



NRC Publications Archive Archives des publications du CNRC

Laboratory simulation of waves

Mansard, Etienne P. D.; Miles, Michael D.

For the publisher's version, please access the DOI link below./ Pour consulter la version de l'éditeur, utilisez le lien DOI ci-dessous.

Publisher's version / Version de l'éditeur:

https://doi.org/10.1142/9789812819307_0040

Handbook of Coastal and Ocean Engineering, pp. 1107-1134, 2009-09

NRC Publications Record / Notice d'Archives des publications de CNRC:

<https://nrc-publications.canada.ca/eng/view/object/?id=4f8e64cf-ab65-4b14-8e26-55b9418adba1>

<https://publications-cnrc.canada.ca/fra/voir/objet/?id=4f8e64cf-ab65-4b14-8e26-55b9418adba1>

Access and use of this website and the material on it are subject to the Terms and Conditions set forth at

<https://nrc-publications.canada.ca/eng/copyright>

READ THESE TERMS AND CONDITIONS CAREFULLY BEFORE USING THIS WEBSITE.

L'accès à ce site Web et l'utilisation de son contenu sont assujettis aux conditions présentées dans le site

<https://publications-cnrc.canada.ca/fra/droits>

LISEZ CES CONDITIONS ATTENTIVEMENT AVANT D'UTILISER CE SITE WEB.

Questions? Contact the NRC Publications Archive team at

PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca. If you wish to email the authors directly, please see the first page of the publication for their contact information.

Vous avez des questions? Nous pouvons vous aider. Pour communiquer directement avec un auteur, consultez la première page de la revue dans laquelle son article a été publié afin de trouver ses coordonnées. Si vous n'arrivez pas à les repérer, communiquez avec nous à PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca.

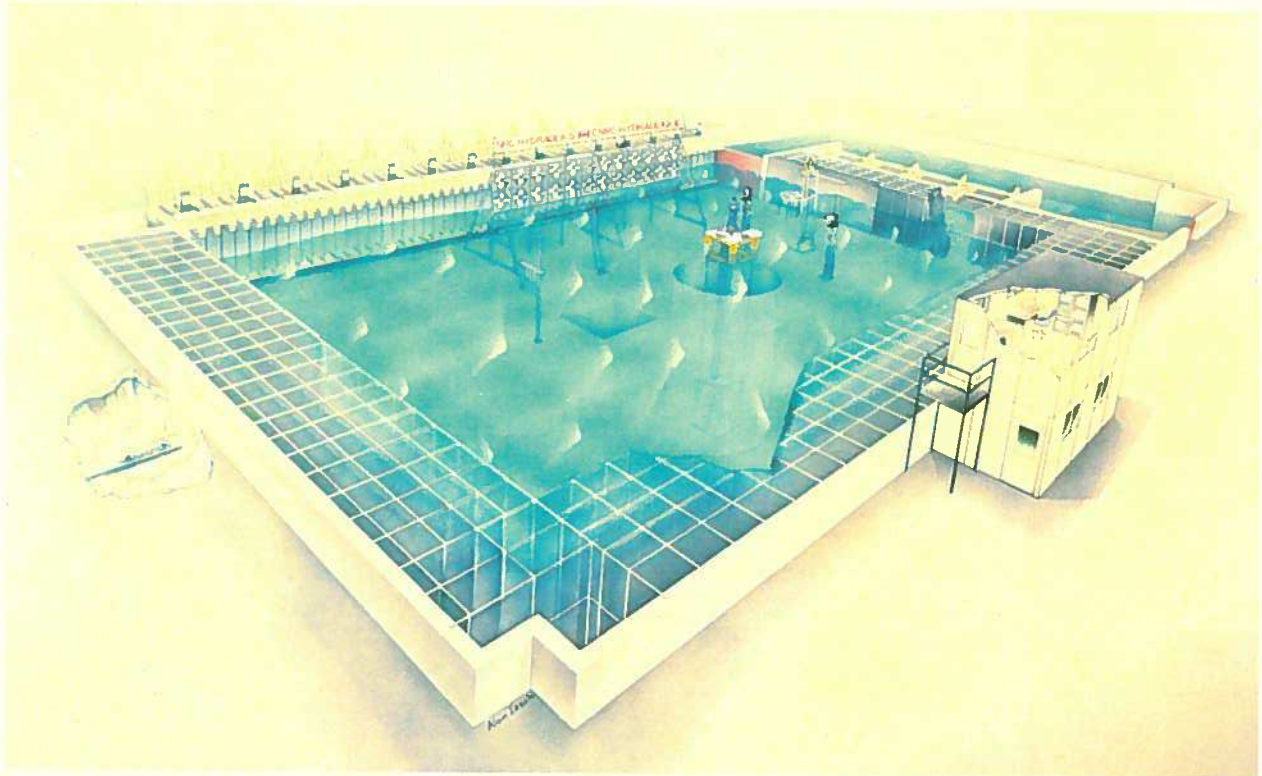


National Research
Council Canada

Conseil national de
recherches Canada

Canada

LABORATORY SIMULATION OF WAVES



Etienne P.D. Mansard and Michael D. Miles

*Canadian Hydraulics Centre
National Research Council Canada
Ottawa, ON, K1A 0R6, Canada*

Published in the Handbook of Coastal and Ocean Engineering
World Scientific Publishing Co.
2008



LABORATORY SIMULATION OF WAVES

Etienne P.D. Mansard and Michael D. Miles¹

*Canadian Hydraulics Centre
National Research Council Canada
Ottawa, ON, K1A 0R6, Canada*

Abstract

During the last two decades, a large number of multidirectional wave facilities have been built around the world. In parallel, wave generation and analysis techniques have also advanced so that it is now possible to ensure realistic wave conditions that mimic the nature in laboratory basins and flumes. With these capabilities, testing of coastal and offshore structures can be carried out with greater accuracy ensuring thereby optimal designs for structures in terms of cost and safety. This paper provides a brief review of the wave generation and analysis techniques that are commonly used for simulating uni- and multidirectional waves and also shares the experience gained at the Canadian Hydraulics Centre of the National Research Council Canada, in this field.

1.0 Introduction

Physical modelling is still the best means of design optimisation of coastal and offshore structures, in spite of the fact that it was speculated in the eighties that the numerical modelling will take over, making physical models redundant. The numerical models have indeed improved over the years that it is now possible to use them to solve a large range of practical problems. However, with the trend of locating marine structures in deeper waters and in more severe environment the numerical models do not seem to cope with some of the environmental factors and/or their associated complexities in a reliable fashion. Hence it is expected that the physical modelling techniques will continue to be relevant. Furthermore, the use of a combination of both numerical and physical modelling techniques in large projects is becoming more common. In such situations, numerical models are used to study the simpler components of the problem while physical models are employed for solving the complex components. There are also some situations where physical models are used to validate the numerical modelling results on some test cases, before applying these tools to find solutions for the entire project.

Waves' being the most important input to any physical model study, correct simulation of waves in laboratory basins is crucial. This paper briefly reviews the wave generation techniques commonly used in major laboratories and shares the experience of the Canadian Hydraulics Centre (CHC) of the National Research Council Canada (NRC) in this field.

¹ Retired staff of the Canadian Hydraulics Centre

2.0 Main Steps in Laboratory Simulation of Waves

The flow chart in Figure 1 lists the main steps involved in laboratory simulation of waves. They include:

- Characterisation or definition of the sea state;
- Synthesis of wave train;
- Preparation of wave machine driving signal;
- Wave generation and data sampling; and
- Wave analysis

A description of the various steps involved in simulating unidirectional waves will be discussed first while the aspects of multidirectional wave generation will be presented subsequently.

3.0 Simulation of Unidirectional Waves

3.1 *Characterisation of the sea state*

The very first step in any model study of coastal and offshore structures is the choice of the design wave conditions to be used for testing purposes. These design wave conditions are often wave conditions that may have a return period of 50, 100 or 1000 years. Longer the return period, higher would be the design wave height and costlier (but safer) would be the structure. It is common to design coastal structures to withstand 1:100 year wave conditions.

Extreme value analysis is used to estimate the wave heights for large return periods from available recorded or hindcast data pertaining to that site. A review of these techniques can be found in publications such as Goda et al (1993), Sarpkaya and Isaacson (1981).

Generally the sea state to be used in a model study is characterised by its significant wave height, peak period and a parametric spectrum. The well-known parametric spectra are:

- Bretschneider spectrum or ITTC (International Towing Tank Conference) or ISSC (International Ship Structure Congress) spectrum;
- Pierson-Moskowitz spectrum that represents fully developed seas;
- JONSWAP spectrum that represents fetch limited conditions;
- Scott spectrum that provides a good fit to observations made in Persian Gulf, North Atlantic and West Coast of India;
- Ochi and Hubble spectrum that represents coexistence of sea and swell; and
- TMA spectrum, an extension of the JONSWAP spectrum for shallow water situations.

The Pierson Moskowitz spectrum is a special case of the Bretschneider spectrum while the JONSWAP spectrum is a modified version of the Pierson Moskowitz spectrum.

3.2 *Synthesis of Wave Train*

3.2.1 *Random Phase Method.*

The technique that is most commonly used to synthesize a wave train from a given variance spectral density (commonly called also as spectrum) is the Random Phase Spectrum Method. It consists of pairing the amplitude spectrum derived from the given variance spectral density with a phase spectrum created by a random number generator and then obtaining a time series of desired length by inverse Fourier transform. Since random phases are used in this procedure, time domain characteristics such as wave grouping, cannot be controlled by this synthesis technique. However, the spectral characteristics are well reproduced, as it uses the amplitude spectrum from the target spectral density.

