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ANALYZED

PROBE for THERMAL CONDUCTIVITY MEASUREMENT OF DRY and MOIST MATERIALS

by
W. WOODSIDE

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Probe for Thermal Conductivity Measurement of Dry and Moist Materials

By W. Woodside*, Ottawa, Canada

ANALYZED

THE TRANSIENT probe or needle method for the measurement of thermal conductivity was originated by Stalhane and Pyk in 1931¹. It was later developed by Van der Held and Van Drunen² who applied the method to the determination of the thermal conductivity of liquids.

The method utilizes a line heat source, i.e., a straight wire through which constant electric current may be passed and a temperature sensitive device, e.g., a thermocouple or resistance thermometer. The 2 elements are embedded alongside each other in the material under test. When the assembly is at a uniform and constant temperature a constant power input is supplied to the heater element and the rise in temperature is recorded during a short heating interval. The rate of rise of temperature is determined by the ability of the test material to conduct heat away from the line heater. The thermal conductivity may be calculated from the temperature-time record and the power input.

The method was further developed by Hooper and Lepper³ and Hooper and Chang⁴ who enclosed the 2 elements in a protective aluminum tube 3/16-in. in diameter and approximately 19 in. in length. This was the first actual probe. The reproducibility of this instrument was said to be 0.5 percent, and it was stated that the probe could measure the conductivity of both dry and wet materials with equal facility and precision. De Vries⁵ also described a probe suitable for measurement of thermal conductivity of soils *in situ*, and now the method is widely used for soil resis-

SUMMARY—The development of the probe or transient *needle* method for thermal conductivity measurement is reviewed. The theory of the method, sources of error and an improved technique for recording the probe temperature rise are described. Experimental results for dry silica aerogel show good reproducibility and agreement with guarded hot plate measurements. Results for snow show a discontinuous variation of conductivity during test. The measured conductivity continuously decreased during tests on moist sawdust and moist clay, suggesting a loss of moisture from the specimen in the vicinity of the probe. This is verified by calculation of the moisture migration caused by the large temperature gradients at the probe surface. It is suggested that temperature gradients, and hence moisture redistribution, in probe tests may be reduced by increasing the probe radius, contrary to the present trend in probe development which is toward smaller probes.

tivity surveys along proposed routes for buried electric cables.

Van der Held and Van Drunen², Lentz⁶, Vos⁷ and Blackwell⁸ have investigated errors in the probe method. The theory of the method is based on the assumption of a perfect line heat source of infinitesimal diameter, and hence errors arise due to the finite diameter of the probe. In an endeavour to reduce errors of this kind to a negligible level, D'Eustachio and Schreiner⁹ constructed a probe approximately 4 in. long with an outside diameter of 0.030 in. Results for dry materials were in good agreement with measurements made by the conventional guarded hot plate method. The probe used in the present work is similar to that used by D'Eustachio

and Schreiner, but with an even larger length to diameter ratio.

Joy¹⁰ used a thin probe to determine conductivities of 3 insulating materials at a variety of moisture contents and at 2 mean temperatures (80 and 8 F). Both thermocouple and resistance thermometer-type probes were used. Joy obtained reproducibility of ± 1 percent for tests on dry materials, and ± 6 percent for tests on moist materials. He concluded that the probe method is suitable when the test specimen is uniformly wetted. The resistance thermometer probe had more integrating ability than the thermocouple type in the cases of non-uniformly wetted specimens.

Mann and Forsyth¹¹ described the construction of 4-in. long probes using hypodermic needle tubing (0.055-in. outside diameter). More recently, Lachenbruch¹² described a probe, containing a thermistor as the temperature-sensing element, for use in the determination of thermal conductivity of frozen soils *in situ*. Saare and Wenner¹³ investigated the effect of the radial position of the thermocouple inside the probe.

Experimental Procedure

The test material was placed, at the desired density, in an aluminum cylinder open at one end. The internal diameter of the cylinder was 6 in. and the internal height was 14.5 in. so that when the probe was inserted in the material at the axis of the cylinder, it was surrounded by 3 in. of material. The reference junction of the probe thermocouple was attached to the outer surface of the cylinder at mid-height. This assembly was placed centrally inside a larger cylinder of internal diameter 1 ft and height 2 ft. The annular space between the 2 cyl-

*Building Services Section, Division of Building Research, National Research Council.

¹Exponent numerals refer to References.

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inders was filled with granulated rock wool insulation. To prevent gain or loss of moisture from the test material, the top of the cylinder was covered with polythene film.

