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Modification of Seismic Input for Fully Discretized Models

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Résumé

La méthode des éléments finis et la solution des équations de mouvement dans le domaine temporel servent souvent à l'analyse sismique des problèmes d'interaction sol-structures. Dans bien des cas, il est raisonnable de supposer que la base du modèle à éléments finis est rigide et, par conséquent, d'appliquer la charge sismique sous forme d'accélérogramme à la base. Les accélérogrammes de base sont calculés à partir des accélérogrammes témoins disponibles, habituellement enregistrés à la surface, qui utilisent l'analyse de déconvolution basée sur les modèles continus de sol. Cependant, contrairement aux prévisions théoriques, lorsque les accélérogrammes de déconvolution sont introduits à la base du modèle à éléments finis analysé dans le domaine temporel, ils ne reproduisent pas exactement le mouvement témoin initial. Cet écart est attribuable aux caractéristiques d'erreur du modèle discret, par exemple la fréquence de coupure, la dispersion et les réflexions parasites.

Les auteurs proposent une méthode simple pour résoudre en partie le problème en modifiant l'accélérogramme de base au moyen d'un modèle unidimensionnel ... que celles du modèle d'i... visqueux à la base. On ... ant à la verticale.

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TECHNICAL NOTE

MODIFICATION OF SEISMIC INPUT FOR FULLY DISCRETIZED MODELS

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ABSTRACT

The finite element method together with the solution of the equations of motion in the time domain are often used for seismic analyses of soil-structure interaction problems. In many situations it is reasonable to assume that the base of the finite element model is rigid and hence apply the seismic loading in the form of an accelerogram at the base. Base accelerograms are calculated from available criteria accelerograms, usually recorded at the surface, employing deconvolution analysis based on continuum soil models. However, contrary to theoretical predictions, when the deconvolved accelerograms are input at the base of the finite element model analyzed in the time domain, they do not exactly reproduce the original criteria motion. This discrepancy is attributed to the error characteristics of the discretized model such as cut-off frequency, dispersion and spurious reflections.

A simple method is proposed to partly overcome this problem by modifying the base accelerogram employing a one-dimensional soil model which has the same error characteristics as those of the interaction model. This model employs a viscous dashpot at the base. Vertically propagating shear waves are considered.

Key words: computer application, dynamic, earthquake, finite element method, wave propagation (**IGC:** E 8)

INTRODUCTION

The finite element method together with the numerical solution of the equations of motion in the time domain are often used for seismic analyses of soil-structure interaction problems. In many situations, it is reasonable to assume that the base of the finite element model is rigid and hence to apply the seismic loading in the form of an accelerogram at the base. Base accelerograms are determined from available criteria accelerograms, usually recorded at the surface, using deconvolution analysis based on continuum soil models. Contrary to

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theoretical predictions, however, when accelerograms are input at the base of the finite element mesh without the structure, they do not exactly reproduce the original criteria motion at the surface. This is because of one or more of the following factors:

- Upper limit of frequency that a mesh can reproduce accurately
- Dispersion characteristics of the finite elements
- Spurious reflections due to mesh gradation
- Dispersion and accuracy characteristics of the time integration schemes

In the following sections a simple method is proposed to partly overcome this problem by modifying the base accelerogram using a one dimensional soil model having the same error characteristics as those of the interaction model. The one dimensional model, however, has a viscous dashpot attached to its base. Vertically propagating shear waves are considered.

1-D SHEAR WAVE PROPAGATION

For vertically propagating shear waves in an elastic homogeneous medium, the horizontal displacement u, a function of time t and depth x, is governed by the following partial differential equation

$$\partial^2 u/\partial t^2 - V_s^2 \partial^2 u/\partial x^2 = 0 \qquad (1)$$

where $V_s = \sqrt{G/\rho}$ = *shear wave velocity*, G and ρ are the material shear modulus and density, respectively. The general solution of Eq. (1) for harmonic motion of frequency ω can be expressed as

$$u(x, t) = E \exp[i(kx+\omega t)] + F \exp[-i(kx-\omega t)] \qquad (2)$$

and the corresponding shear stress is

$$\tau(x, t) = \rho V_s i\omega [E \exp(ikx) + F \exp(-ikx)] \exp(i\omega t) \qquad (3)$$

where $k = \omega/V_s$ = *wave number*. For a horizontally layered medium, Eqs. (2) and (3) are valid for each layer and x becomes the local depth coordinate of the layer. Coefficients E and F are determined by satisfying the zero stress condition at the free surface and displacement and stress continuity at layer interfaces. This results in a closed form recursive solution like that implemented in the well known computer program SHAKE developed by Schnabel et al. (1972). Transient motions can be handled by the Fourier transform technique.

