equivalent SDF model and that of the interaction system multiplied by $(\omega_0/\omega_1)^2$ agree closely over the complete frequency range, even when frequency-dependent foundation parameters are considered.

INTERACTION MODEL

The interaction system under consideration is shown in Fig. 1. This model is the same as that used by Parmelee (9) and therefore only the main points of the derivation will be presented. For purposes of this derivation both the masses, m_0 and m_1, are circular in plan with radius r. The corresponding differential equations of motion under any arbitrary base disturbance are the horizontal translation of structure

$$m_1 \ddot{U}_H + m_0 \ddot{U}_B + P = 0 \qquad (3)$$

the horizontal translation of top mass

$$m_1 \ddot{U}_H + c\dot{U}_m + kU_m = 0 \qquad (4)$$

and the rotation about point b

$$I_1 \ddot{\Phi} + I_0 \ddot{\Phi} + m_1 h \ddot{U}_H + M = 0 \qquad (5a)$$

$$\text{in which } \left.\begin{aligned} I_0 &= m_0 \frac{r^2}{4} + m_1 \frac{r^2}{4} \\ I_1 &= m_1 h^2 \end{aligned}\right\} \qquad (5b)$$

and dots above a variable represent differentiation with respect to time. The

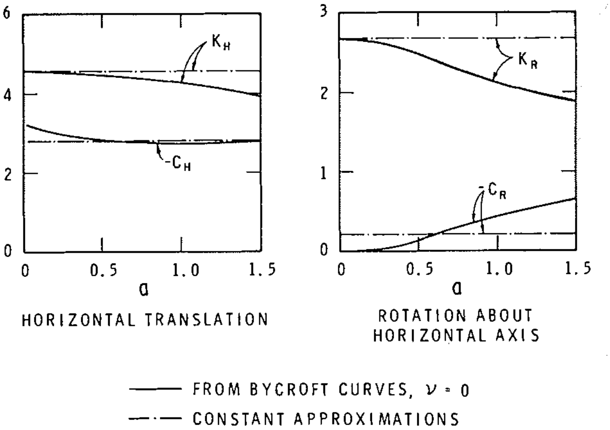

FIG. 3.—SDF FOUNDATION PARAMETERS FOR CIRCULAR BASE

remaining symbols are defined in Fig. 1. Under the influence of a steady-state ground displacement $u_g = We^{ipt}$, the resulting complex amplifications X, Y, and Z of the displacement components U_B, Φ, and U_m are given by

$$\left.\begin{aligned} U_B &= We^{ipt} X = u_g(X_1 + iX_2) \\ \Phi &= We^{ipt} Y = u_g(Y_1 + iY_2) \\ U_m &= We^{ipt} Z = u_g(Z_1 + iZ_2) \end{aligned}\right\} \qquad (6)$$

The forces between the base and the half space are given by

$$P = P_0 e^{ipt} = u_g(X - 1) A \qquad (7)$$

$$M = M_0 e^{ipt} = u_g YB \qquad (8)$$

in which A and B = dynamic stiffness coefficients that relate the generalized forces and the corresponding displacements under sinusoidal excitation. For a circular base

$$A = Gr(K_H + i\,a\,C_H) \quad \text{.......................} \quad (9)$$

$$B = Gr^3(K_R + i\,a\,C_R) \quad \text{.......................} \quad (10)$$

in which $K_H = \dfrac{f_{1H}}{(f_{1H})^2 + (f_{2H})^2}$; $C_H = \dfrac{-\dfrac{f_{2H}}{a}}{(f_{1H})^2 + (f_{2H})^2}$;

$$K_R = \frac{f_{1R}}{(f_{1R})^2 + (f_{2R})^2}; \quad C_R = \frac{-\dfrac{f_{2R}}{a}}{(f_{1R})^2 + (f_{2R})^2} \quad \text{.............} \quad (11)$$

G = shear modulus of ground; r = radius; a = nondimensional frequency = pr/V_s; p = circular frequency, in radians per second; V_s = shear wave velocity of ground; and $i = \sqrt{-1}$. Terms K_H and K_R are horizontal and rotational stiffness factors and C_H and C_R, horizontal and rotational damping factors for the circular footing on the elastic half space, shown by the solid lines in Fig. 3. The values for f_1 and f_2 used herein are those obtained by Bycroft for a circular disc on an elastic half space (2).

