# Ground failure under the action of a track grouser

**Publisher's version / Version de l'éditeur:**

*Technical Memorandum (National Research Council of Canada. Division of Building Research); no. DBR-TM-2, 1945-09-01*

CONFIDENTIAL

NATIONAL RESEARCH COUNCIL OF CANADA

ASSOCIATE COMMITTEE ON SOIL AND SNOW MECHANICS

# TECHNICAL MEMORANDUM NO. 2

# GROUND FAILURE UNDER THE ACTION OF A TRACK GROUSER

(A contribution from the Directorate of Vehicles and Small Arms)


## Summary

The application of Rankine's Theory of Earth Pressure to the action of soil under a grouser is discussed; model experiments are described which cast some doubt upon the accuracy of this application, and a suggestion of an alternative theoretical solution is advanced.


Ottawa, Canada,
September, 1945.

# NATIONAL RESEARCH COUNCIL OF CANADA

## ASSOCIATE COMMITTEE ON SOIL AND SNOW MECHANICS

### Technical Memorandum No. 2

### Ground Failure under the Action of a Track Grouser

There is some analogy between the action of a grouser and that of a vertical diaphragm displaced horizontally in the ground. This analogy has been applied to the problem of earth failure under the action of a track grouser, as considered in the light of the Rankine Theory of Passive Earth Failure.

Since the shape of the failure surface beneath a track link and the normal stresses thereon determine the track "drawbar pull", it was considered worthwhile to investigate the whole question with a view to ascertaining whether the Rankine Theory could not be applied to this grouser problem in the same way as it may be applied to the problem of a retaining wall, subject to the same limitations.

### (2) Pattern of Failures

It can be shown that a blade thrust into an earth medium and displaced slightly in a horizontal direction causes a ground failure, the pattern of which has been explained on the basis of theoretical soil mechanics (see Fig. 1:a,b,c,).

In order to illustrate the problem the better, a few basic points of the Rankine Theory (as propounded by Terzaghi) may be quoted. If a diaphragm ab (Fig. 2) is tilted in such a manner that it takes the position marked by the line $ba_1$, the surface of failure of the ground will be the line bc inclining to the horizontal at an angle $45-\phi/2$ where $\phi$ is the angle of internal friction.

The state of the earth inside the triangle $ba_1c$ is called Passive Rankine State. If there is friction between the diaphragm and the earth or if a different type of displacement than tilting is introduced, the lines of ground failure will be curved as shown in Fig. 3. The passive Rankine state will then be limited to the triangle adc only, whereas the portion abd of the shear pattern is composed of curved lines (logarithmic spirals).

### (3) Theoretical Considerations with regard to the Displacement of a Grouser.

During the transition of the mass from its original (elastic state) to the passive Rankine state the soil within the critical sections undergoes compression in a horizontal direction. "At any depth N below the surface (Fig. 2) the width $\Delta x$ of the sheared area is equal to the width x of the wedge at that depth times the horizontal compression "e" per unit of length which is required to transfer the soil at depth N from its original state of elastic equilibrium to the plastic equilibrium. Hence, at depth N the width of the shaded area equals ex. The value "e" depends not only on the type of soil and its density and on the depth N, but also on the final state of stress.

Hence the only general statement which can be made concerning the shaded area is that its width must increase from zero at a point b to a maximum at point a."*

For a uniform homogeneous cohesionless mass (sand) "e" does not depend on depth N. On this assumption, if we admit no friction between the diaphragm and soil the shaded area aba (Fig. 2) will be a triangle. If friction is considered (Fig. 3) then the shaded area which determines the required diaphragm displacement must be a non-regular figure $aba_1$, since "e" is now variable with depth. It is therefore obvious that any displacement which involves a shear pattern as described by Fig. 3 should also involve lateral yielding of the diaphragm because of its initial regular shape until the final shape $aba_1$ is obtained. If this condition is not satisfied the shear pattern as represented above will not be justified. In practice the grouser does not yield. Therefore the final shear pattern which is the result of a horizontal parallel displacement of the grouser is the resultant of an indefinite number of momentary patterns the potential origin of which are any transitory positions of the grouser in its passage from initial to final states.

The displacement of the diaphragm parallel to its original position is connected also with the so-called "arching" which adds more complexity to the whole picture.** Any conclusions, therefore, which were based on the Rankine Theory would not be justified, except possibly for an extremely small slip. This, however, would cause negligible ground failure. It is

---

* (Terzaghi - "Theoretical Soil Mechanics", New York, 1943, pp. 42-44).