Another method that uses the random phases, but has no complete control over the spectral characteristics, is called Random Complex Spectrum Method. Its synthesis procedure is as follows: A Gaussian distributed white noise complex spectrum, with a standard deviation of 1, is first generated and then filtered using the amplitude spectrum derived from the target spectral density. Subsequent inverse Fourier transform results in a time series of desired record length. Unlike the previous method, the waves produced by this technique will not match exactly the desired spectral density. Obviously, this method does not also exercise control on the time domain characteristics. However both these synthesis methods based on random phases have their own proponents. Funke & Mansard (1987) describe the rationale associated with each of these methods.

One of the requirements during the synthesis of a wave train from a spectral density is the choice of the length of wave train to be created. Since the Gaussian distribution is used to describe the probability distribution of prototype water surface elevations and the Rayleigh distribution is used to describe the distribution of wave heights, the longer the wave train, the better is the fit with the above functions. However, long wave trains can increase the aggregate testing time and thereby the cost of a physical model study. Hence a suitable compromise is required.

The length of wave train is also characterized by the number of waves contained in that train and choices may range between 200 to 1000 waves in a wave train. Often smaller lengths of wave trains (or smaller number of waves) are chosen to conduct sensitivity studies, such as penetration of wave energy in a harbour basin. Longer records are used where it is critical to ensure an appropriate distribution of wave heights. For instance, in breakwater stability studies, simulation of appropriate values of $H_{1/10}$ and H_{max} are important. In general, longer is the wave record larger would be the wave heights such as $H_{1/10}$ and H_{max} . Studies using relatively short records (i.e. 200 waves) in shallow or breaking wave situations could potentially result in wrong interpretation of the test results (see Readshaw et al, 1987). In CHC, wave records containing at least 1000 waves are used in most studies.

3.2.2 *Other Methods of Synthesis*

Besides the random phase method described above in Section 3.2.1, several other approaches could be used to establish the target wave train shown in Figure 1. They include:

- Use of Prototype Data;

- Synthesis of a Grouped Wave Train; and
- Synthesis of Episodic or Freak waves

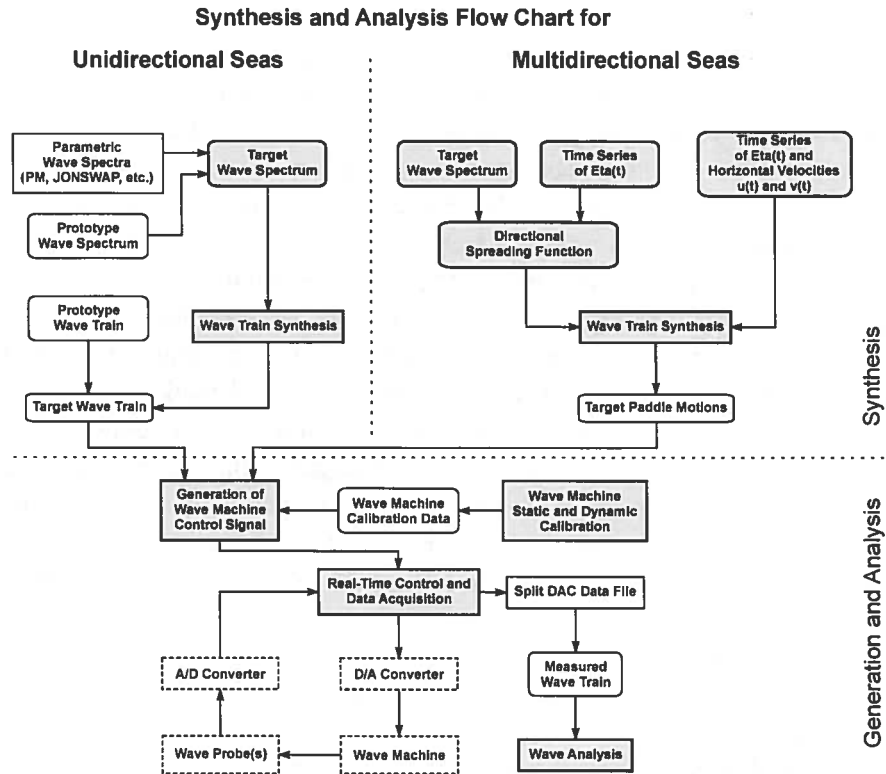


Figure 1: Example of a Flowchart for Laboratory Simulation of Waves

3.2.2.1 Use of Prototype Data

As it can be seen in the flowchart in Figure 1, simulation of prototype wave spectrum or prototype wave train is also an option in the simulation procedure. The reproduction of prototype wave data may be the best way of ensuring realistic waves in laboratory basins. However, this is often not practical because of lack of sufficient wave records for the site under consideration. Furthermore, their record lengths are often relatively short (i.e. 20 minute long record every three hours). However, in situations where there are sufficient records measured in the proximity of a site that is being considered for development, prototype wave trains have been used. It is also common to endeavour to find appropriate prototype wave trains when it is required to recreate a specific situation such as damage to a marine structure.

3.2.2.2 *Synthesis of a Grouped Wave Train*

In the late seventies, NRC investigated the importance of wave grouping on the stability of fixed and floating structures. For this purpose, Funke & Mansard (1979) developed a technique based on the concept of Smoothed Intermediate Wave Energy History to create wave trains that can satisfy both the spectral characteristics of the desired spectrum as well as the desired time domain characteristics in terms of wave grouping.

This technique has been extensively used to understand the physical processes associated with the interaction between wave groups and test structures. Johnson et al (1978) successfully demonstrated that wave grouping is an important parameter to be reckoned with in studies of breakwater stability. Mansard & Pratte (1982) illustrated the importance of wave groups and the long waves generated by these groups on the slow drift oscillations of moored ships. Hence, this technique is indeed an appropriate tool for understanding physical processes associated with wave-structure interactions.

3.2.2.3 *Synthesis of Episodic or Freak Waves*

Funke & Mansard (1979) developed a technique that can generate a plunging type of breaking wave at any predetermined location in a flume using the sweep frequency technique. This technique is used to subject test structures to severe breaking waves in order to study the interaction of extreme waves with structures. For instance, it was used to validate the capsizability of a communication buoy which otherwise was considered stable under normal wave conditions used in testing. NRC has also extended this technique to generate 3-dimensional episodic breaking waves in a multidirectional wave basin which have circular wave fronts and are focussed both by frequency and wave direction.

3.3 *Generation of Wave Machine Control Signal*

Before the synthesis and analysis of multidirectional seas, shown in the top right portion of Figure 1 are discussed, the steps shown in the bottom portion of the flowchart dealing with the generation and analysis of waves are discussed below.

3.3.1 *Capability of Wave Generator*

One of the first tasks that is undertaken before simulating a sea state in any laboratory basin or flume is to determine the capability of the wave machine to generate the desired sea state. This is often the critical step in determining the model scale factor that is needed to produce the design wave condition.

Figure 2 shows the performance curves for one of the NRC wave generators, in a 2D flume. The significant wave heights that can be generated in this facility for peak periods ranging from 0.5s to 5.0s are shown here for two different operational water depths. The left envelope of these two curves represents the breaking limit defined as a wave steepness 60% larger than that of a Pierson-Moskowitz spectrum for a fully developed sea. This limit is based on the maximum steepness observed in a sample of

several hundred measured North Atlantic wave records. Waves with significant wave heights beyond this threshold would be subject to breaking and therefore not achievable. The right hand side portion of these curves indicates the influence of the wave machine stroke limit on the wave heights that can be generated. As long period waves require large strokes, the wave height becomes smaller as the wave period increases. These limits are based on the well known Biesel theory that relates the displacement of the paddle required to generate a wave height of a given period, under a given water depth.

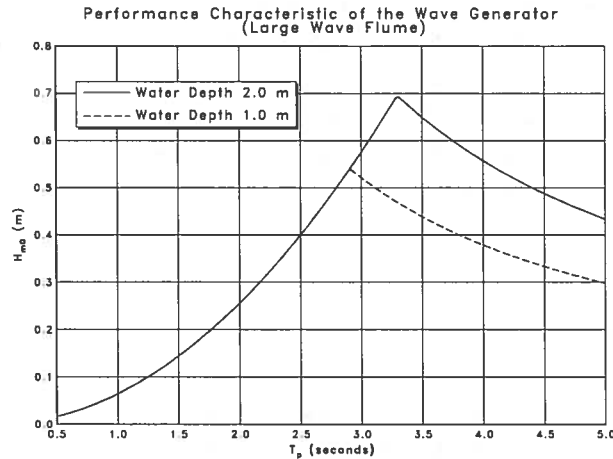


Figure 2: Estimation of the Capability of the Wave Generator

3.3.2 Creation of Wave Machine Control Signal

Correct generation of a wave train in a laboratory facility depends very much on the accuracy of the techniques used to convert the target wave train into an appropriate wave generator control signal. This conversion requires static and dynamic calibration of the wave machine and its associated components such as servo controller and digital to analog filter. The hydrodynamic transfer function that computes the displacement required to produce a desired wave height is based on the well-known Biesel relationship. However the dynamic response of the wave machine components can vary to a large extent from one facility to another and needs to be compensated for, in order to ensure a faithful reproduction of the desired wave train inside the laboratory basin. NRC uses a procedure called dynamic calibration to estimate the compensations required for correcting the dynamic response of the wave machine. This procedure is described below.

The static calibration of the wave machine is first established by driving the wave machine with known and slowly varying inputs of the voltage and then measuring the response in terms of paddle displacement. Then, a time series composed of wavelets of different frequencies is fed into the system and the corresponding displacement of the paddles is monitored and analysed.

Figure 3 presents an example of the drive signal used for dynamic calibration while figure 4 illustrates an expanded version of a portion of this time series (i.e. contained between 180 and 210s).

The cross correlation of the input wave signal and the measured displacements of the paddle provides a complex transfer function that can be used to generate a wave board control signal that can ensure the desired waves.

The phase and the amplitude response of this transfer function are shown in figure 5. The decreasing amplitude response and increasing phase difference found generally in such systems when the frequency increases, can be seen clearly.