Substitution of the preceding relations into Eqs. 3 to 5 and simplification gives:

$$\begin{bmatrix} \left(1 - \dfrac{\omega_0^2}{p^2}\right) & -\dfrac{2\lambda\omega_0}{p} & 1 & 0 & 1 & 0 \\ -\dfrac{2\lambda\omega_0}{p} & -\left(1 - \dfrac{\omega_0^2}{p^2}\right) & 0 & -1 & 0 & -1 \\ 1 & 0 & \left(1 + \dfrac{1}{\alpha} - \dfrac{\omega_H^2}{p^2}\right) & -\dfrac{2\lambda_H\omega_H}{p} & 1 & 0 \\ 0 & -1 & -\dfrac{2\lambda_H\omega_H}{p} & -\left(1 + \dfrac{1}{\alpha} - \dfrac{\omega_H^2}{p^2}\right) & 0 & -1 \\ 1 & 0 & 1 & 0 & \left(1 + \dfrac{1+\alpha}{4\eta} - \dfrac{\omega_R^2}{p^2}\right) & -\dfrac{2\lambda_R\omega_R}{p} \\ 0 & -1 & 0 & -1 & -\dfrac{2\lambda_R\omega_R}{p} & -\left(1 + \dfrac{1+\alpha}{4\eta} - \dfrac{\omega_R^2}{p^2}\right) \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ X_1 \\ X_2 \\ hY_1 \\ hY_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{\omega_H^2}{p^2} \\ \dfrac{2\lambda_H\omega_H}{p} \\ 0 \\ 0 \end{bmatrix} \quad (12)$$

in which $\frac{\omega_H^2}{p^2} = \frac{K_H}{a^2\beta}$; $\frac{\omega_R^2}{p^2} = \frac{K_R}{a^2\beta\eta}$; $\lambda_H = \frac{C_H}{2(\beta K_H)^{1/2}}$; $\lambda_R = \frac{C_R}{2(\beta\eta K_R)^{1/2}}$ (13)

and $\alpha = m_0/m_1$; $\beta = m_1/(\rho r)^3$; $\eta = (h/r)^2$; $\omega_0^2 = k/m_1$; λ = relative interstory damping ratio; and ρ = density of ground. Note that, in general, ω_H^2, ω_R^2, λ_H, and λ_R in Eq. 12 are frequency-dependent quantities; ω_H can be interpreted as the horizontal resonance frequency of the base alone; and ω_R as the rocking frequency of the mass m_1 with moment of inertia $I_1 = m_1 h^2$. Terms λ_H and λ_R are the corresponding relative damping ratios for horizontal and rocking motions, respectively.

Solution for the steady-state amplification vector $T^d_{u_g}$ yields

$$T^d_{u_g} = \begin{bmatrix} Z_1 \\ Z_2 \\ X_1 \\ \\ X_2 \\ hY_1 \\ hY_2 \end{bmatrix} = [6 \times 6 \text{ matrix from Eq. } 12]^{-1} \begin{bmatrix} 0 \\ 0 \\ \frac{\omega_H^2}{p^2} \\ \\ \frac{2\lambda_H\omega_H}{p} \\ 0 \\ 0 \end{bmatrix} \quad \ldots\ldots (14)$$

in which the matrix inversion indicated may be carried out numerically by computer.

Term $T^d_{u_g}$ represents the displacement response vector of the interaction system to a steady-state ground displacement u_g at any frequency and is commonly called the transfer function for the system (4). From the definition of frequency response curve already given it may be seen that for a continuous range of frequencies the frequency response curves representing displacement ratios may be evaluated from Eq. 14.

The transfer function as well as the frequency response curves for other time derivatives may be obtained if Eq. 14 is multiplied by the proper power of ip corresponding to the order of time derivatives represented by the variables. For example, the transfer function $T^d_{\ddot{u}_g}$ for the displacement vector subjected to base accelerations is given by

$$T^d_{\ddot{u}_g} = \frac{1}{(ip)^2} T^d_{u_g} = \left(\frac{-1}{p^2}\right) T^d_{u_g} \quad \ldots\ldots (15)$$

Determination of Eigenvalues for Interaction Structure.—The eigenvalues of the interaction system can be computed by using only the first, third, and fifth rows and columns of the matrix in Eq. 12, corresponding to the real terms in the displacement vector; the right-hand vector is set equal to zero because free vibrations are implied. If frequency-dependent stiffness parameters are present, as in the case now considered, they can be introduced by successively approximating the stiffness parameters corresponding to the eigenvalue computed in the previous cycle.

SPECIFIC RESPONSE CALCULATIONS

Results for specific response calculations are presented to illustrate the use of the equivalent SDF system. Figs. 4 and 5 show relative displacement