** A brief survey of what is involved if the displacement of the face is not according to theory, is made in "Soil Mechanics" by Krynine, New York, 1941, pp 306-313.

therefore felt that theories of grouser spacing developed in accordance with the Rankine Theory, i.e. with the lines of failure inclined to the horizontal at an angle of 45-Ø/2 (Fig. 4), may have very little justification.

(4) The Effect of Supercharge

The above-mentioned conclusion may also be supported from another point of view. The Rankine Theory of Passive Ground Failure concerns a homogeneous, semi-infinite mass under the action of gravity. The gravity force may be increased by a so-called supercharge on the mass under consideration. The supercharge is tacitly admitted to be flexible and yielding in such a manner that it does not affect the shear pattern. In construction work in soil the supercharge on the loaded surface may be caused by earth; under water it may be caused by water. If we consider, however, a flat track shoe as a supercharge the conditions are different. Loading effected by the track shoe can be compared neither with load of a layer of earth, nor a depth of water. While the latter are flexible and yield with the ground without any change in the value of the supercharge, a track shoe is more or less rigid so that the value of supercharge may change while the soil is yielding. This happens not only under the weight of bogie wheels but also under the earthly compression itself.

This can be illustrated by the experiment shown in Fig. 5: a,b,c. The tracked link model loaded with 6 lbs. of weight is displaced horizontally and causes the sand failure as shown by the white layers. This figure shows that the surface of failure similar to that described by Rankine Theory takes place only in the triangle edc (Fig. 5 c). The shear pattern in the section abde is quite different. This can be easily anticipated since section edc is that concerned by Rankine where section abde is not.

Another example shown in Fig. 6:a, b, c, illustrates how much the shape of the shear surface depends on track flexibility and load. In this case a two-link track model is investigated without a hinge in between, one link being loaded with 6 lbs. weight, the other with only 2 lbs. This may be considered as a model illustrating the conditions for a track where the bogie-carrying link is far more loaded than the adjacent links located between two bogies. This experiment shows that the shear surfaces tend to change slope from one link to another and to extend from the lower edge of the grouser approximately horizontally for some distance, and then curve upwards towards the ground surface (that is, if the displacement of the grouser is sufficiently great).

There is little evidence of an alternative, that is, whether the rupture will occur more easily in a horizontal direction or along a surface inclined at an angle 45-Ø/2. The definite tendency to shearing in an approximately horizontal direction is illustrated by Fig. 7:a and b.

In this experiment, only one grouser model has been very slightly displaced and the rupture pattern is by no means sloped at the angle 45-Ø/2, as one would expect from the Rankine Theory. As has been already noted and described in Fig. 5, the abde portion of the shear pattern (Fig. 7b) cannot be explained by the Rankine Theory.

(5) Analogy to Plastic Flow of a Mass Compressed between two Plates.

In an attempt to find some useful suggestions which might be studied in order to find a more appropriate earth failure theory, experiments have been made and typical results are as shown below.

Fig. 8:a,b,c,d illustrates the manner of flow of sand particles under the track shoe as recorded on a smoked glass plate. The flow lines suggest some similarity with the findings of Prandtl and Nadai who investigated stresses and shear patterns of a plastic mass compressed between two rigid plates.*

The experiments confirm this conclusion and the shear pattern under a tracked shoe (Fig. 8) is similar to the theoretical one shown on Fig. 9. The equations of lines of rupture (in polar coordinates) would be similar to those quoted by Nadai, namely:-

$$r = b\sqrt{\frac{c-1}{c-\cos\psi}} \quad e^{\pm\frac{1}{\sqrt{c^2-1}}} \quad \arctan\left(\sqrt{\frac{c+1}{c-1}}\tan\psi/2\right)$$

$$\varphi = \frac{c}{\sqrt{c^2-1}} \arctan\left(\sqrt{\frac{c+1}{c-1}}\tan\psi/2\right) - \psi/2$$

---

* Zeitschrift Fuer Physik Vol XXX, Berlin, 1924 pp 106-124. A brief survey of this theory has been made by Nadai in his book entitled "Plasticity", New York and London, 1931, pp 229-233.

Where b is a constant for which $\varphi = \psi = o$

If the angle $\alpha$ at which both plates intersect is 45° in our case, and value c may be determined from the formula:

$$\alpha = \frac{c}{\sqrt{c^2 - 1}} \arctan \sqrt{\frac{c + 1}{c - 1}} - \frac{\pi}{4}$$

The value $\psi$ is a parameter, which determines the whole family of curves which appears on Fig. 9.