The Real-Time control and Data Acquisition then ensure the appropriate control of the machine and the sampling of data by sensors such as wave gauges.

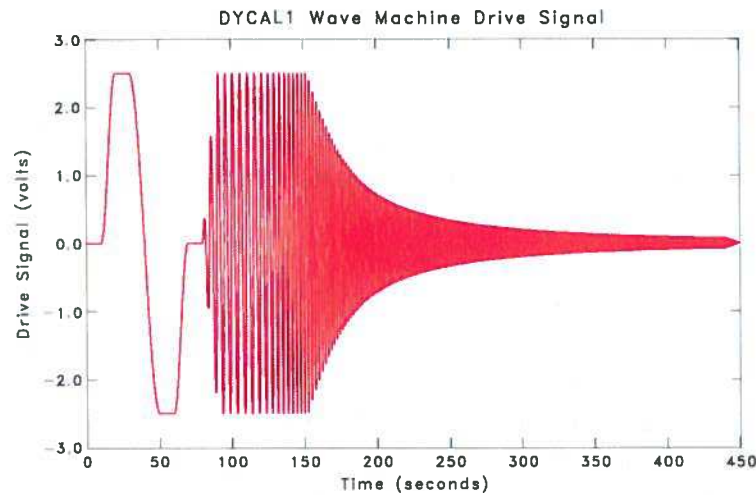


Figure 3: Wave Machine Drive Signal for Dynamic Calibration

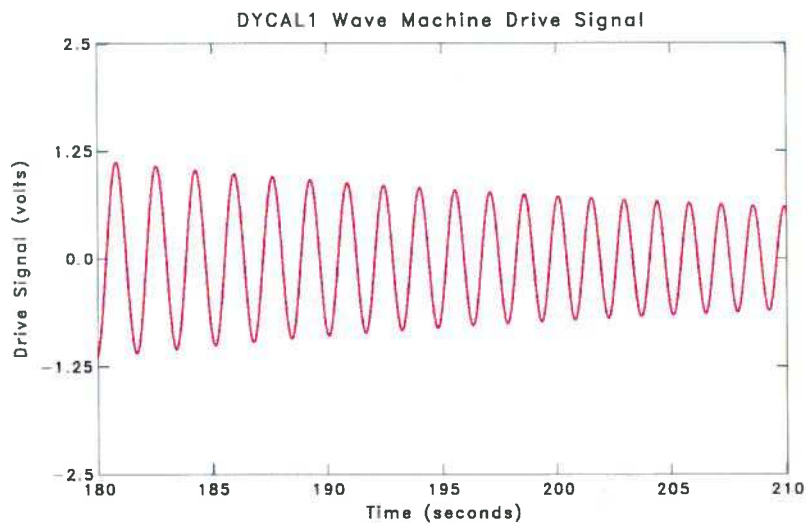


Figure 4: Portion of the Drive Signal presented in Figure 3

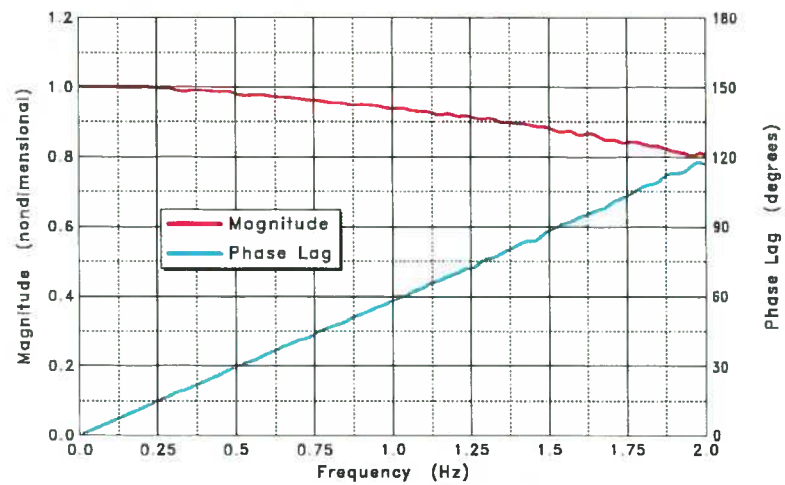


Figure 5: Complex Transfer Function obtained by Dynamic Calibration

3.4 Wave Analysis

3.4.1 Conventional Analysis of Waves

Figure 6 shows a typical wave analysis output of NRC from a test program for one of the wave gauges used in the study. It lists some of the relevant spectral and time domain parameters derived from spectral and zero-crossing analysis. A list of notations of these parameters and their definitions can be found in IAHR/PIANC (1986). The time series of the wave record, the spectrum of the measured time series and the probability distribution of the measured wave heights are also presented. In the same figure, the target spectrum and the theoretical Rayleigh distribution are also overlaid for comparison purposes.

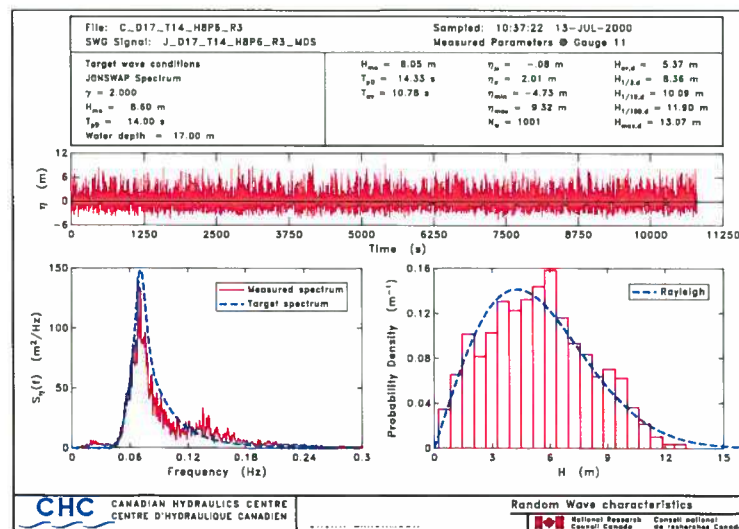


Figure 6: Sample Analysis output of 2D waves

It should be pointed out that although the target spectrum was a JONSWAP spectrum, the measured spectrum displays clearly non-linear sub-harmonic and super-harmonic components on the left and right side of the primary spectrum respectively. This is to be expected given the shallow water conditions of this sea state (see values of the target sea state). A small reduction in the value of H_{m0} from the target value can be attributed to loss of energy during propagation along the basin and/or to some breaking.

In studies where the influence of group-induced long waves is important, the long energy period energy (i.e. sub-harmonics) found in this figure will be analysed in more detail.

Note that in shallow water situations where substantive sub-and super-harmonics are expected to be present, second order wave generation is recommended to be used in order to ensure correct reproduction of these harmonics. The techniques used for this purpose is beyond the scope of this paper. Readers are asked to refer to Barthel et al (1980) and Sand and Mansard (1986) for these techniques.

3.4.2 Reflection Analysis

Reflection Analysis is an important component of the analysis package for laboratory simulation of waves. It is used principally to design wave absorbers that can effectively dissipate the incoming wave energy in order to minimize boundary effects (see Jamieson and Mansard, 1987), but also to estimate the reflection characteristics of coastal and offshore structures.

NRC developed several versions of techniques that can separate the incident and reflection characteristics, both in time and frequency domains (see Mansard et al 1980, 1985, 1987 and Mansard 1994). All these techniques are based on the least-squares method that requires information from 3 probes.

It should also be pointed out that while the above techniques address two-dimensional wave situations, Isaacson et al (1995) have developed techniques for oblique waves. (Research is now underway in several laboratories to develop accurate techniques that can separate incident and reflected wave components in a multidirectional wave situation).

Waves reflected by test structures can propagate back to the paddle and then get re-reflected, corrupting the quality of data that prevails in the experimental set up. In order to overcome this difficulty active absorption techniques have been developed and a description of these techniques is given below.

4.0 Active Wave Absorption

An increasing number of laboratory wave flumes and basins are now equipped with wave machines that can function both as wave generators and absorbers. Such systems are called active wave absorbers to distinguish them from the passive wave absorbers that are also used in basins. Active absorbers are used mainly to improve wave quality by preventing waves reflected by the structure being tested from being re-reflected back towards the structure by the wave machine paddle. Thus, active absorption allows the desired incident wave field to be maintained at the test structure while preventing spurious wave energy

from building up in the section of the flume between the structure and the wave generator. Active absorption can also be used to greatly reduce the stilling time between tests in a flume by removing low frequency waves, which would otherwise persist for some time. Similarly, it can also prevent spurious resonant oscillations in a basin or flume during long duration tests.

There are two main types of active wave absorption systems. The first method uses an array of wave elevation probes located a few metres in front of the wave machine. Real-time reflection analysis of the wave probe signals is used to separate the incoming and outgoing wave trains and the incoming wave component at the position of the wave machine is then computed by linear dispersion theory. The corresponding paddle motion to absorb the incoming wave is then computed by using the linear hydrodynamic transfer function with compensation for the amplitude and phase response of the wave machine's servo control system. The main advantage of this method is that it can be easily installed on existing wave machines without designing a new servo control system. The propagation delay between the wave probe array and the paddle gives the wave machine time to respond so existing analog servo controllers with relatively large phase lags can still be used. No corrections are required for evanescent waves because the wave probes are located some distance from the paddle. One disadvantage of this method is that it relies on linear dispersion theory so nonlinear shallow water waves will introduce phase errors that reduce the absorption capability.