In Eq. (2) the first term represents a wave traveling in the negative x-direction (upward) and is called the incident wave I; the second term represents a wave traveling in the positive x-direction (downward) and is called the reflected wave R. Kunar and Rodrigues (1980) explain the physical meaning of these two parts when they are used as a rigid-base input accelerogram as follows:

- The incident component I propagates upwards to the surface to reproduce the criteria motion. Then it is reflected back towards the base (note: the incident part which is on its way from the surface back to base is in fact what is called the reflected part in Eqs. (2) and (3)).
- When the incident component arrives back at the base, it is reflected back to the surface with the same magnitude but with opposite sign.
- The incident part reflected at the base, however, does not propagate back to the surface because it cancels out with the reflected component R of the base input which is of equal but opposite magnitude.

The problem of concern here is that when base accelerograms obtained by deconvolution procedures, which usually employ continuum models, are used as input for discretized models, the following error is introduced: The incident component of the input I is distorted while propagating up and down in the discretized model and consequently it does not cancel out exactly with the reflected component R of the input. This, in turn, produces a surface motion different from the original criteria motion.

The distortion in the incident part of the motion is mainly due to the dispersion characteristics of the space and time discretizations employed. Fig. 1 shows dispersion curves for the one-dimensional, constant strain finite element in conjunction with some time integra-

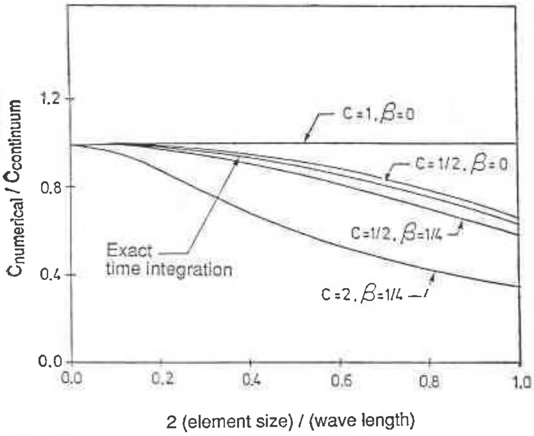

Fig. 1 Dispersion curves of the constant strain line element (Lumped mass) with time integration performed using Newmark's β scheme: $\beta=0$ (central difference scheme); $\beta=1/4$ (trapezoidal method) at different Courant numbers C (after Belytschko and Mullen, 1978)

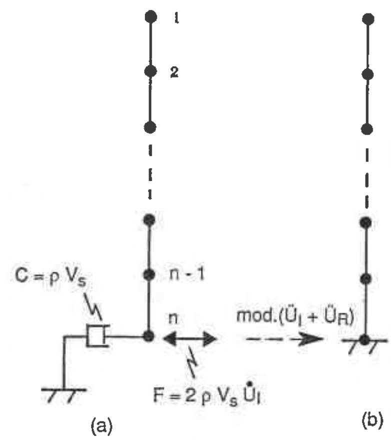

Fig. 2. Half-space model uging constant strain line element with (a) viscous base (b) fixed base

tion schemes (after Belytschko and Mullen, 1978).

The vertical axis of Fig. 1 represents the ratio between the wave speed in the discretized system and that in the continuum; whereas the horizontal axis represents the ratio between the element size and the wave length (note: multiplying this ratio by 2 is done to scale the horizontal axis between 0 and 1 as the allowable value for this ratio should not exceed 1/2). In Fig. 1 also, the Courant number, C, is defined as $C=\Delta t$ *(wave velocity)/(element size)*, where Δt is the size of the time increment. The curve marked as *Exact time integration* refers to the semi-discretized case in which only space discretization is performed; no time discretization is performed, i.e. time integration errors are not present.