The system was allowed to attain thermal equilibrium before each test. This is indicated by zero emf in the thermocouple circuit.

To take full advantage of the entire width of the recorder chart, a portable precision potentiometer was used to provide a constant bias emf in the thermocouple circuit. Thus the recorder operates as zero-right or zero-left instead of zero-centre. The initial temperature rise from 0 to 2 minutes was not used to calculate thermal conductivity. Since the greater part of the absolute temperature rise occurs during these first 2 minutes, it is convenient to use the potentiometer emf source to bias out the initial rise, increase the preamplifier gain by a factor of 4, and re-adjust the bias emf so that at approximately 1.8 min the recorder pen is at zero. With this procedure, used only in the more recent tests, the temperature-rise curve between 2 and 10 min after the start of the test, i.e., the interval used for calculation of conductivity, is recorded on the full 9.5-in. width of the chart. This is possible since the calculation requires only the difference in temperature rises at two times, and not the absolute values of the rises. A typical recorder trace is shown in Fig. 1. Fig. 2 shows a recorder trace from a test in which the bias arrangement just described was used.

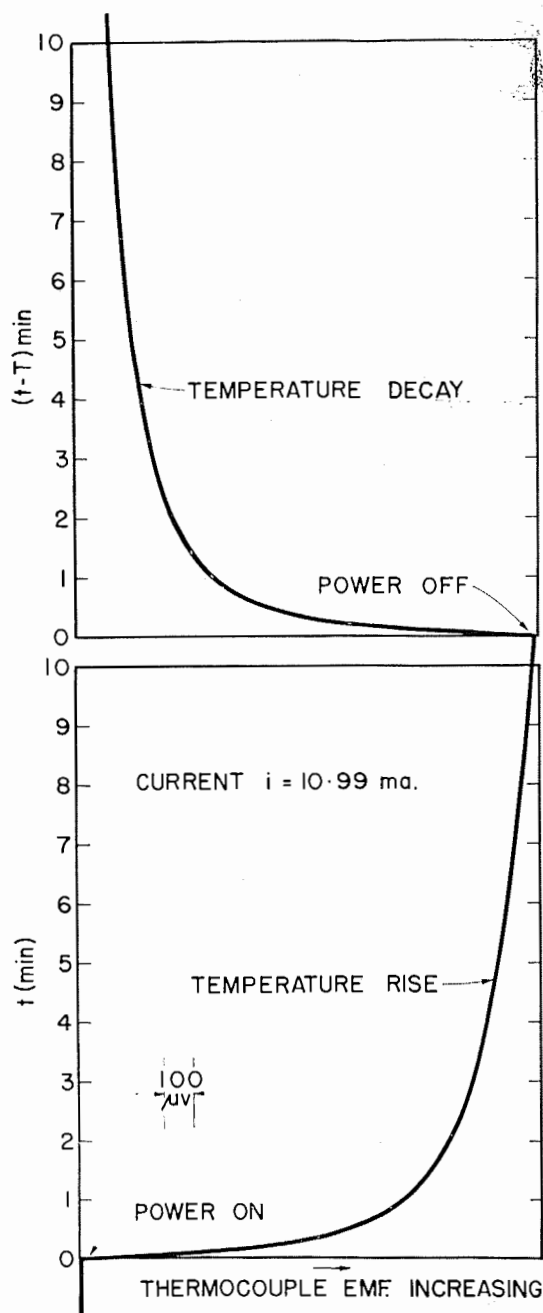
Experimental Results for Silica Aerogel

Dry silica aerogel was packed in the test container at a density of 5.20 lb per cu ft. Fig. 1 shows the recorder trace obtained from a test for which the heating current was $i = 10.99$ mA. The preamplifier gain was $\times 5$ (scale multiplier $\times 40$). The power input corresponding to this probe current is $Q = 0.3260$ watt per ft.

Fig. 3 shows the temperature rise θ plotted against $\ln(t + t_0)$, t_0 being -0.25 min. The results fall on a straight line for t as low as 0.5 min.

values of $[(Q/4\pi k)[C' + \ln(t + t_0)] - \theta$ were plotted against $[\ln(t - T + t_0)]$, (Fig. 3). The 2 sets of results are almost superimposed.

Fig. 1 — Recorder trace of probe temperature rise and decay for test on dry silica aerogel



The slope of the straight line (temperature-rise data) is 5.325, giving a value for k of 0.199₅ Btu in. per (hr) (sq ft) (F deg). In this test the power was switched off at time $t = T = 10.5$ min, and the temperature decay was also recorded. From the extra-polated temperature-rise data,

The slope of the straight line (temperature-decay data) is 5.410, and the calculated thermal conductivity is 0.196₃ Btu in. per (hr) (sq ft) (F deg) agreeing very well with the conductivity determined from the temperature-rise data. The lower conductivity for the decay curve may be partly

explained by the lower mean temperature during the measurement interval of the decay curve.