The second method uses a wave elevation probe mounted on the face of the paddle and moving with it. This is the method most commonly used on segmented wave machines with one wave probe on each segment. The expected wave elevation at the paddle is subtracted from the total measured wave elevation and the residual wave elevation is assumed to be due to the incoming wave to be absorbed. The commanded paddle motion required to absorb the incoming wave is then computed by linear wave theory. The digital servo control system for this method must be very carefully designed and tuned so that there is very little phase lag over the full frequency range because the paddle motion must respond immediately to the measured incoming wave signal. The control system must also compensate for the amplitude and phase of the evanescent waves since the wave probe is mounted on the paddle. This is especially important at higher frequencies. Careful tuning may also be required to ensure a stable control system since phase lag must be kept to a minimum.

Depending on the wave frequency and the distance between the wave machine and the reflecting structure being tested, the paddle motion required for absorption may be in phase with the motion required for wave generation. Consequently, the paddle motion required for simultaneous wave generation and absorption is larger than that required for wave generation alone although no additional power is needed. When designing new wave machines, the paddle stroke should typically be 30% larger than that required for wave generation alone if active absorption is to be used. The wave machine controller must also ensure that the paddle motions do not exceed the mechanical stroke limits and the percentage of active absorption control may have to be reduced when generating large waves to stay within the limits of the machine.

The two main active wave absorption methods that have been used on segmented wave machines are known as quasi-3D and fully-3D. In the quasi-3D method, the direction of the incident waves approaching the wave machine is not measured but is set a priori based on the configuration of the basin and the structure being tested. The resulting paddle motions will have the correct phase when absorbing

an oblique wave but the amplitude may be in error. However, since the amplitude varies as $\cos(\theta)$, the quasi-3D method still works very well in most cases since the amplitude error is small when the incident waves are within 20 degrees of the estimated direction. In fully-3D systems, the angle of the incident waves is measured directly so that the correct amplitude can be used. This can either be done by using a wave probe array in the basin or by using signals from wave probes mounted on several adjacent segments to determine the incident wave angle. Schäffer and Klopman (1997) provide a good review of the various active absorption techniques that have been used for segmented wave machines.

The Canadian Hydraulics Centre of the NRC developed a 2D active wave absorption system for wave flumes in 1999 that uses an array of three wave probes to measure the incoming and outgoing waves. This system provides a convenient and inexpensive way to add active absorption to an existing wave machine since the original servo controller can be used. CHC has recently also developed a new digital active wave absorption control system for segmented wave machines that uses a wave elevation probe mounted on each paddle. This system uses two drive signals per segment, which define the paddle motion for wave generation and the expected wave elevation at the probe including the evanescent component. The controller subtracts the expected wave elevation from the measured wave elevation to obtain the elevation of the incoming wave to be absorbed. Both of these methods have been tested experimentally using a special wave flume equipped with two wave generators as shown in Figure 7. In wave absorption tests, the waves are generated by wave machine A and absorbed by wave machine B. Standard reflection analysis of data from a wave probe array is then used to measure the reflection coefficient of the active wave absorber. In other tests, wave machine B performs simultaneous generation and absorption while wave machine A remains stationary to provide a reflecting boundary.

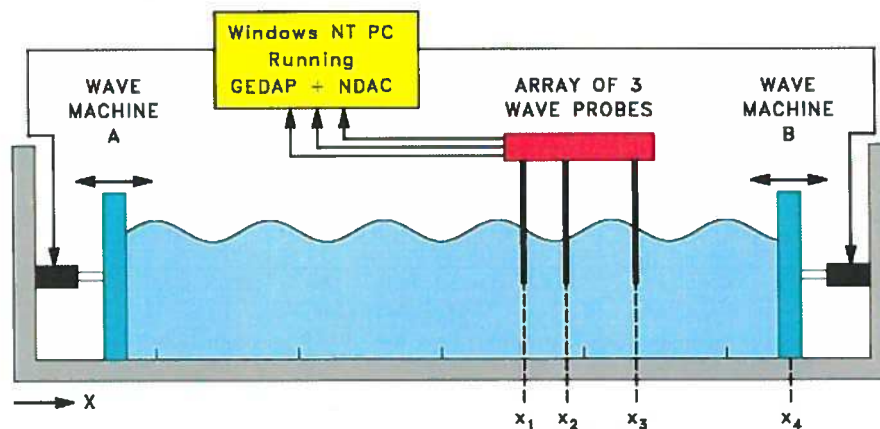


Figure 7: Flume Configuration for Active Wave Absorber Tests

Some regular wave test results for the two types of active absorbers are shown in Figure 8. The 3-probe array method has good performance with an average reflection coefficient of about 10% over the main range of wave periods from 1.0 to 3.0 seconds. However, the reflection coefficient increases at longer wave periods due to phase errors associated with nonlinear shallow water waves. The new digital

controller has very good performance over the full range of wave periods with an average reflection coefficient of 5%.

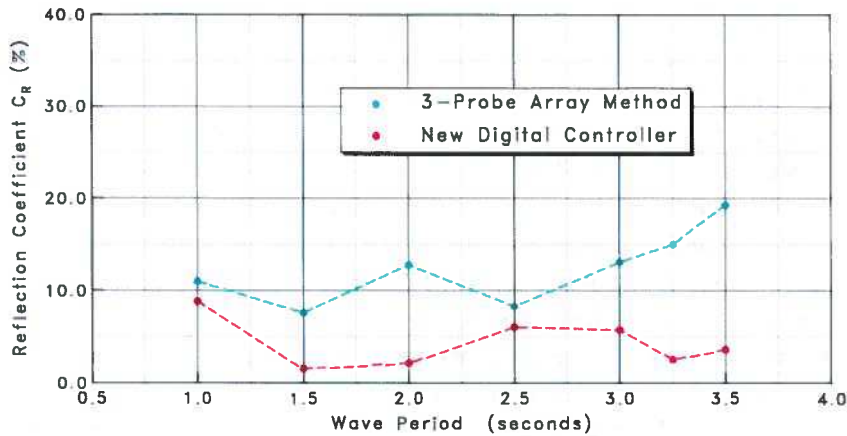


Figure 8: Active Wave Absorption Test Results for Regular Waves

Some irregular wave test results for the new digital controller with a JONSWAP spectrum with a peak period of 2 seconds are shown in Figure 9. It can be seen that the system also has excellent performance in irregular waves with an average reflection coefficient of 4%.

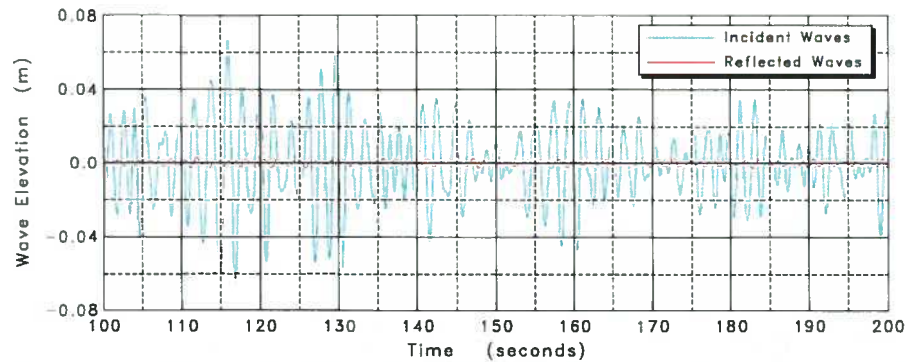


Figure 9: Irregular Wave Test Results for a JONSWAP Spectrum with $T_p = 2.0$ s

The results of a test of simultaneous generation and absorption are shown in Figure 10. In this case, waves were generated at a resonant frequency of the flume. It can be seen that the wave height steadily increases when active absorption is turned off but a stable standing wave is quickly established and maintained when active absorption is on.

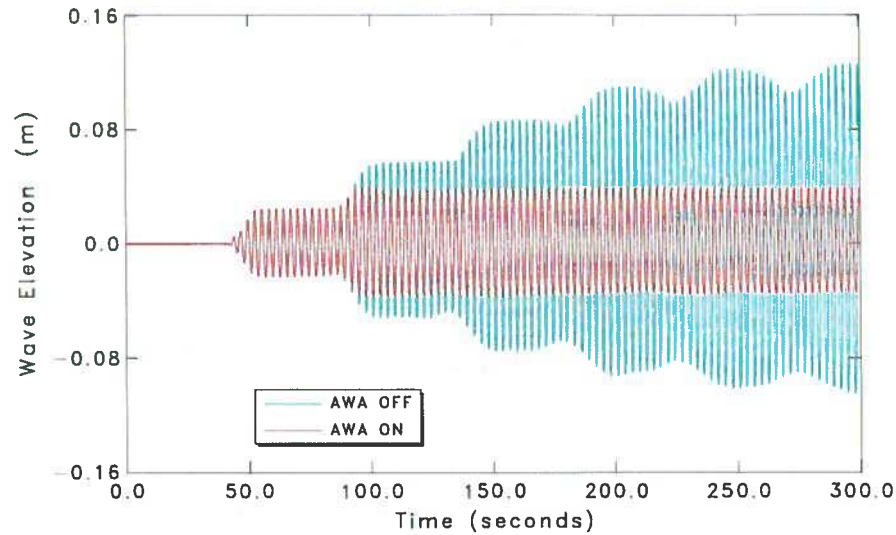


Figure 10: Resonant Standing Wave Test Results for $T = 2.947$ seconds

Figure 11 presents an example of the influence of Active Wave Absorption in terms of stilling a flume. The two curves show the elevations measured by a probe when the active absorption was turned off and also when it was turned on. During these tests, the wave generation was stopped at $t = 100$ s. Data was sampled for another 260 seconds. The two measured wave trains are almost identical for this first 100s, which means that the AWA has accurately identified that there is virtually no incoming waves to absorb during that period. The periods from 100 to 160s consists mainly the first reflection of waves by the generator A (see Figure 7). During this period, the AWA on wave height is approximately half as large as the AWA off, indicating the good absorption when AWA is on. Finally a comparison of the results for the period 160 to 360s shows that the AWA system has very effectively reduced the residual wave energy in the flume.