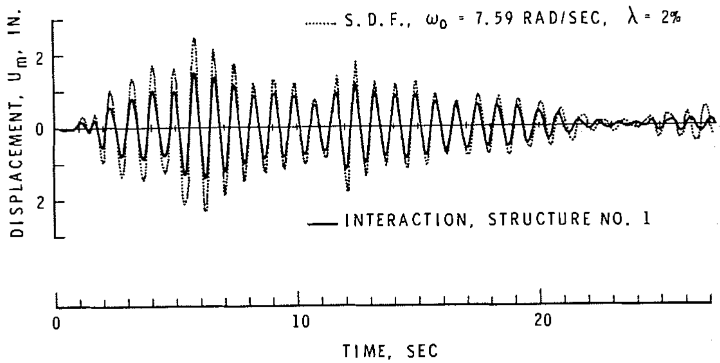

FIG. 4.—RELATIVE DISPLACEMENT RESPONSE FOR STRUCTURE NO. 1 SUBJECTED TO EL CENTRO 1940, N-S COMPONENT

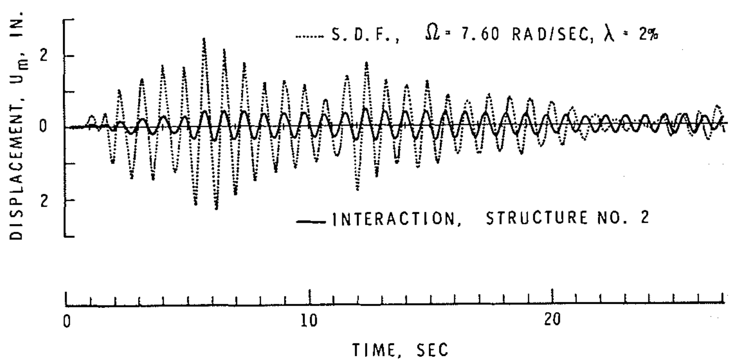

FIG. 5.—RELATIVE DISPLACEMENT RESPONSE FOR STRUCTURE NO. 2 SUBJECTED TO EL CENTRO 1940, N-S COMPONENT

responses for Structures No. 1 and 2, with their parameters as given in Table 1. The base disturbance consists of 27 sec of the record of the El Centro, California, 1940 earthquake, N-S component. For Structure No. 1 the solid line in Fig. 4 represents the response of the interaction system, obtained by the Fourier transform method with Eq. 14 used as the transfer function. The dotted line represents the response of an SDF system with natural frequency Ω = 7.59 rad per sec and interstory damping λ = 2 %. The interaction response using the equivalent SDF with $\omega_1 = 7.59$ rad per sec, $(\omega_1/\omega_0)^2 = (7.59/10.0)^2 = 0.576$, and $\lambda_e = 1.37$ % gives results that are indistinguishable from the true interaction response throughout the full 27 sec. The SDF response was obtained by a numerical integration procedure (8).

Similar calculations for Structure No. 2 are presented in Fig. 5. The solid line represents the interaction response obtained by numerical integration using the equivalent SDF model shown in Table 1*b*. The dotted curve again represents the relative displacement for the SDF with $\lambda = 2$ % and $\Omega = 7.60$ rad per sec.

PARAMETER STUDY

A parameter study has been carried out with the two ranges of parameters shown in Table 2 to examine the behavior of the interaction system. Poisson's ratio ν and density of the ground ρ have been kept constant at zero and 120 lb per cu ft, respectively. A variation in the two parameters would be reflected primarily in changes in the shear wave velocity V_s and the rocking stiffness k_Φ, both of which are variables in this study.

Parameter Set A.—Parameter Set A, Table 2, includes structures such as elevated storage tanks and water towers and other tower structures that can be idealized as a lumped mass supported by a spring.

Reduction of Resonance Frequency.—For Parameter Set A, Table 2, the reduction of resonance frequency for the interaction system is shown in Fig. 6. The ordinates indicate the reduction in resonance frequency relative to fixed-base frequency, whereas the abscissa is the ratio of static rocking frequency to the fixed-base natural frequency. Rocking frequency

$$\omega_\Phi^2 = \frac{k_\Phi}{I} \qquad (16)$$

in which $k_\Phi = 8\,Gr^3/[3(1-\nu)]$ = rocking stiffness of circular plate under static conditions and I = total moment of inertia $= m_1 h^2 + m_1(r^2/4) + m_0(r^2/4)$ for the circular configuration of top and bottom mass.

The lower, heavier curve in Fig. 6 corresponds to the theoretical relation of frequency reduction for a single-story building, considering only rocking and relative displacement (7). This curve also establishes the theoretical lower bound for fundamental frequency reduction of the three-degree-of-freedom interaction system. Actual values of frequency reduction for the aforementioned ranges of Parameter Set A, Table 2 fall within the region bounded by the two curves.

Magnitudes of Resonance Peaks.—In addition to the factor ω_1/ω_0, the other parameter required for a complete quantitative description of the equivalent SDF model is the equivalent damping ratio λ_e. For particular structural parameters and foundation properties, λ_e may be found from a parameter study of resonance amplitude peaks and Eq. 2. Detailed results are presented for the Bycroft foundation model and a foundation having constant stiffness and damping coefficients.

Bycroft Foundation Model.—For the foundation properties presented in Fig. 3, Figs. 7 and 8 show the ratio of peak amplitudes M_I of the frequency response curve for the interaction system to the peak amplitudes M_S of the SDF, fixed-base system. The abscissa is $a_0 = \omega_0 r/V_s$, in which ω_0 = natural frequency of fixed-base structure. Terms M_I and M_S represent the relative displacements U_m at the resonance frequency $\Omega = \omega_1$.