Tests were also performed on silica aerogel with different power inputs. The results are shown in Table 1.

The measured conductivities in the second column of the table were calculated from Equation A-4 between

Table 1 — Variation of measured conductivity with power input

Power input Q , watt/ft	Measured conductivity Btu in./hr (sq ft) (F)	Mean Temperature, F	Max. temp. gradient at probe surface, F deg per in.
0.00366	0.165	70.6	13.7
0.00632	0.169	71.0	23.5
0.01400	0.185	72.1	52.2
0.02927	0.186	74.2	109.1
0.05168	0.190	77.3	192.6
0.3260	0.199s	129.4	1214.0

than that normally experienced with guarded hot plate measurements: Van der Held has also observed that in the case of materials easily penetrated by thermal radiation, thermal conductivities measured by a transient method such as the probe are higher than conductivities measured by steady-state methods. In a series of

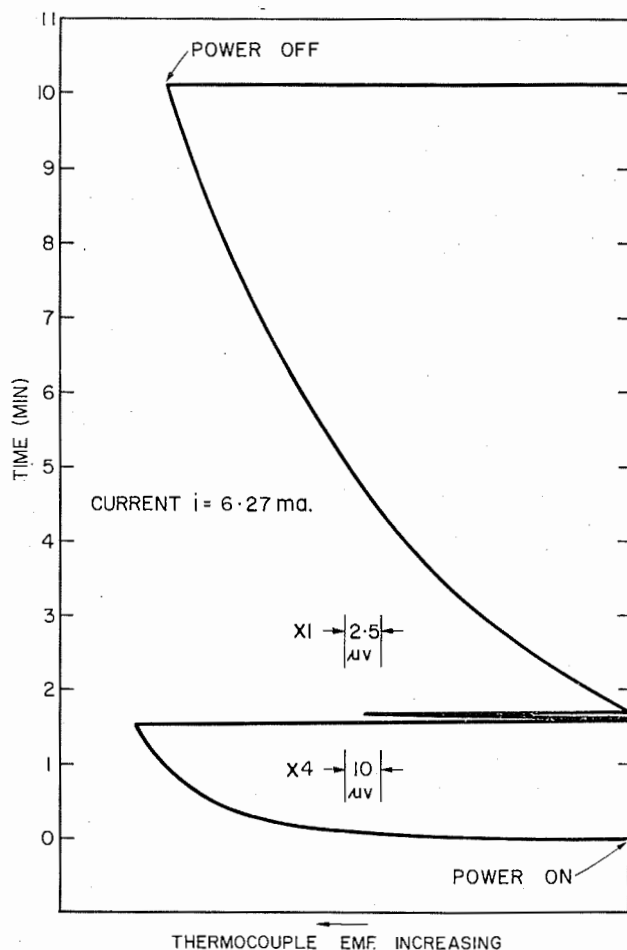


Fig. 2 — Recorder trace obtained with use of bias emf.

times of 2 and 10 min. The values shown are the averages of 7 values calculated during this time interval. The mean temperatures shown in the third column are calculated, assuming an initial temperature of 70 F, and taking the average of the temperature rises at 2 and 10 min.

*Equation A-4 is developed in the Appendix, and is $k = 0.649_2 \{ \bar{r}^2 \log [(t_2 - 0.25)/(t_1 - 0.25)] \} / (V_2 - V_1)$. In the Appendix also are Equations A-1, A-2, A-3 and A-5. It is planned to publish the Appendix with the paper in the TRANSACTIONS.

Conductivity of silica aerogel at 70 F and density of 5.57 lb per cu ft has also been measured with an 8- by 8-in. guarded hot-plate apparatus, the value being 0.167 in good agreement with the probe values 0.165 and 0.169 (Table 1).