In the following section a simple method is proposed to modify base accelerograms so that incident waves reflected off the free surface are cancelled at the base by the reflected part of the input motion.

MODIFICATION PROCEDURE

If viscous dashpots are attached to the base of the soil model, only the incident part of the input accelerogram is required; the reflected part must be omitted. The incident part will be absorbed by the dashpots after it is reflected off the free surface. In the case of vertically propagating shear waves, Joyner and Chen (1975) have shown that the dashpot coefficient is ρV_s and the seismic loading is applied in the form of an effective stress $2\rho V_s \dot{U}_I$, as shown in Fig. 2 for the half-space modeled by line elements. $\dot{U}_I$ is the particle velocity for the incident wave at the base. This actually corresponds to applying an accelerogram equal to twice the incident part in Eq. (2) at the fixed end of the dashpot. Then the problem can be stated in terms of relative motion between the free nodes and the fixed end of the dashpot as follows

$$[M]\{a^r\}+[C]\{v^r\}+[K]\{d^r\}=-[M]\{2\ddot{U}_I\} \quad (4)$$

where $[M]$, $[C]$ and $[K]$ are the mass matrix, damping matrix and stiffness matrix, respectively; $\{a^r\}$, $\{v^r\}$ and $\{d^r\}$ are the relative acceleration, velocity and displacement vectors, respectively. $\ddot{U}_I$ is the particle acceleration for the incident wave. Absolute motion is obtained by adding $2U_I$, $2\dot{U}_I$, $2\ddot{U}_I$ to the relative motion d^r, v^r, a^r, respectively.

If space and time discretizations used for the one-dimensional soil model with viscous dashpots at the base are exactly the same as those used for the two or three-dimensional soil–structure model, then the response at node (n) in Fig. 2 (a) is the desired modified base motion. When this modified base motion is used for the soil column but without the dashpot, the incident and reflected parts will cancel, almost exactly, as demonstrated by the numerical examples in the next section. In summary, the modification procedure consists of the following steps:

1. Perform a deconvolution analysis, using the SHAKE program for instance, to obtain the base incident motion $\ddot{U}_I$ that corresponds to the given surface motion.
2. Run a one dimensional model analysis using the base input $2\ddot{U}_I$ in order to obtain the modified base input $mod.(\ddot{U}_I+\ddot{U}_R)$. The one dimensional model should employ a viscous bottom boundary and it should have the same space and time discretizations used for the soil–structure interaction model.
3. Run the two or three dimensional model of the problem at hand with a rigid base at which the specified motion is given by $mod.(\ddot{U}_I+\ddot{U}_R)$.

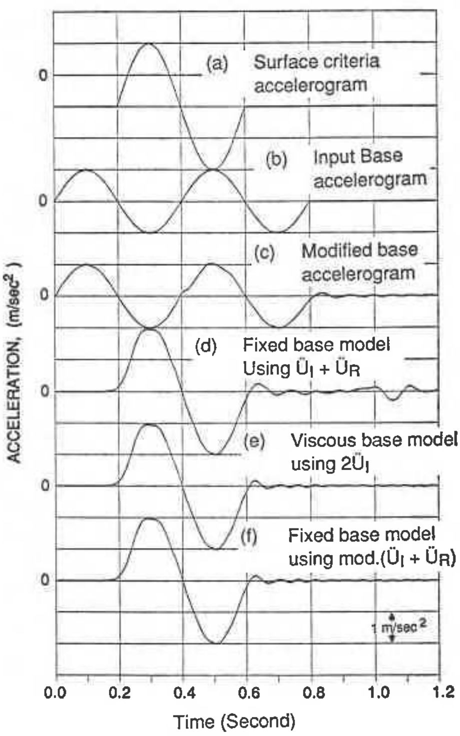

Fig. 3. Acceleration input and output results of example (1) using Newmark's time integration method

VERIFICATION EXAMPLES

For vertically propagating shear waves with planar wave front, the half–space can be modeled by a series of line elements. In the following analyses, one–dimensional constant strain line elements are employed as shown in Fig. 2. Each element has length 4 m, cross-sectional area 1 m^2, density 1.8 t/m^3 and shear modulus 288000 kN/m^2. A total of 20 elements are used and lumped mass formulation is adopted. The seismic loading is input in the form of an accelerogram at the fixed base. All computations were performed in double precision.