With small values of a_0 it may be seen from Figs. 7 and 8 that for tall structures the peak amplitude of the frequency response curve exceeds that of an SDF system with the same amount of structural damping. In order to interpret some influences of the foundation properties it is useful to refer to the

TABLE 2.—RANGES OF PARAMETERS

Variable (1)	Parameter set A (2)	Parameter set B (3)
ω_0, in radians per second	5 to 20	5 to 20
m_1, in pounds per second squared per inch	1,000 to 4,000	100,000 and 400,000
m_0, in pounds per second squared per inch	1,000	100,000 and 400,000
h, in feet	20 to 80	40 to 80
r, in feet	15 and 20	60
λ, as a percentage	1, 2, and 5	2
V_S, in feet per second	300, 500, and 800	500, 800, and 1600

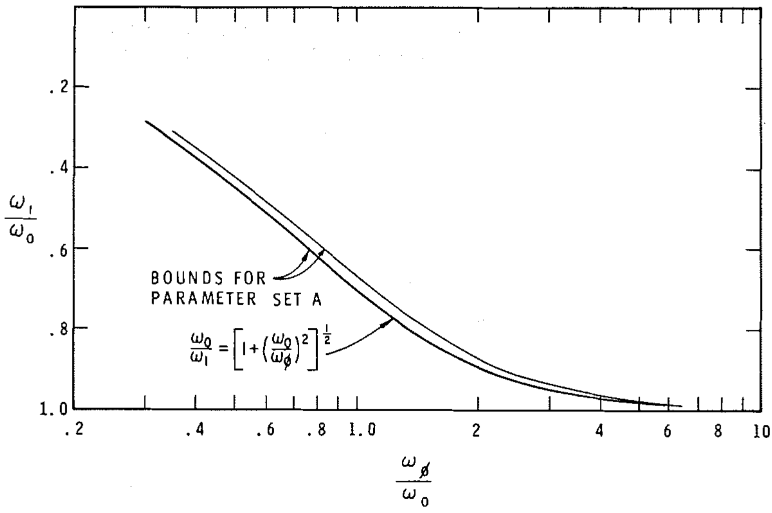

FIG. 6.—REDUCTION IN RESONANCE FREQUENCY FOR INTERACTION SYSTEMS, PARAMETER SET A, TABLE 2

SDF stiffness and damping terms presented in Fig. 3. It may be observed that in the rocking mode the foundation damping coefficient for small values of a is practically zero; therefore, the large values of peak resonance amplitudes are not unexpected.

The curve for h = 20 ft in Fig. 7 exhibits behavior slightly different from that for taller structures. This is due to the increased influence the hori-

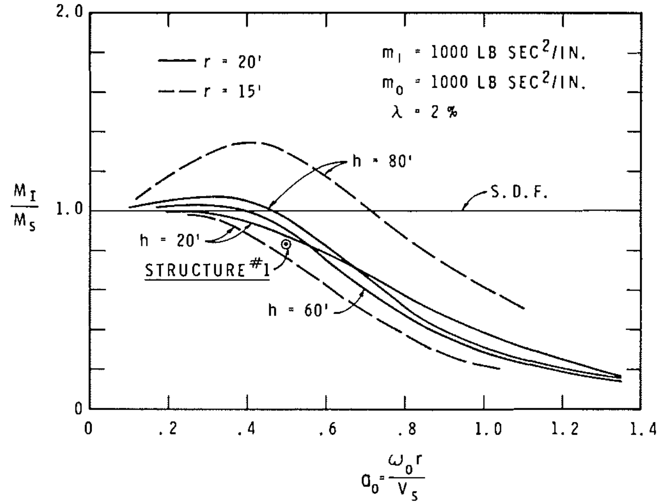

FIG. 7.—MAGNITUDES OF PEAK RESONANCE CURVES FOR RELATIVE DISPLACEMENTS, m_1 = 1,000 LB SEC SQUARED PER IN., BYCROFT FOUNDATION

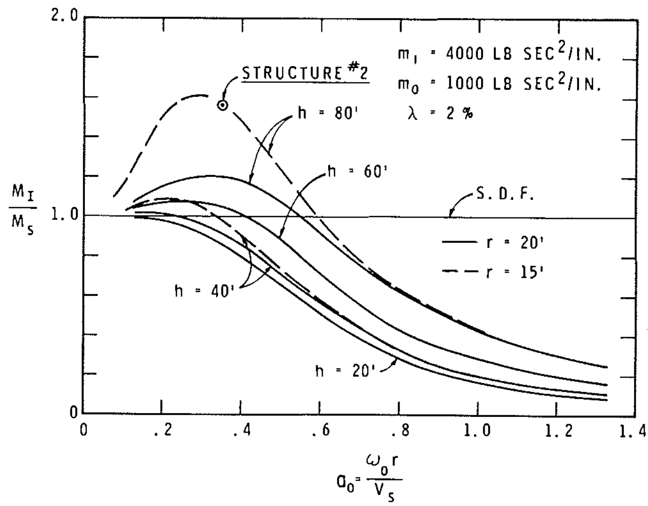

FIG. 8.—MAGNITUDES OF PEAK RESONANCE CURVES FOR RELATIVE DISPLACEMENTS, m_1 = 4,000 LB SEC SQUARED PER IN., BYCROFT FOUNDATION