(6) Summary

The conditions required to cause earth failure according to the Rankine Theory have been analyzed. It is concluded that they cannot be applied to the case of a slipping track shoe with a grouser and that a two-set family of straight lines does not form the correct picture of the real shear pattern. Consequently, any considerations based on the Rankine Theory, as for example to the problem of spacing track grousers, can have little practical application.

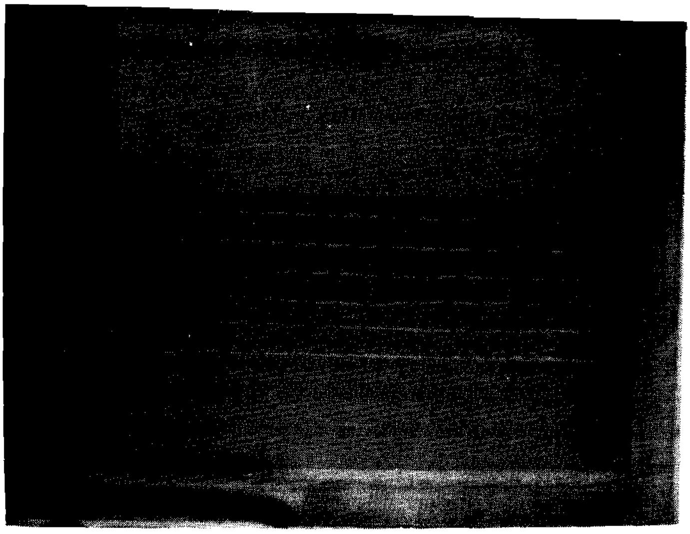
Experiments made with models indicate that the final shape of earth failure depends not only on the soil but also on other factors such as track pitch, track tension and load distribution.

The above experiments also suggest that the analytical method of finding the shear pattern of a mass compressed between two plates is applicable here. Consequently the shear pattern of a particular track link will be a two-set family of logarithmic spirals, the general equation of which has been noted. The experiments also suggest that further study of the fundamentals of grouser action may usefully be considered. The Committee has arranged for such an experimental program to be carried out during the winter of 1945-46. Results will be published as early in 1946 as is possible.

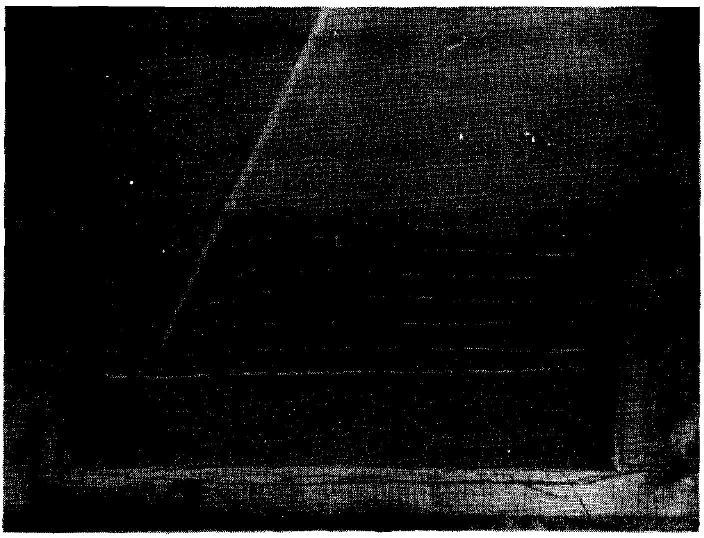
First Draft: August 1945.
Prepared by: M.G.B.
Checked by: R.F.L.
Approved: 24-31. VIII.'45
No. of Copies: 100
Issued. 2.x.'45.

/PB

FIG. 1.