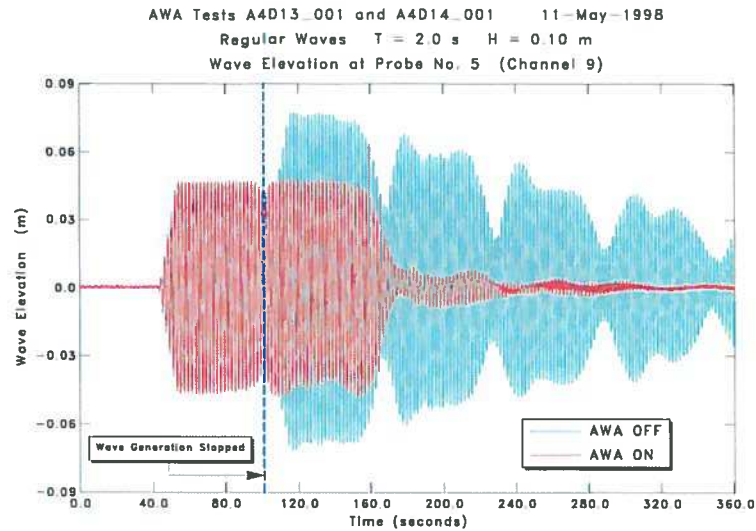


Figure 11: Validation of the Active Wave Absorption Technique

5.0 Simulation of Multidirectional Waves

The importance of testing under multidirectional seas has been described in many publications such as Funke & Mansard (1992), Mansard (1998), Hoklie et al (1983), Franco et al (1996) and Stansberg et al (1997) and many multidirectional wave facilities have been built around the world during the last 15-20 years. In parallel, numerous techniques have been developed to simulate multidirectional waves for laboratory model investigations. The next sections will provide a brief review of the techniques used for the generation and analysis of multidirectional seas. It will also describe a numerical model that is used at NRC for designing experiments in an existing multidirectional wave basin or for designing new facilities.

The flowchart given earlier in Figure 1 provided also a sketch of the different steps involved in the generation of multidirectional seas. As it can be seen from that figure, the synthesis part of the target wave train is different for generation of unidirectional and multidirectional seas, while the generation component through the wave machine is similar. Wave analysis will of course be different for the two cases.

5.1 Generation of Multidirectional (or 3D) Waves

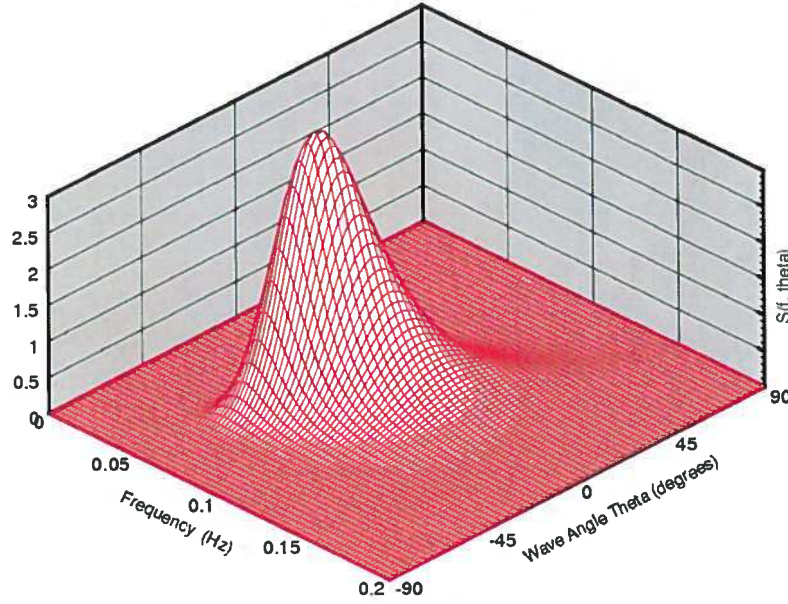


Figure 12: An Example of a Directional Spectrum

The multidirectional spectral density of a sea state is given by:

$$S(f, \theta) = S(f) \cdot D(f, \theta) \quad (1)$$

Where $S(f)$ is a spectral density and $D(f, \theta)$ is the directional spreading function satisfying the relationship.

$$\int_0^{2\pi} D(f, \theta) d\theta = 1 \quad (2)$$

Figure 12 shows an example of a multidirectional spectrum. The main step involved in the generation of multidirectional seas is the choice of a directional spreading function, which describes the mean direction and the angular distribution of energy.

The most commonly used spreading function is of the following form where Γ is the gamma function, θ_0 is the mean wave direction, and s is the spreading index. This function can either be the same for all frequencies or the parameters θ_0 and s may vary with frequency.

$$D(f, \theta) = \frac{\Gamma(s+1)}{\sqrt{\pi} \Gamma(s+1/2)} \cos^{2s}(\theta - \theta_0) \quad \text{for } |\theta - \theta_0| < \pi/2 \quad (3)$$

Figure 13 shows an example of a directional spreading function, which is non-uniform over the different frequency ranges.

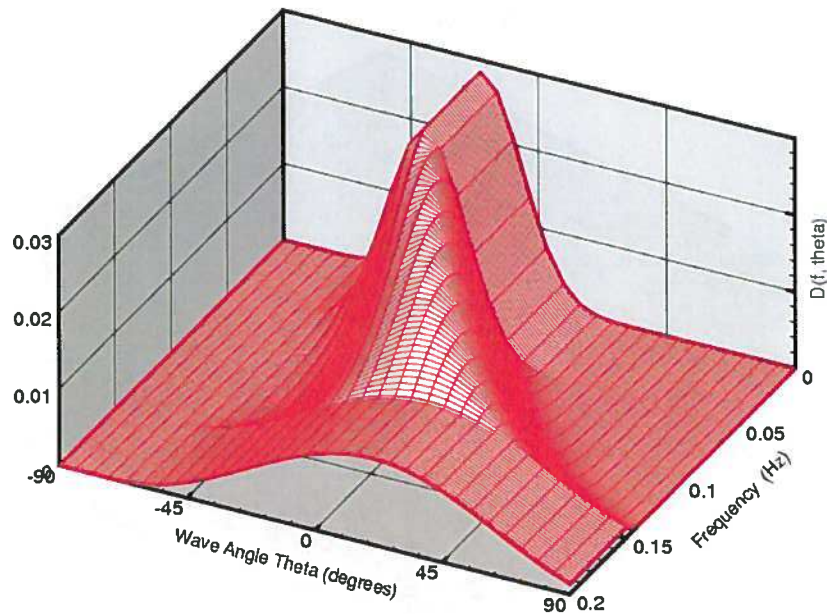


Figure 13: Non-Uniform Spreading Function

Several techniques can be used to synthesize a time series from the directional spectral density given in the equation 1. Amongst them, the most commonly used techniques include Single Summation and Double Summation Models. Details of these two models can be found in publications such as Miles & Funke (1989) and Miles (1989).

Amongst these two techniques, many laboratories prefer the use of the Single Summation Model. In this model, each frequency component can only travel in one direction, thus ensuring a spatially homogeneous field in the basin for typical record lengths used in laboratory testing. Whereas, in the double summation model, multiple wave directions exist at each discrete frequency resulting in a non-homogeneous wave field in the test section. Jefferys (1987) and Miles & Funke (1989) discuss the relative merits and demerits of these models.

In the flow chart presented in Figure 1, it is shown that the desired target multidirectional wave train could also be synthesized by combining a unidirectional wave record $\eta(t)$ and a target spreading function. Alternatively, a time series of the water surface elevation $\eta(t)$ and its associated orthogonal velocities $u(t)$ & $v(t)$ can also be used directly in the wave synthesis, since the values of $u(t)$ & $v(t)$ describe adequately the directional characteristics of the waves.

The various synthesis techniques available at NRC for multidirectional wave generation can be summarized as follows:

- Single Summation Method with equally spaced wave angles for specified wave spectrum and spreading function
- Single Summation Method with random wave angles for specified wave spectrum and spreading function
- Single Summation Method with random wave angles for specified wave train and spreading function

- Double Summation Method with equally spaced wave angles for specified wave spectrum and spreading function.
- Discrete FFT wave synthesis for specified time series of wave elevation $\eta(t)$ and horizontal velocities $u(t)$ and $v(t)$.

The above methods perform not only the synthesis of water surface elevation but also compute the required paddle motions based on the snake principle method of Sand & Mynett (1987). These paddle motions are then compensated for dynamic and static transfer functions as in the case of unidirectional waves. The Single Summation Method with random wave angles is most commonly used since it provides the most accurate frequency spectrum but equally spaced angle methods provide a more accurate spreading function at the expense of some spectral distortion. Test durations corresponding to one hour full-scale are adequate when the spreading function does not vary with frequency. However, test durations of five hours or more full-scale are required to accurately reproduce the directional spectrum in cases such as hurricane seas where the spreading function has large variations with frequency. For example, see Cornett and Miles (1991).

5.2 Analysis of Multidirectional Waves

5.2.1 Analysis Techniques

A comprehensive review of the multidirectional wave analysis was carried out under the auspices of the Maritime Hydraulics Section of the International Association for Hydraulic Engineering and Research by Benoit et al (1997). The different methods of analysis can be classified according to the following categories:

- Fourier Decomposition Method;
- Fitting of Parametric Models;
- Maximum Likelihood Methods;
- Maximum Entropy Methods;
- Bayesian Directional Method; and
- Deterministic Analysis Methods.

Amongst these methods, the Maximum Likelihood Method (MLM) and Maximum Entropy Method (MEM) are those that are commonly used. The instrumentation that best corresponds to these methods is either a wave probe array or an η - u - v sensor.

The experience gained at NRC, which used these two popular methods, is described below.