The increase in measured conductivity with increasing power input may at least be partly attributed to the increase in mean temperature. However the increase is much greater

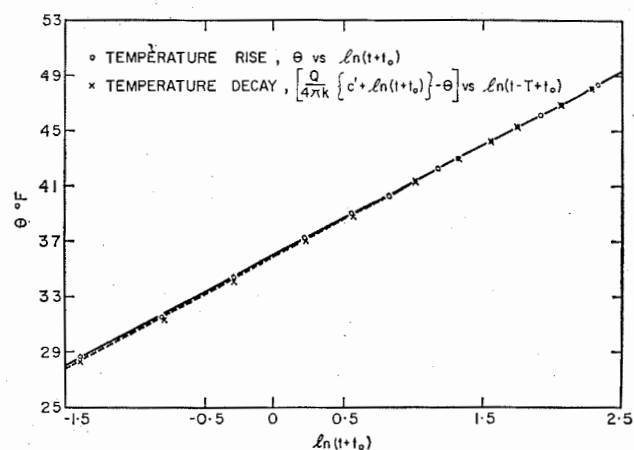


Fig. 3 — Temperature rise and decay plotted against the logarithm of the time for test on silica aerogel

papers^{14, 15, 16} Van der Held has discussed the contribution of radiation to heat transfer in materials under both transient and steady-state conditions. It appears therefore that radiative heat transfer in silica aerogel

Nomenclature

- θ = temperature rise of probe, Fahrenheit.
- Q = probe heat input per unit length, Btu per (hour) (foot).
- r = radial distance from probe axis, feet.
- r_0 = probe radius, feet.
- t = time from start of heating, hour.
- T = duration of probe heating, hour.
- t_0 = probe time correction, hour.
- τ = time for complete drying of volume element of specimen adjacent to probe, hour.
- V = probe thermocouple emf corresponding to temperature rise, microvolts.
- i = probe heating current, milli-amperes.
- p = water vapor pressure in pores of specimen, inches mercury.
- k = thermal conductivity of test specimen, Btu per (hour) (foot) (Fahrenheit degree).
- a = thermal diffusivity of test specimen, square feet per hour.
- ρ = dry density of test specimen, pounds per cubic foot.
- c = specific heat of test specimen, Btu per (pound) (Fahrenheit degree).
- μ = water vapor permeability of test specimen pounds per (hour) (foot) (inches mercury).
- m = initial moisture content of test specimen, fraction of dry weight.

is appreciable under transient conditions.

Thermal diffusivity α of the test specimen may be estimated from the probe data, providing the effective radial distance, r , between the probe heater and thermocouple is known. It will be assumed that $r = 0.01$ in. (outside diameter of probe = 0.020 in.). From Fig. 3, it may be seen that when $\ln(t + t_0) = 0$,

$$\theta = (Q/4\pi k) C' = 36.00$$

since

$$Q/4\pi k = 5.325, C' = 6.76$$

but

$$C' = -0.577 - \ln(r^2/4\alpha),$$

therefore

$$r^2/4\alpha = 6.51 \times 10^{-4},$$

and

$$\alpha = 0.016 \text{ sq ft per hr.}$$

Wilkes and Wood¹⁷ give for silica aerogel at 147 F a value for α of 0.020 sq ft per hr, the density ρ being 4.0 lb per cu ft. Since the density in the probe tests was 5.2 lb per cu ft, and $\alpha = k/\rho c$ the probe diffusivity value agrees well with the value given by Wilkes and Wood.

Experimental Results for Moist Sawdust

Several experiments were performed on moist sawdust with the probe, the moisture content and the probe power input being varied. Equation A-4 for the calculation of thermal conductivity from probe data is based upon heat conduction theory. It is known that heat transfer processes other than pure conduction occur in moist porous materials under a temperature gradient. However, it is assumed that the equation may be applied to the calculation of an effective thermal conductivity in the case of moist materials. In the tests on moist sawdust, the measured conductivity was found to vary during the time of the test. Fig. 4 shows a typical plot of conductivity vs. time after start of test. The conductivities

shown on this graph are those calculated for times 0.5 to 1.0 min, 1.0 to 1.5 min, 2.0 to 3.0 min, 3.0 to 4.0 min, and so on. The conductivity apparently has a high initial value, quickly falling to a more or less constant value. This effect may be caused by the partial drying out of the test specimen in the immediate vicinity of the probe. Results are shown in Table 2. The conductivities tabulated are average values for the interval 2.0 to 10.0 min.

Experimental Results for Snow

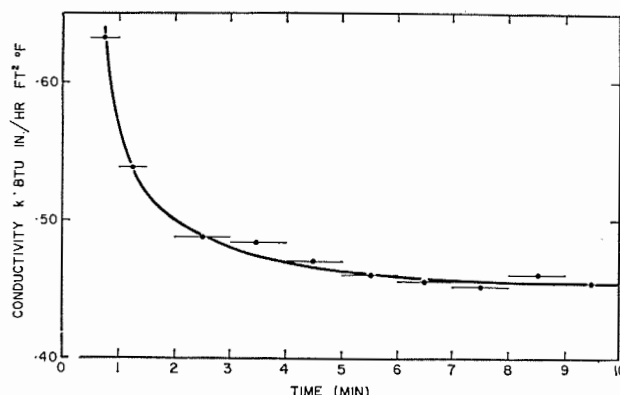
For this series of tests, the test assembly was placed in a cold room¹⁸,

are the averages of 7 values calculated during the interval between 2 and 10 min from the beginning of each test. The initial temperatures given in the first column of the table are the air temperatures in the cold room during each test.