A) BLADE THRUST INTO THE SOIL



B) BLADE TILTED

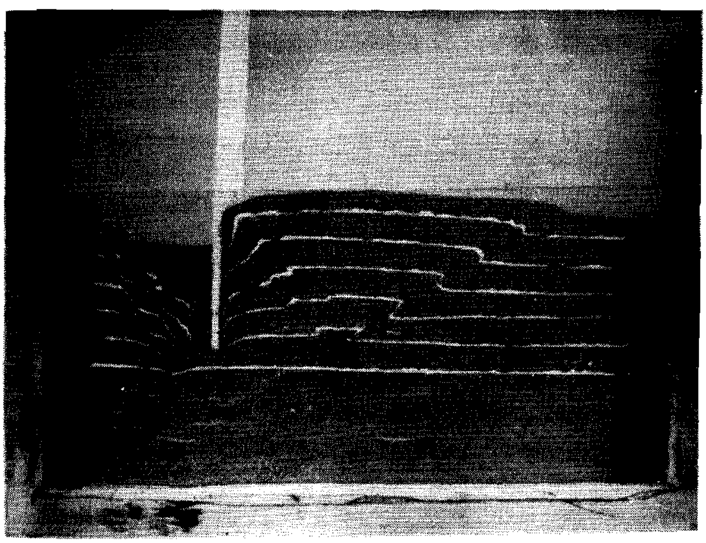
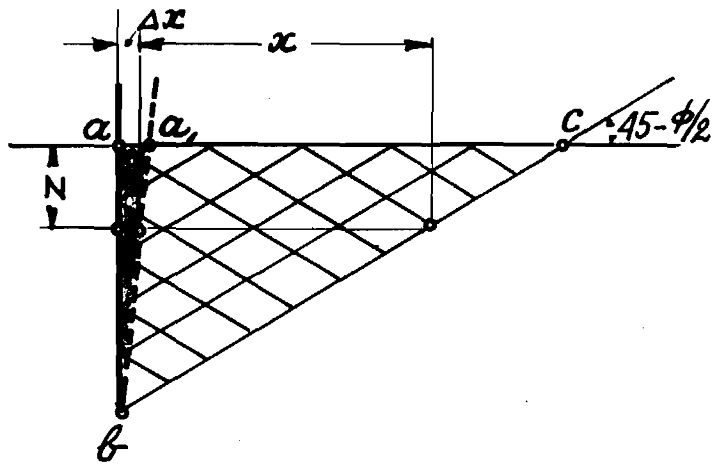
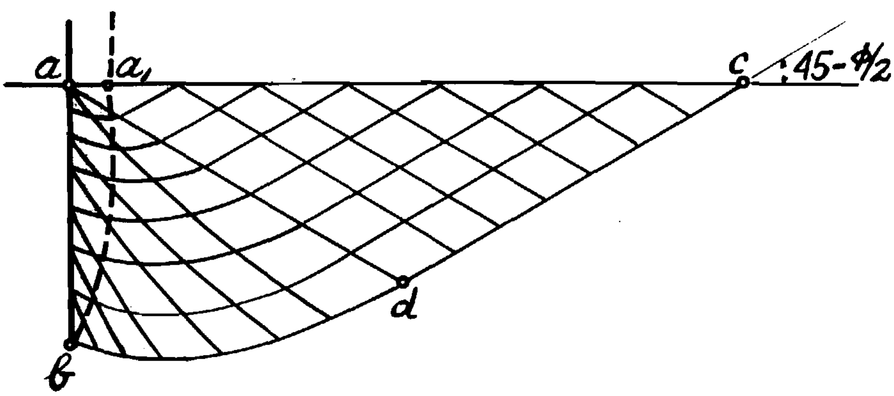


FIG. 2.



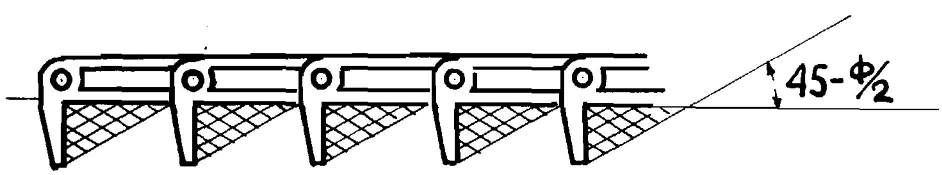

NOTE SIMILARITY WITH FIG. 1B

FIG. 3.