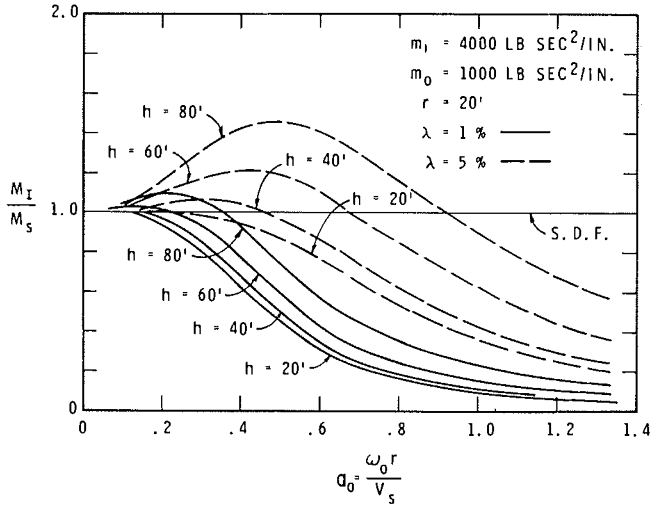

FIG. 9.—MAGNITUDE OF PEAK RESONANCE CURVES FOR RELATIVE DISPLACEMENTS, m_1 = 4,000 LB SEC SQUARED PER IN., λ = 1 % AND 5 % BYCROFT FOUNDATION

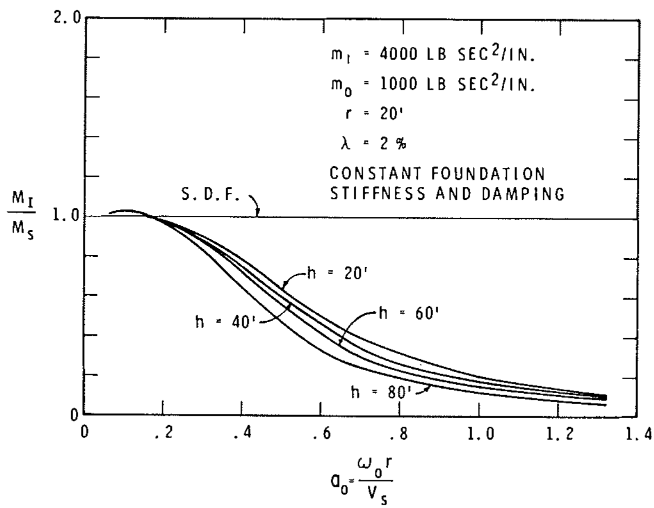

FIG. 10.—MAGNITUDES OF RESONANCE PEAKS FOR RELATIVE DISPLACEMENTS, m_1 = 4,000 LB SEC SQUARED PER IN., CONSTANT SDF FOUNDATION STIFFNESS AND DAMPING

zontal base motion assumes for structures with small height-to-width ratios. For taller structures rocking predominates, so that all curves will be mainly influenced by the damping curve for rotation, C_R, shown in Fig. 3. Consequently, all curves in Fig. 7 for the taller structures can be expected to have the same general shape. In Figs. 8, 9, and 10, this different behavior for h = 20 ft is not in evidence, because top mass m_1 is four times as large as that in Fig. 7. This increases the moment of inertia and again results in a predominance of rocking over horizontal base motion, even for h = 20 ft.

The effects of varying interstory damping are demonstrated in Fig. 9 for λ = 1 % and 5 %, using the Bycroft foundation parameters in Fig. 2. The general variation of peak amplitudes is similar, although the values for λ = 5 % are larger. This may be explained by referring to the C_R curve in Fig. 3. For a given value of a or a_0, foundation damping represents a smaller proportion of the overall system damping for λ = 5 % than for λ = 1 %. Consequently, the ratio of resonance peaks M_I/M_S for a given value of a_0 can be expected to be larger for the higher values of interstory damping.

Constant Foundation Parameters.—An assumption frequently adopted in investigating the effects of foundation flexibility is that stiffness and damping properties are independent of frequency. Such an approximation is shown in Fig. 3 by the dash-dotted lines. The parameter study for the ratio M_I/M_S, using constant foundation parameters, is shown in Fig. 10. It may be seen that the peak values in the range of small a_0 have been substantially reduced in comparison with the peaks for the frequency-dependent foundation parameters shown in Fig. 8.

This would be expected because with smaller values of a_0 more damping is present in the rocking motion under the constant approximation than for the frequency-dependent case. The effect of the approximation, however, on the reduction of natural frequency is quite small. For the parameter range studied the largest increase in the fundamental resonant frequency over the case with frequency-dependent parameters is 4 % and occurs for $\omega_1 r/V_s$ = 0.35. The deviations decrease for smaller values of $\omega_1 r/V_s$. For this approximation of constant foundation properties the bounds for frequency reduction in Fig. 6 are therefore still valid.