Table 2 — Conductivity results for sawdust

Dry density lb per cu ft	Moisture content % dry wt	Power input watt per ft	Mean temp F	Measured conductivity Btu in. per (hr) (sq ft) (F deg)
13.00	3.14	0.1017	78.2	0.468
13.00	3.14	0.2425	83.5	0.471
13.00	3.14	0.5330	103.0	0.479
13.00	6.32	0.1186	79.0	0.505
13.00	6.32	0.2631	87.0	0.505
13.00	6.32	0.5838	105.0	0.508
13.84	12.8	0.1063	77.0	0.649
13.84	12.8	0.3223	85.4	0.635
13.84	12.8	0.7793	107.0	0.621

Fig. 4 — Variation of measured thermal conductivity with time during probe test on moist sawdust



the probe power and thermocouple leads being taken out to an adjacent laboratory where the auxiliary equipment was situated. During the majority of these tests, the cold room was operated at a temperature of +15 F; the power input to the probe was limited in order that the temperature rise at the probe should not exceed 10 F deg.

Snow was first passed through a No. 6 sieve (opening 0.131 in.) and then placed in the test cylinder at a density of 17.83 lb per cu ft (0.286 gm per cc). Results of the tests on snow are shown in Table 3.

The measured conductivity varied appreciably from one test to the next, and also during the course of a single test. The measured conductivities reported in the last column of the table

Table 3 — Thermal conductivity of snow (0.286 gm per cc at different temperatures)

Initial temp F deg	Power input watt per ft	Tempera- ture rise F deg	Maximum temp gradient F deg per in.	Measured conductivity Btu in. per (hr) (sq ft) (F deg)
+15	0.242	6.0	110	1.51
+15	0.054	1.3	27	1.39
+15	0.054	1.3	27	1.54
+15	0.237	6.0	110	1.59
+13	0.329	7.5	161	1.40
+13	0.270	6.0	185	1.01
-5	0.237	6.0	110	1.16
-5	0.542	13.3	202	1.84
-33	0.229	5.0	99	1.61
-33	0.231	5.0	99	1.63

On averaging the conductivities obtained for the several tests at each temperature, the results indicate that the thermal conductivity of snow increases with decreasing temperature. Jakob¹⁹ reports that the thermal conductivity of ice increases with decreasing temperature, and is 15.5 Btu in. per (hr) (sq ft) (F deg) at

32 F, 17.3 at -13 F, and 19.2 at -58 F. Thus, although the effective thermal conductivity of the air in the pore spaces in snow decreases with decreasing temperature, the net effect is an increase in the conductivity of snow with decreasing temperatures.

In the probe tests on moist sawdust the measured conductivity varied con-

tinuously with time during test. This corresponds to 3 different values of thermal conductivity: from 1.8 to 4.5 min $k = 1.21$; from 4.5 to 14.7 min $k = 1.85$; and from 14.7 to 25 min $k = 1.16$ Btu in. per (hr) (sq ft) (F deg). The times when a change in conductivity occurs are not the same from one test to the next and,

values shown in Table 3. The values of α range between 0.0132 and 0.0151 sq ft per hr.

Experimental Results for Moist Leda Clay

Results of probe measurements on dry Leda clay are reported by Woodside and de Bruyn²¹. Two measurements were made on Leda clay at a moisture content of 10.2 percent by dry weight, the dry density being