Both the Maximum Likelihood Method (MLM) and the Maximum Entropy Method (MEM) are based on cross spectral analysis of the various sensor signals. The MLM method assumes that $D(f, \theta)$ can be expressed as a linear combination of the cross spectra whereas the MEM method estimates $D(f, \theta)$ by maximizing an entropy function subject to cross spectra constraints. However, the MLM method, which in fact is easier to implement, tends to provide an estimate of the spreading function wider than the target function.

The NRC technique developed by Nwogu et al (1987), and based on MEM technique using η - u - v data, was found by Benoit et al (1997) to be superior to other methods.

This technique was later adapted to work with data from an array of wave gauges, partly because of the fact the current meters could be more expensive than wave gauges and also susceptible to errors caused by contamination of the current-turbulent fluctuations produced at the same frequencies as the wave induced kinematics.

The wave probe array consisted of 5 probes arranged in a trapezoidal fashion and the water surface elevation data from the probe array was used to resolve the directional characteristics of the sea state.

More recently, this method was modified to use the wave slopes derived from the water surface elevation measured by these gauges in the MEM analysis rather than using directly the water surface elevations. This modification was triggered by the fact that the previous method was efficient only over a limited frequency range where the energy contained in the spectrum was substantial. Computational effort in terms of convergence of solution was also relatively high. It was also sensitive to some extent, to the spacing between gauges.

The new method, is analogous to the η - u - v method since it uses the water surface elevation η , and the orthogonal surface slopes $\frac{d\eta}{dx}$ and $\frac{d\eta}{dy}$. Figure 14 shows a definition sketch of the 5-probe array (Cornett et al, 2005).

Gauges A, B, C & D are located on the circumference of a circle having a radius R and gauge E is located at the centre of the circle. The water surface elevation is derived from the gauge E, while the orthogonal slopes $\frac{d\eta}{dx}$ and $\frac{d\eta}{dy}$ are derived from the wave elevation differences between gauges A & C and gauges B & D respectively.

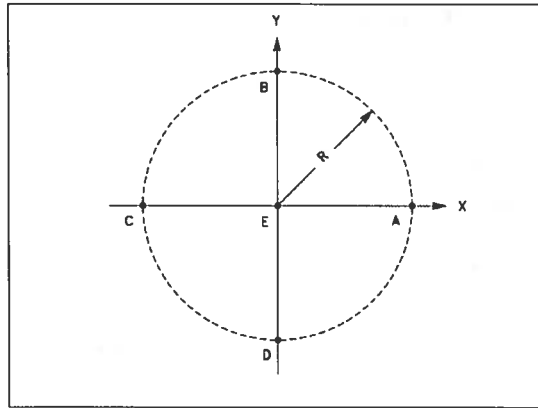


Figure 14: Definition Sketch for the Wave Probe Array

Numerical simulations were undertaken to test this technique (called as $5\text{-}\eta$ MEM Method at NRC) on synthesized multidirectional waves with a JONSWAP spectrum and a Cos^{2s} spreading function. The traditional well-proven technique, called η - u - v MEM Method that uses η - u - v data was used for comparison purposes. Figure 15a shows the results of this comparison. The mean direction, the standard deviation of the spreading function and the spreading function at the peak frequency estimated by these two techniques ($5\text{-}\eta$ MEM and η - u - v MEM) are compared with the corresponding target (or imposed) function. The two analysis techniques ensure a good agreement with the target spreading function. In fact

the $5\text{-}\eta$ MEM result even comes closer to the target. Similarly, the values of standard deviation of the spreading function and the mean values of the direction resolved by these two techniques are nearly indistinguishable and also agree well with the target values.

Following the good performance of $5\text{-}\eta$ MEM, additional investigations were undertaken to investigate the accuracy of the analysis if only 4 gauges were used instead of 5. In this case, the water surface elevations from gauges A, B, C & D will be averaged rather than using the information from gauge E.

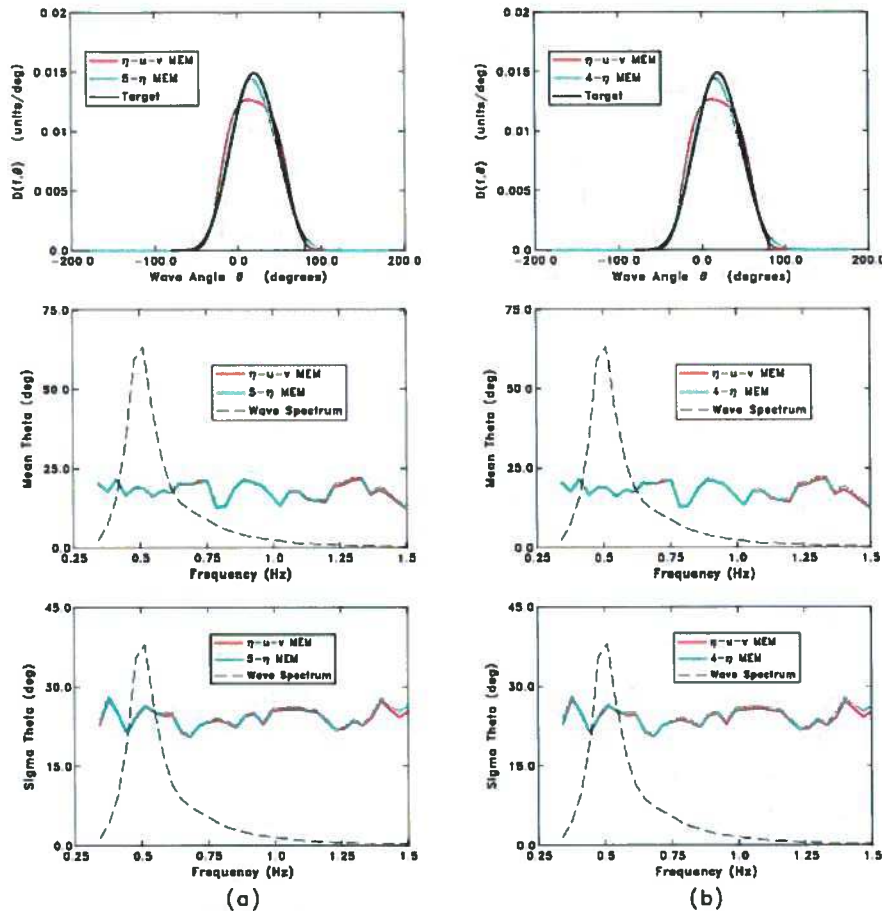


Figure 15: Numerical Validation of Wave Slope Method using data from 4 and 5 probe arrays.

Figure 15b shows comparisons obtained by a 4-probe array, similar to the ones presented previously in Figure 15a. These figures show clearly that a 4-probe array is quite adequate for resolving the directional characteristics of the sea state. The 4-probe array will be less expensive than a 5-probe array and in fact can lend itself for better structural support through the use of a supporting rod in the middle, instead of probe 5. The preferred NRC method is therefore the 4-probe array slope method called $4\text{-}\eta$ MEM.

The optimal probe spacing required (i.e. value of R) as a function of the peak period wave length (L_p) in order to offer reliable results was also investigated through numerical simulations.

The results of these investigations suggested that a value of $R/L_p \approx 0.02$ would be adequate to yield reliable results. More details on these investigations can be found in Cornett et al (2005).

5.2.2 Laboratory Validation

Extensive investigations were undertaken to validate this technique through basin tests even under severe situations such as breaking and non-linear waves. Figure 16 shows a sample result from these investigations carried out under 2 different water depths. Unlike in Figure 15, where the values of standard deviation and mean direction were close to the expected values even in high to low frequency parts of the spectrum (where the relative energy content is lower than at the peak of the spectrum), the measured data shows that the accuracy of the results is poor in those regions. This is to be expected since the analysis accuracy depends highly on the relative magnitude of signal/noise ratio. Generally in the high and low frequency parts of the spectrum, one expects this ratio to be high and therefore lesser accuracy of the analysis results. Furthermore in this case, as it can be clearly seen from the variance spectral density measured at 10m water depths, waves were highly non-linear, whereas the entire analysis technique is based on linear theory.

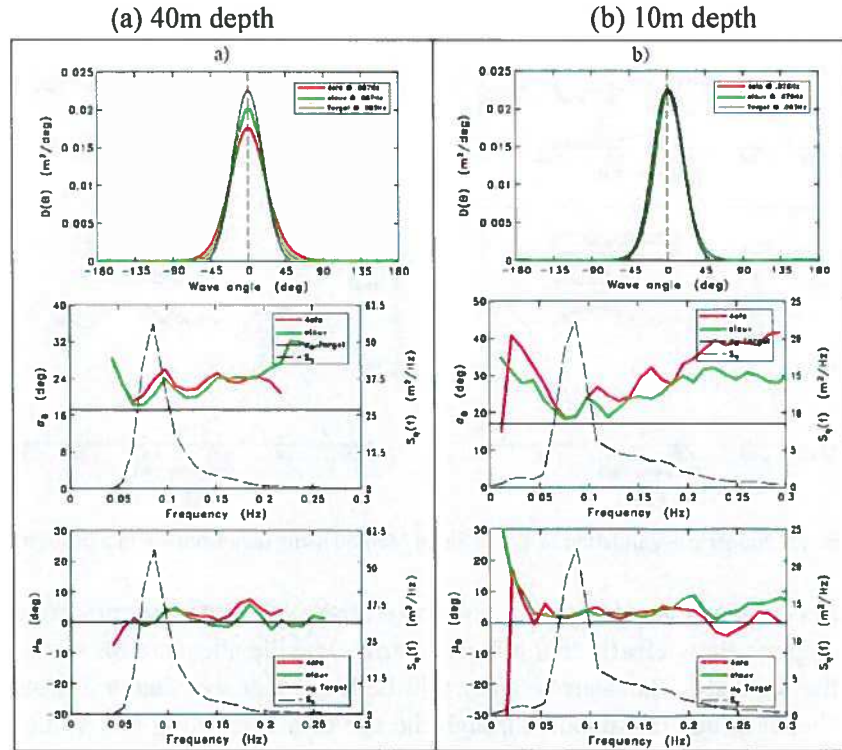


Figure 16: Directional Wave Analysis Results for a set of Wave Basin Data (Cornett et al , 2005)

Given below are the conclusions that were drawn from basin studies conducted at NRC by Cornett et al (2005):

The surface slopes MEM method (i.e. $4\text{-}\eta$ MEM) performs about as well as the $\eta\text{-}u\text{-}v$ MEM method over a wide range of wave conditions and water depths. Tests conducted with bi-modal seas, non-linear shallow water waves and even breaking waves indicate that both methods perform reasonably well, even under these challenging conditions. The surface slopes MEM method offers a number of operational advantages such as calibration compared with alternative methods, and is a viable alternative for accurate and reliable directional wave analysis in the laboratory. However, the optimal array radius being dependent on the dominant wave length could be perceived as a disadvantage of the surface slopes method.