Parameter Set B.—Parameter Set B, Table 2 corresponds to structures such as nuclear reactor containment vessels.

Reduction of Resonance Frequency.—The reduction of resonance frequency for Parameter Set B is shown in Fig. 11. The same abscissa and ordinate are chosen as for Parameter Set A. Frequency reductions are larger than those for Set A shown in Fig. 6 owing to the increasing influence of the relative horizontal base displacement U_B (for Parameter Set A the relative base displacement had negligible influence). As the structure becomes taller, frequency reductions approach the theoretical curve for frequency reduction considering base rocking only. This latter curve is shown in Fig. 11 in dashed lines.

Magnitudes of Resonance Peaks.—For Parameter Set B plots of magnitudes of resonance peaks are presented in Fig. 12. The equivalent damping factor, λ_e, can be found therefrom, as described previously. It may be seen that resonance peaks for all values of a_0 are less than or equal to those of the SDF system with the same resonance frequency and interstory damping. This implies that for the structural parameters considered the interstory displacement of the interaction system is less than or possibly equal to the re-

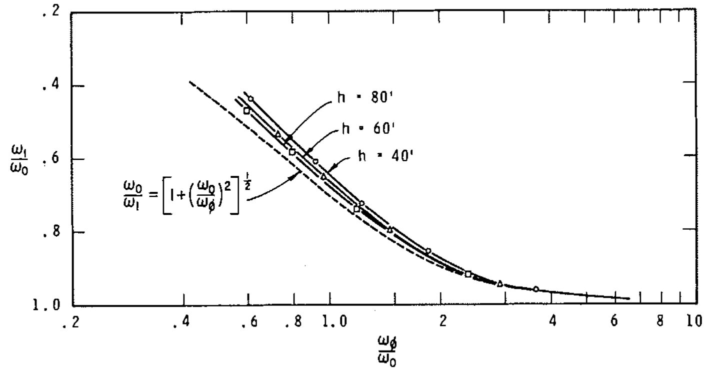

FIG. 11.—REDUCTION IN RESONANCE FREQUENCY FOR INTERACTION SYSTEMS, PARAMETER SET B, TABLE 2

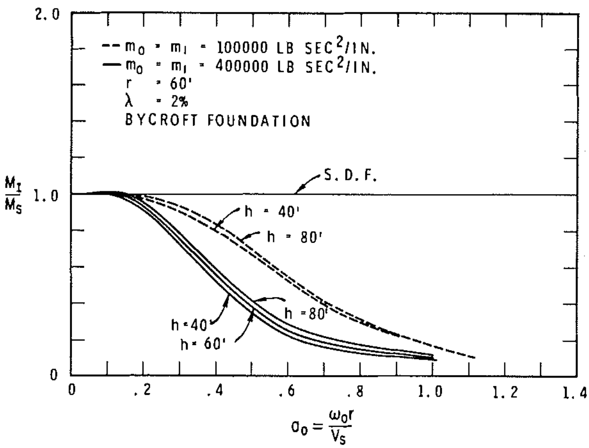

FIG. 12.—MAGNITUDES OF RESONANCE PEAKS FOR RELATIVE DISPLACEMENTS, $m_1 = m_0$ = 100,000 AND 400,000 LB SEC SQUARED PER IN., λ = 2 %, BYCROFT FOUNDATION

sponse of the corresponding SDF system for any type of ground disturbance.

Response due to Random-type Disturbances.—Note that peak amplitudes of the frequency response curves for relative displacement do not, by themselves, give any measure of the response that may be expected from a random-type base input; in the disturbance, frequency components other than those at the resonance frequency of the structure may predominate.

DETERMINATION OF MAXIMUM RESPONSE FROM SPECTRA

With the aid of the equivalent SDF model, the maximum response for an interaction system may be determined from established response spectra of known earthquakes or other disturbances. The procedure is illustrated herein with Structure No. 2, Table 1, for the response spectrum of the El Centro, 1940 earthquake, N-S component (1), shown in Fig. 13.

1. Find reduced fundamental frequency ω_1 (either from computations outlined previously or from the graph in Fig. 6); ω_1 = 7.60 rad per sec.
2. Determine peak amplitude M_I/M_S of frequency response curve from Fig. 8; M_I/M_S = 1.51.