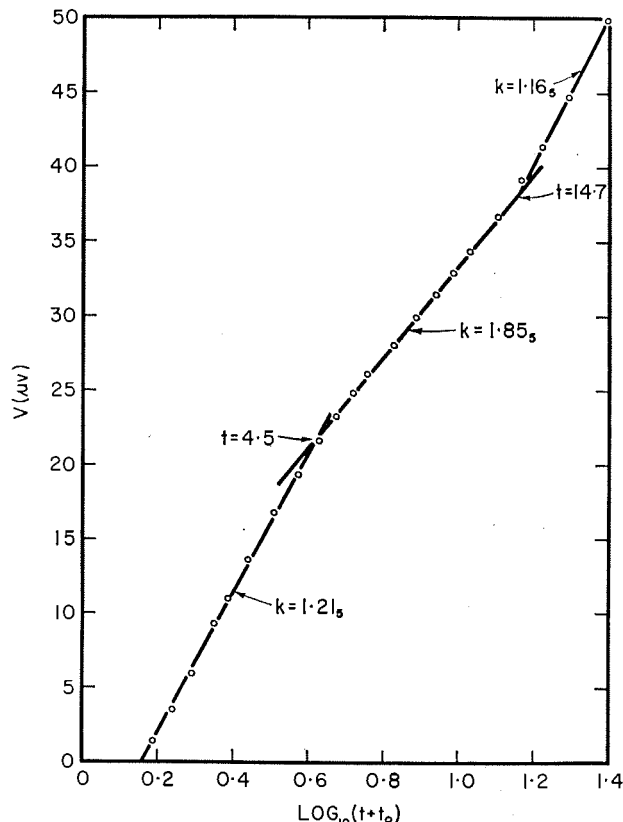


Fig. 5 — Probe thermocouple emf plotted against log (time) for test on snow

tinuously with time during test. A variation in conductivity during test was also observed in the tests on snow; the variation was discontinuous, however. Fig. 5 shows the microvolt reading of the probe thermocouple plotted against $\log(t + t_0)$ for a probe test on snow which was allowed to run for 25 min instead of the usual 10. For a normal material this graph would be a straight line, (e.g., Fig. 3 for silica aerogel) the slope of which determines the thermal conductivity. In the present case, 3 straight lines each of a different slope, may be drawn

as is evident from Fig. 5, the transition from one value of conductivity to another appears to be a sharp one.

Yosida²⁰ observed this same phenomenon during measurements of the thermal conductivity of snow, the method of measurement being very different from the one described here. The tests lasted about 30 min. Yosida attributed these changes in conductivity to changes in the snow structure brought about by the heat flow through the snow.

The thermal diffusivity α also may be calculated from the conductivity

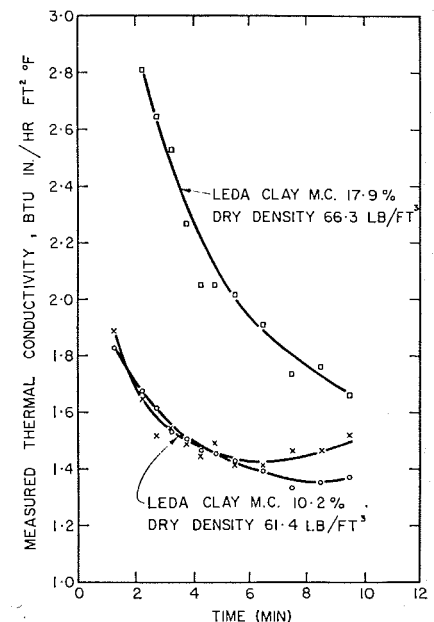


Fig. 6 — Variation of measured thermal conductivity with time during probe tests on moist Leda clay

61.4 lb per cu ft. The average measured conductivities were 1.46 and 1.47 Btu in. per (hr) (sq ft) (F deg). The variation of measured conductivity with time for these 2 measurements is shown in Fig. 6.

Five measurements were performed on Leda clay at a moisture content of 17.9 percent by dry weight, the dry density being 61.4 lb per cu ft. The average measured conductivities varied from 1.52 to 2.13 Btu in. per (hr) (sq ft) (F deg), the conductivity apparently increasing from one test to the next. An example of the very large variation in conductivity during the course of a single test is shown in Fig. 6.

Discussion

Probe measurements on dry materials: The thermal conductivity probe method appears to be satisfactory for measurements on dry materials. The tests performed on dry silica aerogel gave results with excellent reproducibility, in good agreement with guarded hot plate data.

Several advantages of the probe method over the conventional guarded hot plate method are apparent. Testing time is of the order of 15 min compared with at least 24 hr for normal hot plate measurements. This does not mean, however, that 4 tests may be performed in 1 hr since the sample must be allowed to attain thermal equilibrium before the start of each test. The thermal diffusivity of the specimen may be estimated from probe data provided that the radial distance between thermocouple and heater is known.

Probe measurements may be performed with little auxiliary equipment, the basic necessities being a storage battery, portable potentiometer, stop watch and dewar flask. However, the measurements are more accurate and more convenient when a recording potentiometer with pre-amplifier and a variable bias emf source, such as those described, are available. Compared with guarded hot plate equipment, the apparatus is compact, portable and relatively inexpensive. In some instances, probe measurements might be performed on a test material *in situ*.