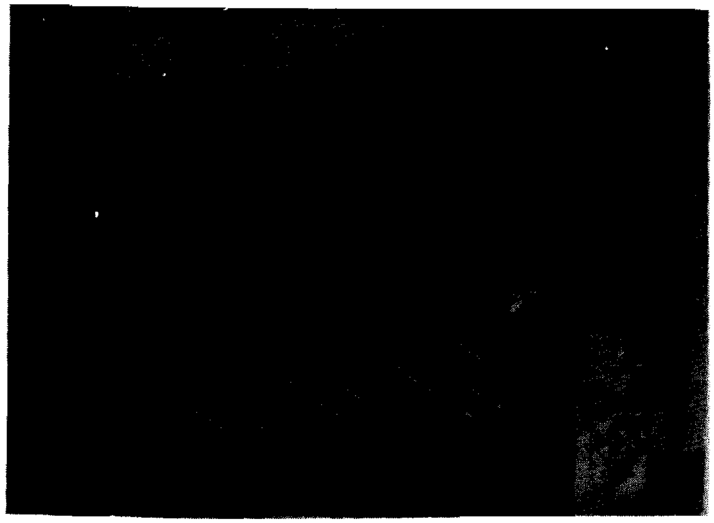



NOTE SIMILARITY OF SHEAR PATTERN WITH FIG. 1C

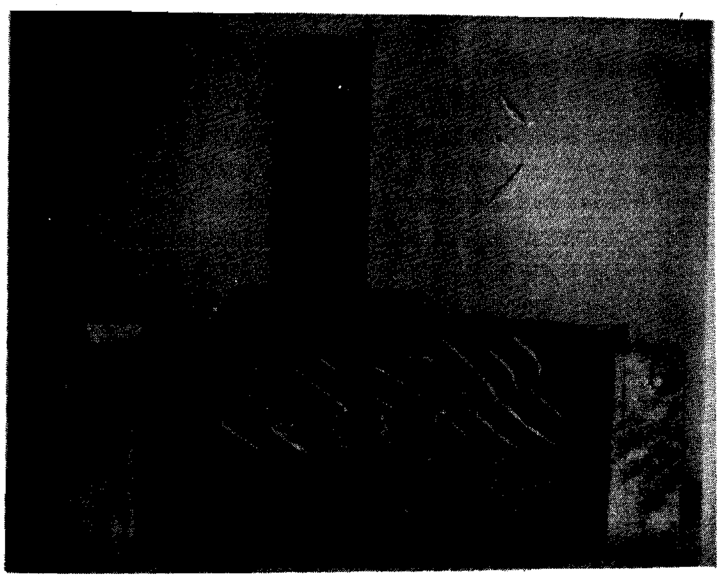
FIG. 4.



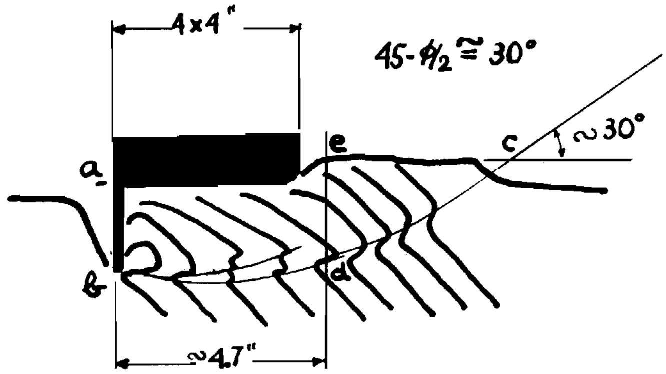
# FIG. 5.



A TRACK SHOE MODEL BEFORE DISPLACEMENT.

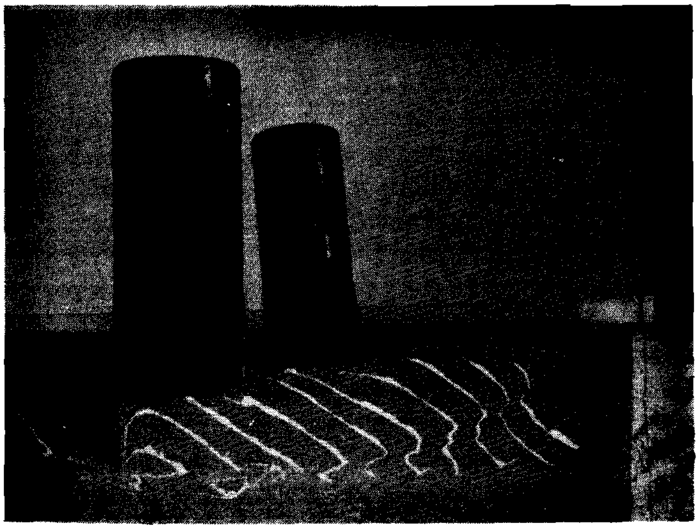


B TRACK SHOE MODEL AFTER DISPLACEMENT.