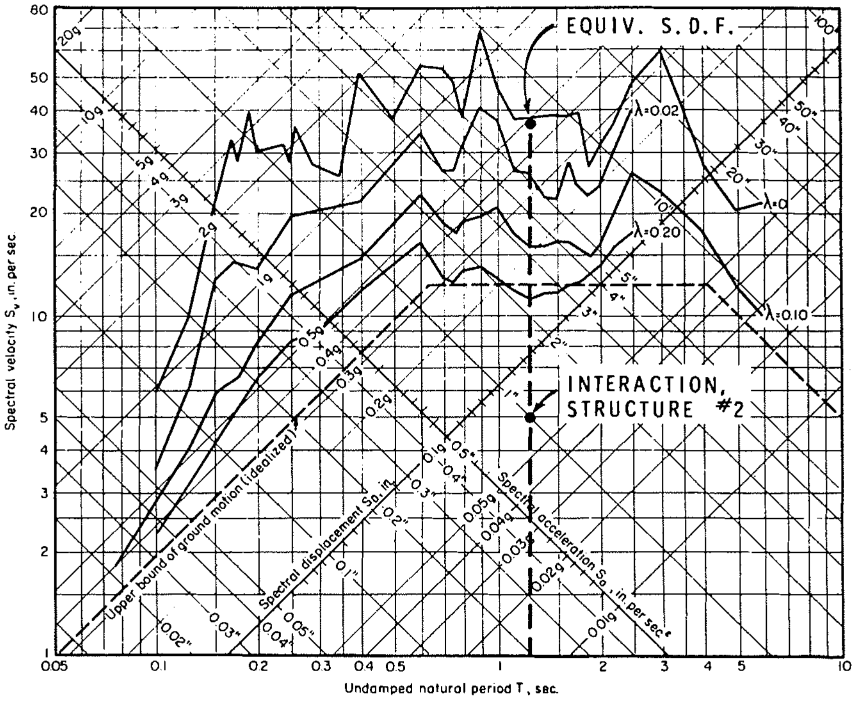

FIG. 13.—ELASTIC RESPONSE SPECTRA, 1940 EL CENTRO EARTHQUAKE, N-S COMPONENT (1)

3. Multiply amplitude M_I/M_S by $(\omega_0/\omega_1)^2$ = 6.95 and determine effective damping ratio from Eq. 2. By direct proportionality with the amplitude for λ = 2 %

$$\lambda_e = (0.02)\,\frac{1}{(1.51)(6.95)} = 0.00192 = 0.192\ \% \qquad (17)$$

4. Enter response spectrum with damping ratio λ_e and natural frequency ω_1 (or the corresponding period T = 1.21 sec) and read maximum spectral response; $S_D \simeq$ 7 in.
5. Divide spectral value by $(\omega_0/\omega_1)^2$ to obtain true maximum interaction response; maximum relative displacement $\simeq$ 1 in. This value agrees with the response calculation shown in Fig. 5.

The procedure is only slightly more complex than that using the response spectrum for SDF systems. With the aid of the equivalent SDF model it may thus be possible to construct a modified response spectrum to account for the influence of structure-ground interaction.

A useful approximation in the application of the response spectrum follows. For structures with small amounts of interstory damping (say up to 5 %) the equivalent damping, λ_e will be small for reasonably large reductions of frequency, as is evident from Eq. 2. Consequently, the spectrum curve for zero damping can be used and the spectral is divided by response by $(\omega_0/\omega_1)^2$. This will give a conservative estimate of interstory response for all cases. The approximation will improve with smaller interstory damping and larger frequency reduction ratio $(\omega_0/\omega_1)^2$.

Generalization of Maximum Response Comparisons.—A general conclusion regarding the response magnitude of interaction systems can be obtained from an examination of Figs. 7 to 10 and Fig. 12, which indicate that over a considerable range of values of a_0 the peaks of the frequency response curves are smaller than those of the SDF oscillator with the same natural frequency. For these cases the maximum response of the interaction system is a priori less than that of the SDF case. Where the resonance peaks exceed those of the SDF case, the response of the interaction system may exceed that of the SDF oscillator, particularly under steady-state excitations with frequency close to the resonance frequency of the structure. Considering, however, the random nature of the earthquake excitation, the contributions over the whole frequency response curve have to be included.

By means of a numerical summation technique it was found that the areas under the frequency response curves of the interaction systems studied are smaller than or equal to the corresponding areas for SDF systems with the same natural frequency and interstory damping. If the disturbance is idealized as weakly stationary, then on the basis of random vibration theory (6) it can be demonstrated that the mean relative displacement response for the interaction systems considered will be less than or equal to the response of an SDF oscillator with the same natural frequency. A similar conclusion for a more limited range of parameters was reached by Perelman, et al. (12) by means of response calculations with artificially generated earthquakes. For any particular interaction structure and a given base motion this can be verified with the aid of the equivalent SDF model and the response spectrum.

CONCLUSIONS

A method of analysis is presented that utilizes the transformation of a single-story interaction structure into an equivalent SDF model to determine the response of interaction systems under earthquake-type disturbances. The equivalent SDF model permits the use of numerical integration and response spectrum techniques in determining relative displacements. This approach avoids some of the drawbacks of previous studies in which either a random-type or a steady-state disturbance was used alone.