Tests may be performed with only a small temperature rise; for example, the temperature rise in several of the tests on silica aerogel was less than 1 F. This is an advantage for determining the thermal conductivity of materials at different mean temperatures, especially when the k vs temperature variation is non-linear. In the guarded hot plate, temperature differences of 30 to 40 F are normally applied across the specimens.

All the tests described in this paper have been performed on granular materials, and as the probe was easily inserted good thermal contact was obtained. In the case of non-granular materials, e.g., wood, concrete, and fibrous insulation, it would be more difficult to obtain good thermal contact. Also many insulating materials are anisotropic, e.g., wood, fiberboard, and many rock wool batt insulations. Such materials have vastly different thermal conductivities in directions parallel to and perpendicular to the *grain* or plane in which most of the fibers are oriented. Since the probe is a radial heat flow method, such materials introduce special problems. Specimens may be arranged so that tests may be performed first with the probe perpendicular (k_1) to the direction of the fibers of the material and, secondly, with it parallel (k_2) to the fiber direction. The conductivity k for heat flow perpendicular to the plane in which the fibers are oriented is then $k = (k_2)^2/k_1$. This procedure has been successfully applied to fiberboard by Joy¹⁰.

The length to diameter ratio of a probe should be high so that the instrument approximates an ideal line heat source. Large probes require large specimens, however, and small probes tend to be comparable in diameter to the particles or cells of some porous materials.

Probe measurements on moist materials: In the determination of the thermal conductivity of moist materials, many problems arise. Most investigators in this field realize that moisture movement is not eliminated in probe measurements¹⁰ but assume that the moisture redistribution during the short testing interval is negligible^{3, 4, 5, 9}. For example, Hooper and Lepper³ have stated that examination of the specimens following test indicated no substantial alteration in the moisture distribution. This is in marked contrast to measurements on moist materials with the guarded hot plate apparatus and other steady-state

methods for measuring conductivity^{21, 22, 23}. If a moisture redistribution should develop during a probe test, however, it would be difficult to detect since the greater part of the moisture content gradient would occur close to the probe surface.

The decrease in measured conductivity with time during the test interval, which was observed in the tests on moist sawdust and moist clay, indicates that (a) some drying out of the specimen close to the probe does occur; and/or (b) application of the heat conduction theory to the heat transfer mechanism which occurs in moist porous media is not justified.

Since the layers of test specimen close to the probe have the largest effect on the conductivity measured by the probe, any drying out of these layers would produce a large decrease in the measured conductivity. It would seem to be of interest, therefore, to calculate the rate of vapor flow induced by the temperature gradient. Such a calculation of necessity involves several assumptions and approximations.

The water vapor flow is given by the equation (Fick's law) $W = -\mu A dp/dr$, where μ is the vapor permeability of the specimen, A is the effective area for flow, and (dp/dr) is the vapor pressure gradient in the pore spaces of the specimen. Consider an element of test specimen in the shape of a concentric cylindrical shell surrounding the probe, of unit length and radius r . Both A and (dp/dr) will be evaluated at the outer surface of this shell. Thus $A = 2\pi r$.

Now

$$dp/dr = (d\theta/dr) (dp/d\theta)$$

But by Equation A-5 the steady-state temperature gradient is

$$d\theta/dr = -Q/2\pi rk$$

This is the overall temperature gradient in the material at radius r . The temperature gradient in the pore spaces of the material, however, may

be many times greater than this overall gradient^{24, 25}, and it is the gradient in the pores which determines the corresponding vapor pressure gradient. For the present purpose, however, the overall gradient will be used. Therefore,

$$\begin{aligned} W &= \mu 2\pi r (Q/2\pi rk) (dp/d\theta) \\ &= \mu (Q/k) (dp/d\theta) \end{aligned}$$

The initial weight of moisture contained in the volume element is

$$M = \pi (r^2 - r_0^2) \rho m$$

where r_0 is the probe radius, ρ is the dry density of the test specimen, and m is the initial moisture content of the specimen expressed as a fraction of the dry weight.