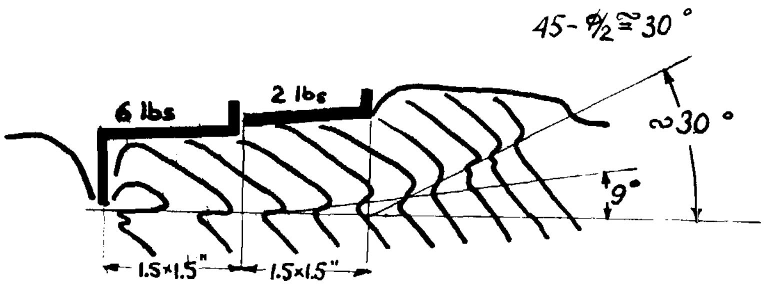


# FIG. 6.

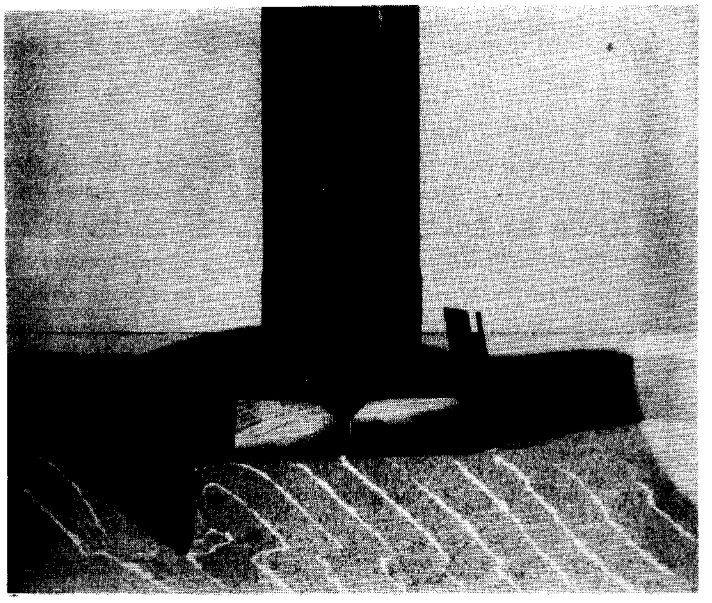
A TWO-LINK TRACK MODEL WITHOUT WEIGHTS BEFORE DISPLACEMENT



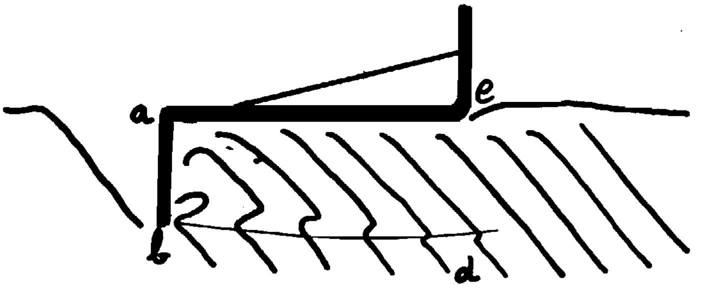
B. TWO-LINK TRACK MODEL AFTER DISPLACEMENT



# FIG. 7.

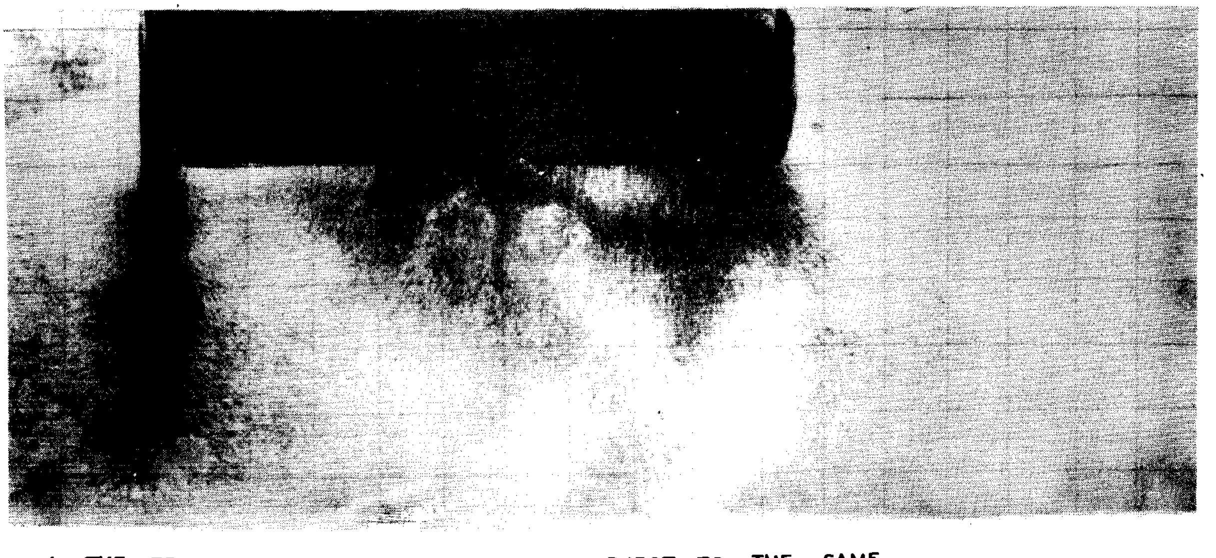


A ONE-LINK TRACK MODEL AFTER VERY SLIGHT DISPLACEMENT




B SHEAR SURFACE AFTER FIG. 7A.