From an extensive parameter study it is shown that for tall slender structures the most important interaction parameter is the ratio of rocking frequency of the structure to its fixed-base natural frequency. This ratio permits the simple determination of the natural frequency of the interaction structure, the constant multiplication factor for conversion to the equivalent SDF model, and the equivalent damping ratio.

For massive structures with a low height-to-base ratio, the relative base displacement as well as rocking motion influence the frequency reduction, and consequently the ground-structure interaction effects are also influenced significantly.

From a study of resonance peak amplitudes it is shown that under steady-state disturbances the interstory displacement of the interaction structure can be larger than that for an SDF with the same resonance frequency and interstory damping. This is the case for tall slender structures and for values of the nondimensional frequency a_0 up to about 0.6 with the Bycroft foundation. In contrast, for interaction structures under random-type excitations the interstory displacement can be expected to be equal to or less than that for an SDF oscillator with the same natural frequency and interstory damping.

ACKNOWLEDGMENT

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APPENDIX II.—NOTATION

The following symbols are used in this paper:

A = dynamic stiffness for horizontal displacement on half space;

$a = pr/V_S$ = nondimensional frequency;

$a_0 = \omega_0 r/V_S$ = nondimensional SDF resonance frequency;

B = dynamic rocking stiffness on half space;

C_H, C_R = SDF damping coefficients for horizontal and rocking displacements, respectively;

c = interstory damping coefficient;

f_{1H}, f_{2H}, f_{1R}, f_{2R} = variables for steady-state dynamic behavior of weightless disc on elastic half space for horizontal and rocking motion;

G = shear modulus of ground;

g = subscript denoting ground;

h = story height;

$I = I_0 + I_1$ = total moment of inertia, also subscript designating interaction system;

I_0 = moment of inertia of top and bottom mass;

I_1 = second moment of mass m_1 about base = $m_1 h^2$;

K_H, K_R = SDF stiffness coefficients for horizontal and rocking displacements, respectively;

k = story stiffness;

k_Φ = static rocking stiffness;

M, M_0 = moment on base under arbitrary and steady-state motion, respectively;

M_e = peak magnitude of frequency response for equivalent SDF model;

M_I, M_S = peak magnitude of frequency response for interaction system and SDF oscillator, respectively;

m_0 = base mass;

m_1 = top mass;

P, P_0 = horizontal force on base under arbitrary and steady-state motion, respectively;

p = frequency, in radians per second;

r = radius of base;

S = subscript designating SDF system;

S_D = spectral displacement;

T = period, in seconds;

$\mathrm{T}_{u_g}^{d}$ = transfer function for displacement vector d and ground displacement u_g;

U_B = total base displacement of interaction system;
U_H = total displacement of top mass of interaction system;
U_m = relative interstory displacement of interaction system or SDF system;
$u_g, \ddot{u}_g$ = steady-state ground displacement and acceleration, respectively;
u = relative horizontal displacement of base mass with respect to free-field ground motion;
V_s = shear wave velocity of ground;
W = amplitude of steady-state ground disturbance;
X, Y, Z = complex amplification factors for base displacement, rocking and interstory displacement, respectively;
$\alpha = m_0/m_1$ = mass ratio;
$\beta = m_1/\rho r^3$ = nondimensional top mass;
$\eta = (h/r)^2$ = aspect ratio squared;
λ = relative interstory damping ratio;
λ_H, λ_R = relative damping ratios for horizontal and rocking motion, respectively, of base mass;
λ_e = equivalent SDF damping ratio;
ν = Poisson's ratio of ground;
ρ = mass density of ground;
Φ = angular variable, in radians;
Ω = natural frequency of SDF oscillator, in radians per second;
ω_H = resonant frequencies of disc on elastic half space for horizontal motion, in radians per second;
$\omega_R = I_1 = m_1 h$ = resonant rocking frequency for circular structure with moment of inertia, in radians per second;
$\omega_0 = (k/m_1)^{1/2}$ = natural frequency of fixed-base structure, in radians per second;
ω_1 = fundamental frequency of interaction system, in radians per second; and
$\omega_\Phi = (k_\Phi/I)^{1/2}$ = rocking frequency.

8422 STRUCTURE-GROUND INTERACTION IN EARTHQUAKES

KEY WORDS: dynamics; earthquakes; elasticity; engineering mechanics; foundations; responses; structural analysis; structural engineering; vibration

ABSTRACT: A method of analysis is presented for determining elastic structure-ground interaction effects of single-story structures under earthquake loads. The method derives an equivalent SDF model for the relative story displacement, incorporating the effects of horizontal and rotational base flexibility in a structure. The equivalent SDF model then permits the determination of the seismic interaction response directly from established or assumed response spectra, or from response calculations of ordinary SDF systems. Sample calculations are presented showing responses for the interaction system, the equivalent SDF model, and an SDF system with the same natural frequency as the interaction structure. A study of a wide range of parameters under earthquake-type disturbances establishes that for the interaction systems considered interstory displacements are reduced in relation to an SDF system with identical fundamental frequency and interstory damping.

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