Fig. 3 presents the input and results of the first example as follows:

(a) Criteria surface acceleration: one cycle sine wave with period $T=2H/V_s=0.4$s, where H is the depth to the fixed base.

(b) Exactly deconvolved base acceleration $\ddot{U}_I+\ddot{U}_R$

(c) Modified base acceleration as proposed $mod.(\ddot{U}_I+\ddot{U}_R)$

(d) Surface output acceleration using discretized fixed base model and exactly deconvolved base accelerogram $\ddot{U}_I+\ddot{U}_R$

(e) Surface output acceleration using discretized model with viscous base and the incident part of the exactly deconvolved base accelerogram $2\ddot{U}_I$

(f) Surface output acceleration using discretized fixed base model and modified base accelerogram $mod.(\ddot{U}_I+\ddot{U}_R)$

Newmark's time integration scheme (Bathe and Wilson, 1976) is used with the following parameters: $\beta=0.25$, $\gamma=0.5$ and time step $\Delta t=0.01$ second. Comparison of curves (d) and (f) in Fig. 3 indicates the improvement of output surface acceleration by using the modified base accelerogram.

The same problem presented above is solved again but using the central difference time

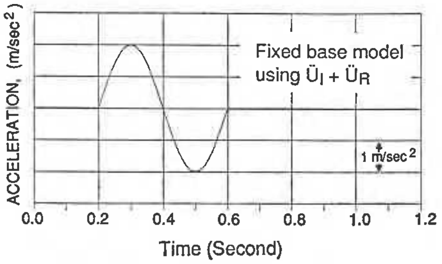

Fig. 4. Output surface acceleration using central difference time integration with Courant number=1

integration scheme (Bathe and Wilson, 1976). With Courant number $C=1.0$ and lumped mass formulation, this discretized system is completely non-dispersive (see Fig. 1). Fig. 4 shows the surface output acceleration of the fixed base model using the unmodified base accelerogram $\ddot{U}_I+\ddot{U}_R$. The output is exactly the same as the criteria surface acceleration. Although this example applies only to one-dimensional wave propagation using linear displacement elements in a uniform mesh and time integration with Courant number $C=1.0$, the results demonstrate that a satisfactory time history can be obtained when wave distortion

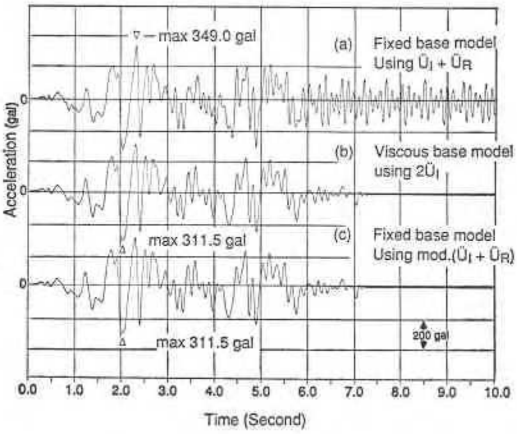

Fig. 5. Output surface acceleration of example (2) using Newmark's time integration method

is prevented.

In the second example, SHAKE program (Schnabel et al., 1972) is used to deconvolve the first 10 seconds of the EL CENTRO earthquake (frequencies above 10 Hz are removed). The resulting base accelerogram is then applied to the half-space model used in the first example. The surface output acceleration is shown in Fig. 5 for the following models:

(a) Discretized fixed base model using unmodified SHAKE output base accelerogram $\ddot{U}_I+\ddot{U}_R$.

(b) Discretized model with viscous base using the incident part of SHAKE output base accelerogram $2\ddot{U}_I$.

(c) Discretized fixed base model using modified SHAKE output base accelerogram $mod.(\ddot{U}_I+\ddot{U}_R)$.

Comparison of curves (a) and (c) in Fig. 5 demonstrates the advantage of the proposed modification procedure for the base accelerogram.

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