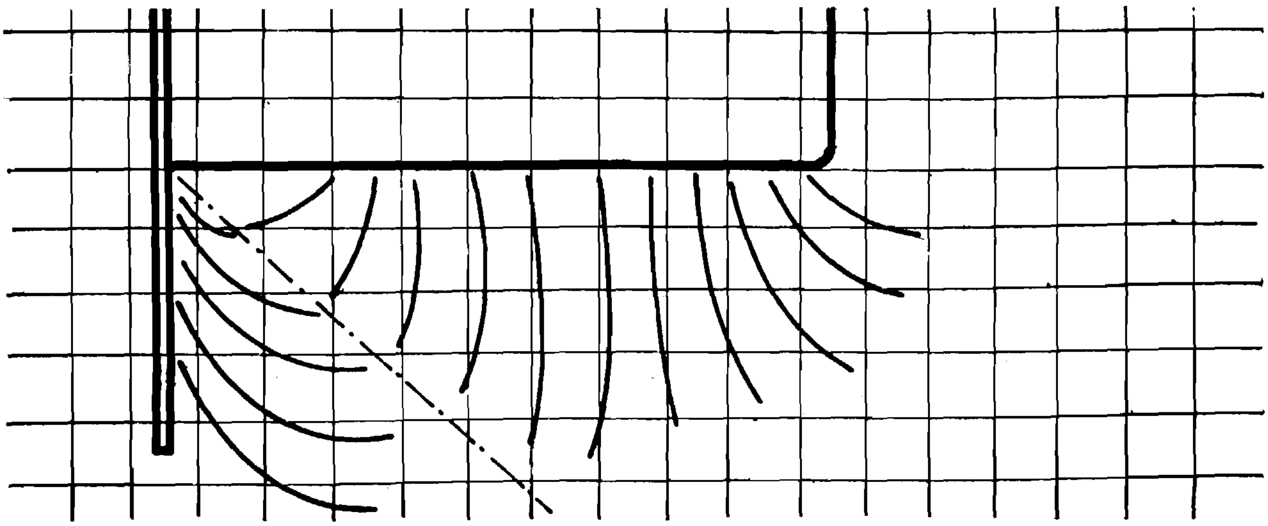
# FIG. 8.



A THE TRACK SHOE AND GROUSER ARE SUBJECT TO THE SAME UNIT PRESSURE. MODEL HAS NOT BEEN DISPLACED.

FIG. 8

B ELONGATION OF SPIRALS SHOWN AT FIG. 8 A ATTRIBUTED TO THE DISPLACEMENT OF THE MODEL



C SHEAR PATTERN AFTER FIG. 8 A.