Thus the time τ required to dry out completely the volume element considered is

$$\begin{aligned} \tau &= M/W \\ &= [\pi (r^2 - r_0^2) \rho m k] / [\mu Q (dp/d\theta)] \end{aligned}$$

If a volume element of radius $2r_0$ is considered,

$$\tau = [3 \pi r_0^2 \rho m k] / [\mu Q (dp/d\theta)] \quad \text{..(1)}$$

This equation is now applied to the probe test on Leda clay with 10.2 percent moisture content by dry weight and 61.4 lb per cu ft dry density. The variation of measured conductivity with time for this test is shown in Fig. 6, and $r_0 = 0.01$ in., $m = 0.102$, $\rho = 61.4$ lb per cu ft. The vapor permeability μ of this clay was measured by the dry cup test, yielding a value for μ of 33.9 perm. in. (grains in. per (hr) (sq ft) (in. Hg)). If the relative humidity within the pores of the clay is 100 percent at 85 F which is the mean temperature of the probe test, then

$$(dp/d\theta) = 0.0394, \text{ in. Hg per F deg}$$

Substituting these values in Equation 1, the value for τ is found to be 10.8 min. Thus the first layer of clay (of thickness equal to the probe radius)

should lose all its moisture in approximately 11 min., and hence the measured thermal conductivity should decrease appreciably during the 10-min. test interval, as was observed.

Since the foregoing calculation leads to a conclusion contrary to the view now held by many workers in this field, the assumptions involved in the calculation are re-examined.

1. It was assumed that the steady-state temperature gradient is applied instantaneously and maintained during the test. The gradient at the probe surface is essentially the steady-state gradient as soon as the higher terms in the $I(x)$ series may be neglected, which is after 0.5 min for the probe described.

2. The calculation neglected the fact that as the volume element dries out, the relative humidity and hence the vapor pressure in the pores, decreases, thus reducing the flow.

3. It has been assumed that the only mechanism for moisture flow is thermally actuated vapor diffusion. Capillary forces, however, may tend to return liquid moisture to the drying regions.

4. Finally, as stated before, the temperature and vapor pressure gradients in the pore spaces may be several times higher than the corresponding overall gradients.

The effects due to 1, 2 and 3 tend to be wholly or partially compensated for by 4, so that the calculation leads to a result which is at least correct to an order of magnitude.

It may be concluded, therefore, that in some instances thermal conductivity probe measurements may produce moisture content gradients in the vicinity of the probe just as severe as those produced by steady-state hot plate measurements. In measuring thermal conductivity of moist materials, the probe sets up large temperature gradients for a short duration, whereas the guarded hot plate imposes relatively low gradients for a long duration. The ideal method, which would maintain as uniform a moisture distribution as possible, would appear to be one which is fast and uses small temperature gradients.

Equation 1 indicates that τ will be small, and hence the moisture content

gradients set up will be large, when (a) probe radius is small; (b) power input to the probe is large; (c) mean temperature is high (i.e., when $dp/d\theta$ is large); and (d) conductivity of the test specimen is small.

Factors (a), (b) and (d) also lead to high temperature gradients in the specimen. Thus it appears that moisture content redistribution in probe tests may be reduced by increasing the size of the probe and by using smaller power inputs.

From the point of view of probe design, the theory requires that the probe radius be as small as possible; for testing moist materials, however, it appears desirable that the probe radius be large. Thus the recent trend in probe development towards smaller probe diameters (large length to diameter ratios) appears to be in the wrong direction for the testing of moist materials.

Conclusions

1. It has been shown that the temperature gradients set up in the vicinity of a thin probe during test are exceedingly high, being of the order of hundreds of Fahrenheit degrees per inch.

2. The results of probe tests on dry silica aerogel have shown that both reproducibility and agreement with guarded hot plate results are good (better than 1 percent) when low probe power inputs are used.

3. Results of tests on moist sawdust have shown a steady decrease in measured conductivity during the 10-min test interval.

4. Results of tests on snow have shown a variation in measured thermal conductivity during test similar to that observed by Yosida who used an entirely different method of measurement.

5. In tests on moist clay, a steady decrease in measured conductivity during test was observed.

6. The probe method appears satisfactory for dry materials. The steady decrease in measured conductivity during test observed in the measurements on moist sawdust and moist clay suggests a steady drying out of the material in the vicinity of the probe.

7. A rough calculation indicates that in the test on 10 percent moisture content clay, a layer of clay equal

in thickness to the probe radius, should completely dry out during the short 10-min test. Thus the moisture redistribution during test does not appear to be negligible as is commonly assumed, at least for tests using thin probes.

8. One way by which the temperature gradients and hence moisture redistribution in probe tests, may be reduced is to increase the probe radius, contrary to the present trend in probe development which is toward smaller and smaller probe sizes. Since errors due to the finite size of a probe may be corrected for, it would seem advantageous to use larger probes and avoid some of the difficulties in testing moist materials.

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