Readers are asked to refer to Cornett et al (2005) for more detailed information on the validation tests.

6.0 Numerical Modelling of Multidirectional Wave Basins

Numerical models have been developed for evaluating design options for new multidirectional wave basins but they can also be very useful tools for designing experiments in existing wave basins. They provide an effective way to select the best wave generation method and basin configuration in order to optimize the quality of the generated waves in a designated region of the basin for a particular type of model test. One such numerical model named WAGEN was originally developed by Isaacson et al (1990) for the case of oblique unidirectional regular waves generated by one or more segmented wave machines in a basin of constant depth. It is a linear diffraction model, which uses a large number of point wave sources to represent the action of the wave machine segments and the other fully reflecting or partially reflecting boundaries of the basin. Hiraishi et al (1992) provides some results on the experimental validation of this model in a multidirectional wave basin.

The WAGEN model was subsequently extended by CHC to cover the case of multidirectional irregular waves and wave machines with active wave absorption. It uses an iterative technique to compute the primary wave field and the secondary wave fields produced by partial reflection of the primary waves from the passive wave absorbers in the basin as well as the re-reflection of any incident waves from the segmented wave generators.

In the case of regular waves, WAGEN computes the wave height and the horizontal u and v wave velocity components over a specified $x\text{-}y$ grid in the basin for a target wave train defined by period, wave height and wave propagation angle. In the case of multidirectional irregular waves, the target wave field is typically synthesized by approximately 2000 wave components with individual frequencies, amplitudes, directions and phases. The amplitudes and phases are computed by the Random Phase method for a specified wave spectrum and the wave directions are selected at random based on the cumulative distribution defined by a specified directional spreading function. The velocity potential $\Phi(x,y)$ is first computed for each of these wave components and the final velocity potential for the total wave field is obtained by linear superposition. WAGEN then computes the significant wave height H_s , the mean wave direction θ_m and the directional spreading width σ_θ as functions of x and y on the basin grid. Time series

of wave elevation $\eta(t)$ and velocities $u(t)$ and $v(t)$ can also be computed at specified points in the basin so that directional wave spectra can be computed by the Maximum Entropy Method.

Most multidirectional wave basins have a segmented wave machine on one side and passive wave absorbers on the other three sides. The passive absorbers are essential to prevent reflected waves from propagating back to the model test site but they also cause substantial variations in wave height due to wave diffraction. There are also wave height variations due to reflection from the passive absorbers. WAGEN can be used to compute the wave height variation over the basin so that the location and orientation of the structure being tested can be chosen to minimize the effects of diffraction and reflection. In some cases, partial length guide walls are used to reduce diffraction and WAGEN is also very useful for selecting the best length and position for such devices. For example, diffraction effects can be reduced by using intentional reflection off a short guide wall at one end of a segmented wave generator (i.e. extending from $X=0$ to 5 m in this case) for the case of unidirectional regular waves propagating at an angle of 30 degrees relative to the x-axis of the basin with a target wave height of 0.2 m (see Figure 17). The wave generator is located on the left side of the basin. It can be seen that the corner reflection method has significantly reduced the size of the diffraction zone on the lower right side of the basin. The increased wave height near the origin due to the corner reflection technique is also evident.

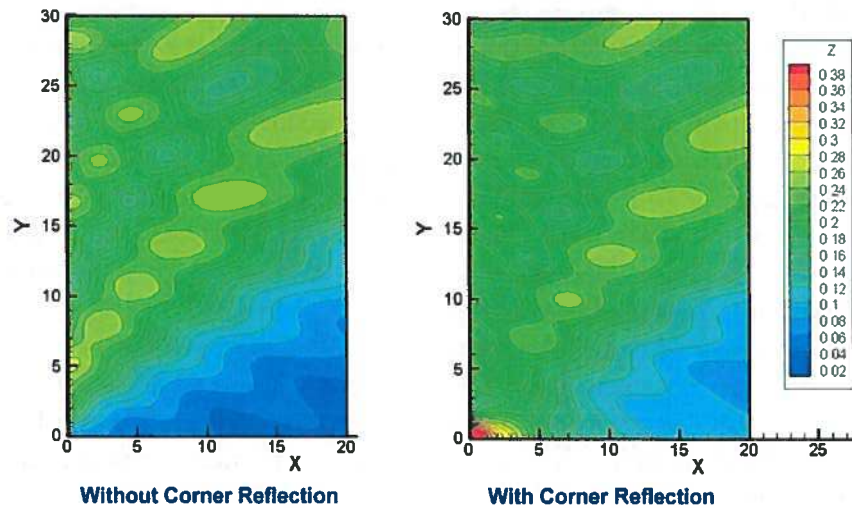


Figure 17: Contour plots of Regular Wave Height with and without Corner Reflection

A contour plot of the normalized significant wave height computed by WAGEN for the case of a multidirectional JONSWAP spectrum with a spreading width of $\sigma_0 = 30$ degrees and a mean direction of 0 degrees is shown in Figure 18. The segmented wave generator is located on the left side of this basin and passive absorbers with a reflection coefficient of 0.1 are installed on the other three sides. These results show a useful working area about 5 m by 5 m at the centre of the basin where the wave height is quite homogeneous but there are large variations in other parts of the basin due primarily to diffraction.

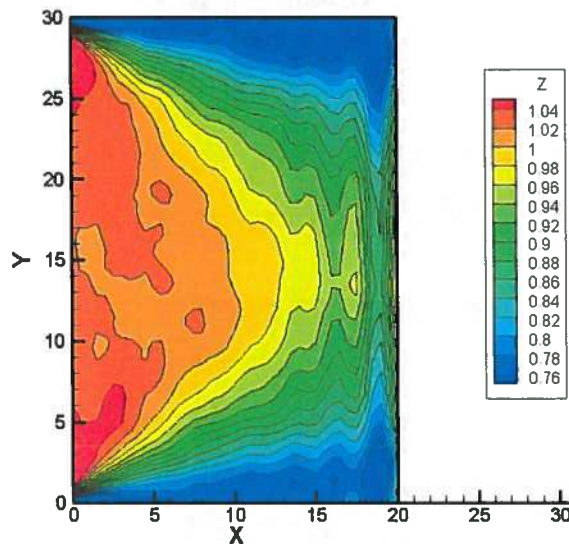


Figure 18: Normalized Significant Wave Height for Irregular Multidirectional Waves

Multidirectional wave basins designed for testing moving ship models usually have segmented wave machines installed on two adjacent sides to reduce the size of the diffraction zones. The normalized significant wave height computed by WAGEN for a basin with segmented wave machines installed on the bottom and left sides are shown in Figure 19. These waves have a JONSWAP spectrum with a spreading width of 30 degrees and a mean wave direction is 90 degrees relative to the x-axis. It can be seen that the use of two wave generators has greatly increased the size of the useful testing area where the wave height is homogeneous. These results also show how the wave height uniformity can be further improved when active wave absorption is used to prevent waves generated by the bottom wave machine from being reflected off the other wave machine on the left side.

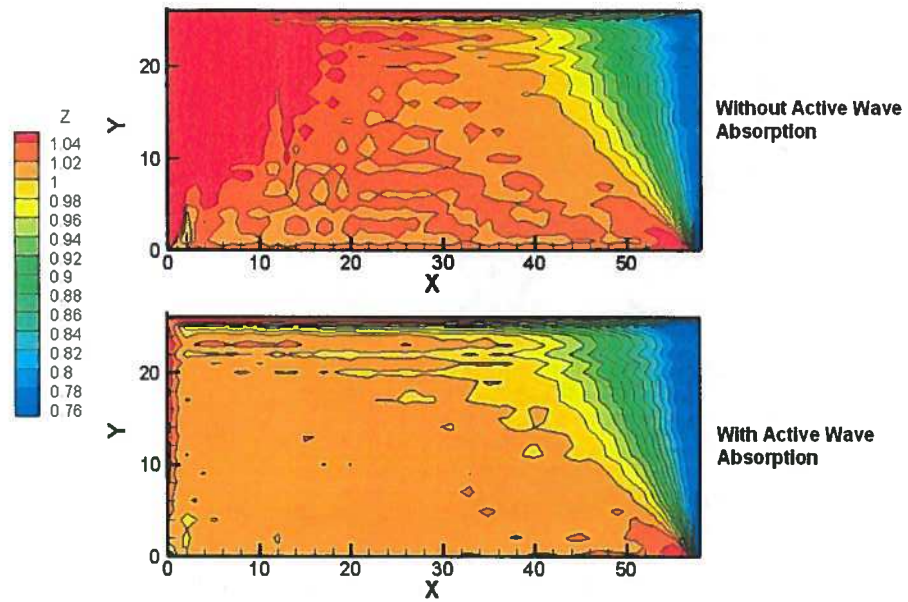


Figure 19: Normalized Significant Wave Height for a Basin with two Wave Generators

The corresponding mean wave direction computed by WAGEN for this case is shown in Figure 20, which also demonstrates the improvement in wave quality due to active wave absorption.