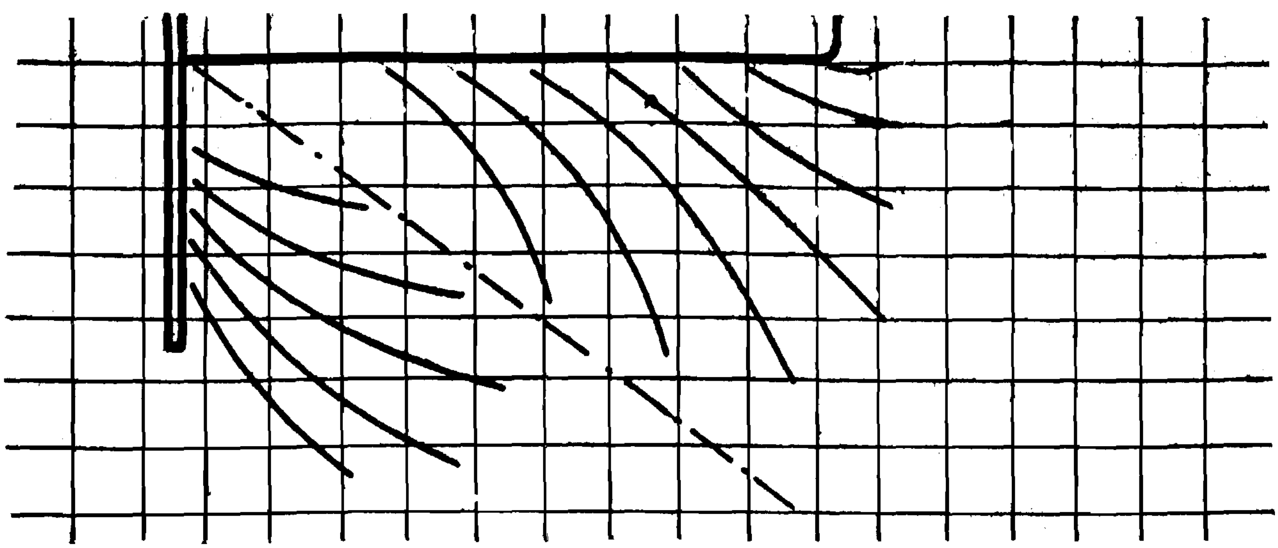
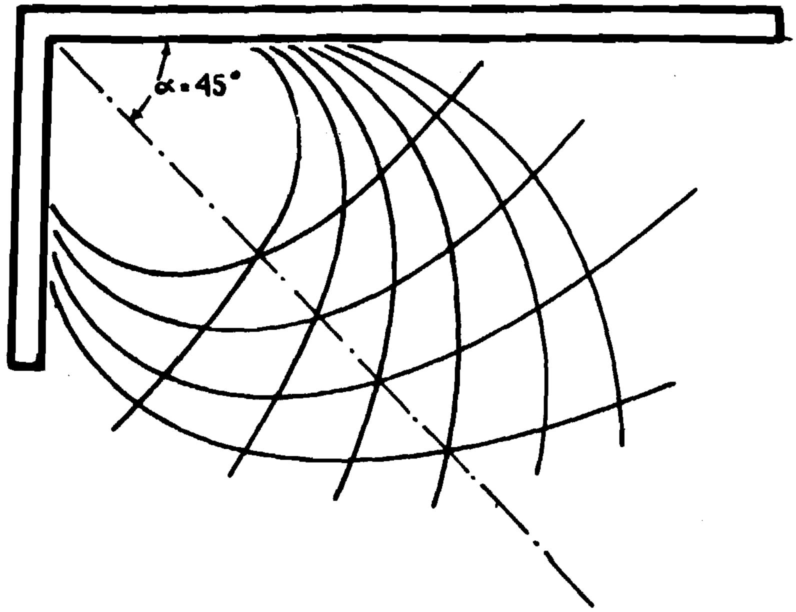


FIG. 9.




THEORETICAL SHEAR PATTERN

ASSOCIATE COMMITTEE ON SOIL AND SNOW MECHANICS

TECHNICAL MEMORANDUM NO. 2: DISTRIBUTION LIST

(a) Through Secretary of Committee, N.R.C.

President, N.R.C.
Other Members of Committee (6)
Committee File
Secretary Treasurer, N.R.C.
Director, Division of Mechanical Engineering, N.R.C.
Officer in Charge, Research Plans and Publications, N.R.C.
Central Records, N.R.C.
Library, N.R.C.

Scientific Liaison Officer, London, England: (4)
N.R.C. London Office
C.M.H.Q. London
British Mud Crossing Committee (2)

S.A.E. Advisory Mud Crossing Committee (Mr. R. Sackett, Secretary, Detroit) (5)

U.S. Dept. of Agriculture Soil Tillage Laboratory, Auburn, Ala.

U.S. Waterways Expt. Station, Vicksburg, Miss. (Dr. G. H. Matthes, Director)

(b) Through D.V.S.A. (Ottawa)

D.V.S.A. Library, Ottawa
D.O.R. Library, Ottawa
D.S.D. (W), Ottawa (2)
D.M.O.&P., Ottawa
D.E.D., Ottawa

Commander, Camp Borden

Commanding General, ADT Branch, USAAF, Orlando, Florida.

C.M.H.Q. London (6)
C.M.H.Q. - S.D.W.
S.T.T.
D.T.D.
T.T.2
D.R.A.C. - War Office
D.G.F.V.R.&D. - M. of S.

C.A.S. Washington (7)
Combined Operations Beach Trafficability Committee (Lt. Col. Sturdevant, U.S. Marines Chairman) (2)
U.S. Engineer Board, Fort Belvoir (2)
Aberdeen Proving Grounds (for attention Major C. S. Mackenzie)
Australian Military Mission (2)

<u>Canadian Liaison Officer, Detroit</u> (5)
C.O.C. Detroit (2)
B.A.S. (Attention Brigadier Ross) (3)

Canadian Military Attache, Australia