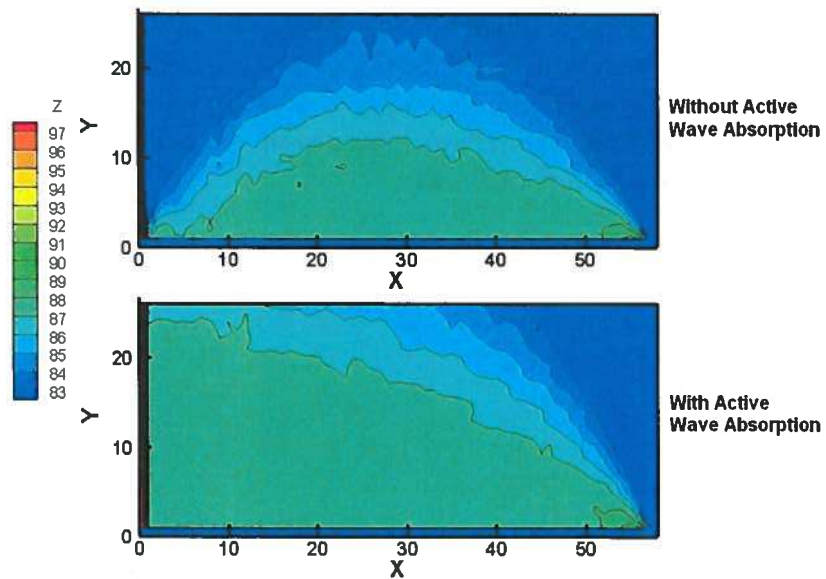


Figure 20: Mean Wave Direction for a Basin with two Wave Generators

7.0 Conclusions

Wave simulation technology has made great strides in the last two decades that it is now possible to ensure realistic ocean waves inside laboratory environments for testing purposes. Sophisticated analysis techniques ensure that the desired characteristics of the waves are faithfully reproduced. Advanced tools are also available for properly designing experiments in laboratory basins and for ensuring improved wave quality by preventing waves reflected by structures being re-reflected back to the structure by the wave generator. Further research is required to develop techniques that can analyse non-linear multidirectional waves, separate the incident and reflected components from co-existing multidirectional waves and also to perform true 3D Active Wave Absorption.

8.0 References

- Barthel, V., Mansard, E.P.D., Sand, S.E. and Vis, F.C., (1983). Group Bounded Long Waves in Physical Models. *Ocean Engineering*, Vol. 10, No. 4, pp. 261-294.
- Benoit, M, Frigaard, P., Schäffer, H., (1997). Analysing Multidirectional Wave Spectra: A Tentative Classification of Available Methods. *Proc. IAHR Seminar on Multidirectional Waves and their Interaction with Structures*, 27th IAHR Congress, San Francisco, USA. Edited by Etienne Mansard.
- Cornett, A., Miles, M. and Pelletier, D., (2005). Measurement and Analysis of Multidirectional Waves using Free Surface Slopes. *Proc. 5th International Symposium on Wave Measurement and Analysis*, Madrid, Spain.
- Cornett, A. and Miles, M.D., (1991). Simulation of Hurricane Seas in a Multidirectional Wave Basin. *OMAE Journal*, Vol. 113, pp. 219-227
- Franco, C., Van der Meer, J.W. and Franco, L., (1996). Multidirectional Wave Loads on Vertical Breakwaters. *Proc. 25th International Conference on Coastal Engineering*, Orlando, Florida, USA.
- Funke, E.R.R., and Mansard, E.P.D. (1992). On the Testing of Models in Multidirectional Seas. Presented at the 23rd Int. Conference on Coastal Engineering, Venice, Italy.
- Funke, E.R.R., and Mansard, E.P.D., (1987). A Rationale for the use of the Deterministic Approach to Laboratory Wave Simulation. *Proc. Of the Seminar on Wave Generation and Analysis in Laboratory Basins*, 22nd IAHR Conference, Lausanne, Switzerland.
- Funke, E.R.R. and Mansard, E.P.D., (1979). SPLSH: A Program for the Synthesis of Episodic Waves. National Research Council Hydraulics Laboratory Technical Report LTR-HY-065.
- Funke, E.R.R and Mansard, E.P.D., (1979). On the Synthesis of Realistic Sea States in a Laboratory Flume, National Research Council Hydraulics Laboratory Technical Report LTR-HY-066.
- Goda, Y., Hawkes, P., Mansard, E.P.D., Martin, J.M., Mathiesen, M., Peltier, E., Thompson, E.F. and VanVledder, G.Ph., (1993). Intercomparison of Extremal Wave Analysis Methods Using Numerically Simulated Data. *Proc. Waves '93 Conference*,

New Orleans, U.S.A.

Hoklie, M., Stansberg, C.T. and Werenskiold, P. (1983). Model Tests with a Single Point Mooring System in Short-Crested Seas. Paper 4644, 15th Offshore Technology Conference, Texas, USA.

Hiraishi, T., Mansard, E.P.D., Miles, M.D., Funke, E.R., Isaacson, I. (1992). Validation of a Numerical Diffraction Model for Multidirectional Wave Generation Part 2: Experimental Verification of the Model Results. Presented at ISOPE-92 Conference, San Francisco, June 14-19.

IAHR/PIANC (1986). List of Sea State Parameters. PIANC Supplement to Bulletin No. 52, General Secretariat of PIANC, Rue de la loi 155, 1040 Brussels, Belgium.

Isaacson, M., Papps, D., and Mansard, E.P.D. (1995). Oblique Reflection Characteristics of Rubble mound Structures. Proceedings of the 12th Canadian Hydrotechnical Conference, Ottawa, Canada, Vol. 1, pp.445-454.

Isaacson, M. and Qu, S.Q., (1990) "Predicted Wave Field in a Laboratory Wave Basin", Canadian Journal of Civil Engineering, Vol. 17, No. 6, pp. 1005-1014.

Jamieson, W.W. and Mansard, E.P.D. (1987). An Efficient Upright Wave Absorber. Proc. ASCE Specialty Conference on Coastal Hydrodynamics, University of Delaware, USA.

Jefferys, E.R. (1987). Directional Seas should be Ergodic. Applied Ocean Research, 9, (4).

Johnson, R.R., Mansard, E.P.D. and Ploeg, J., (1978). Effects of Wave Grouping on Breakwater Stability. Proc. 16th International Conference on Coastal Engineering, Hamburg, Germany, pp. 2228-2243.

Mansard, E.P.D. (1998). Directional Wave Generation and its Application. Proc. Of the Symposium Real Sea '98, Taejon, Korea.

Mansard, E.P.D., (1994). On the Estimation of the Incident Wave Train from Reflection Analysis, Proc. of 9th Congress of the Asian-Pacific Division of International Association for Hydraulic Research, 24-26 August, Singapore.

Mansard, E.P.D. and Funke, E.R.R., (1987). On the Reflection Analysis of Irregular Waves. National Research Council Hydraulics Laboratory Technical Report TR-HY-017.

Mansard, E.P.D., Sand, S.E. and Funke, E.R.R., (1985). Reflection Analysis of Non-linear Regular Waves. National Research Council Hydraulics Laboratory Technical Report TR-HY-011.

Mansard, E.P.D. and Pratte, B.D., (1982). Moored Ship Response in Irregular Waves. Proc. 18th International Conference on Coastal Engineering, Capetown, South Africa, pp. 2621-2640.

Mansard, E.P.D. and Funke, E.R., (1980). The Measurement of Incident and Reflected Spectra Using a Least Squares Method. Proc. 17th International Conference on Coastal Engineering, Sydney, Australia, pp. 154-172.

Miles, M.D. and Funke, E.R., (1989). A Comparison of Methods for Synthesis of Directional Seas, A.S.M.E. Journal of Offshore Mechanics and Engineering, Vol. 111, No. 1, pp. 43--48.

Miles, M.D., (1989). A Note on Directional Random Wave Synthesis by the Single Summation Method, Vol. C, Proc. 23rd IAHR Congress, Ottawa, Ont., August, 1989, pp. 243-250.

Nwogu, O., Mansard, E.P.D., Miles, M.D. and Isaacson, M. St. de Q., (1987). Estimation of Directional Waves Spectra by the Maximum Entropy Method. Proc. Seminar on Wave Analysis and Generation, 22nd IAHR Conference, Lausanne, Switzerland, pp. 363-376.

Readshaw, J.S., Baird, W.F. and Mansard, E.P.D., (1987). Shallow Water Wave Generation: An Engineering Perspective. Proc. Seminar on Wave Analysis and Generation, 22nd IAHR Conference, Lausanne, Switzerland, pp. 397-410.

Sand, S.E. and Mynett, A.E. (1987). Directional Wave Generation and Analysis. Proc. IAHR Seminar on Wave Analysis and Generation in Laboratory Basins, 22nd IAHR Congress, Lausanne, Switzerland.

Sand, S.E., Mansard, E.P.D., (1986). Description and Reproduction of Higher Harmonic Waves. National Research Council Hydraulics Laboratory Technical Report TR-HY-012.

Sarpkaya, T. and Isaacson, M. (1981). Mechanics of Wave Forces on Offshore Structures, Van Norstrand Reinhold Company Inc., New York.

Schäffer, H. and Klopman, G., (1997) "Review of Multidirectional Active Wave Absorption Methods", IAHR Seminar on Multidirectional Waves, 27th IAHR Congress, San Francisco, pp. 159-182.

Stansberg, C.T., Krokstad, J.R. and Nielsen, F.G. (1997). Model Testing of the Slow Drift Motion of a Moored Semi Submersible in Multidirectional Waves. Proc. IAHR Seminar on Multidirectional Waves and their Interaction with Structures, 27th IAHR Congress, San Francisco, USA. Edited by Etienne Mansard.

