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A. INTRODUCTION

Lateral or translational motion of structures is often accompanied by rotations of cross sections due to flexural deformations, as illustrated in Fig. 1. These rotations are particularly pronounced on tall towers or chimneys. When full-scale dynamic properties of such structures are measured, transducers deployed for determining lateral motion are thus subjected to rotation as well as translation. It is the purpose of this report to determine the effect of such rotations on the signals obtained from full-scale measurements of lateral motions of chimneys and tower structures.

B. THEORETICAL CONSIDERATIONS

1) General Assumptions and Scope

The rotational effects investigated here are those that are described by the small deflection theory of structures and the linear range of structural and material behaviour. The translational transducer is assumed to be modelled by a single-degree-of-freedom (SDF) oscillator. The results are thus directly applicable to instruments that consist of a mass supported by a spring element such as a piezoelectric crystal. They also apply to transducers such as force-balance or servo accelerometers which are more complex in construction but whose behaviour closely resembles that of an SDF oscillator.

Only transducers that measure horizontal motion are considered here since it can be easily demonstrated that for practical purposes vertical transducers are not affected by small rotations of their base.

2) Amplification Factors for Transducers

The schematic model of a transducer is shown in Fig. 2. Summation of forces in the horizontal direction, neglecting second-order terms, gives:

$$m\ddot{r} + c\dot{r} + kr = -m\ddot{s} + mg\theta \tag{1}$$

where: m = mass of transducer;
c = viscous damping coefficient;
k = spring stiffness;
r = relative displacement of mass;
s = horizontal displacement of transducer base;
θ = angle of rotation of transducer base from horizontal;
g = acceleration due to gravity.

Division by m and simplification gives:

$$\ddot{r} + 2\beta\omega_0\dot{r} + \omega_0^2 r = -\ddot{s} + g\theta \qquad (2)$$

where: $\omega_0 = (k/m)^{\frac{1}{2}}$ = natural frequency of transducer (rad/s);
$\beta = c/(2\sqrt{km})$ = damping ratio for transducer.

For consideration of the angular component of deformation in structures it is useful to distinguish between two cases: i) cases in which the rotation is a known proportion of the translation, and ii) cases in which the rotation is not known a priori, but can be determined by measurement or calculation. These two cases will now be considered in some detail.

i) Rotation is a known proportion of translation. Structures for which the rotational components are a known proportion of the translational amplitude are simple ones, for which a closed form relationship can be found, e.g. a uniform cantilever. For more complex structures, the rotational component can be calculated when the deflected shapes or mode shapes are determined numerically.

For this case, the angular component θ is directly related to the translation, s, under the assumptions of small deflection theory:

$$\theta = \frac{s}{R} \qquad (3)$$

where R can be interpreted as a radius of rotation (see Fig. 1).

The solution of Eq. (2) is of the form:

$$s = A\exp(i\alpha t)$$

$$r = B\exp(i(\alpha t - \phi))$$

$$\theta = \frac{A}{R}\exp(i\alpha t)$$

Substitution in Eq. (2) then gives the ratio of relative displacement of the transducer mass to the displacement of the base:

$$\frac{Be^{-i\phi}}{A} = \frac{\alpha^2 + g/R}{-\alpha^2 + i2\beta\alpha\omega_0 + \omega_0^2} \qquad (4)$$

Definition of the frequency ratio:

$$\Omega = \frac{\alpha}{\omega_0} \qquad (5)$$

and computation of amplitudes gives:

$$\frac{B}{A} = \frac{\Omega^2 + g/R\omega_0^2}{((1-\Omega^2)^2 + (2\beta\Omega)^2)^{\frac{1}{2}}} \qquad (6)$$

Equation (6) shows the relationship between the signal amplitude indicated by the transducer and the horizontal amplitude of motion of the transducer base. Since all terms in Eq. (6) are known, the relative magnitudes of translational signal and rotational signal can be found from the first and the second terms in the numerator, respectively.

Neglecting the second term in the numerator, i.e. the angular rotation term, one obtains the well-known amplitude relationship for the relative displacement of the transducer mass subjected to base displacement:

$$\frac{B}{A} = \frac{\Omega^2}{((1-\Omega^2)^2 + (2\beta\Omega)^2)^{\frac{1}{2}}} \tag{7}$$

Although phase is not considered explicitly, Eq. (4) shows that the inclusion of angular motion produces no additional phase shifts in the transducer as long as θ is in the same positive coordinate direction as s. When θ is in the opposite direction, the angular contribution is subtracted from the translational component.

ii) Proportion of rotational component is not known. For some structures, the amount of rotation that accompanies the translation is not known *a priori* from mechanics or has not been determined by other methods. It is often possible, however, to measure the rotation simultaneously with the translation. This is illustrated in Fig. 3, which shows a structural member rotating and translating about a centre of rotation. The radius of rotation R to the mid-member is given by:

$$R = \frac{s}{\theta} \tag{8}$$

The angle θ is determined by two transducers measuring vertical response A_1 and A_2 and separated by a distance L from which:

$$\theta = (A_1 + A_2)/L \tag{9}$$

With θ from Eq. (9) substituted in Eq. (8), the radius of rotation is not uniquely determined, however, because s is not known. Thus, a process of successive approximations has to be used. As an initial trial, R is determined from the amplitudes of a particular frequency component of the total signal and angle θ. Subsequent substitutions use this improved value of R, and convergence to a satisfactory accuracy should be achieved in two or three iterations.

C. NUMERICAL EVALUATION OF ROTATION EFFECT

1) General Parametric Study

From Eq. (6) it may be seen that the rotational and the translational components are related by the ratio of $g/R\omega_0{}^2$ to Ω^2, which for $\omega_0 = 2\pi f$ can be expressed as:

$$C = \frac{g/R}{(2\pi f)^2} \tag{10}$$

When there is no rotational component at the transducer location, $R = \infty$ and the ratio C is zero. When only rotation and no translation is present, then under small displacement assumptions $R = 0$ and the ratio C is infinity. For intermediate values of R, the natural frequency f of the structure is seen to be a dominant parameter since it appears in the denominator to the power 2. Thus, the rotational component of the signal becomes a significant fraction of the translation for very low structural frequencies. A parameter study of the variation of C as a function of R and f is presented in Table 1.

2) Radius of Rotation From Mode Shapes

The notion of radius of rotation of the horizontal transducer mounting is a useful one for cases where such values are available from structural mechanics or numerical analyses. The radius of rotation can also be determined graphically from the mode shapes of structures that deform predominantly in flexure. The projection onto the null line of the tangent drawn from the transducer location on the mode shape to where this tangent intersects the null line gives the radius of rotation. This conforms to the relationship in Eq. (3) and the illustration in Fig. 1 when $\sin\theta$ is replaced by θ as is consistent with small deflection theory.

3) Numerical Examples of Rotation Effects for Towers and Chimneys

The following examples were chosen from available data for towers and chimneys to demonstrate: a) the method of finding the ratio of rotations; and b) to arrive at representative values for rotational corrections for towers and chimneys. Actual transducer locations have been used wherever possible, although for some structures transducer locations have been assumed as indicated.

i) Emley Moor Television Tower, United Kingdom. Pertinent structural details are given in Ref. 1 and the mode shapes, reproduced from Ref. 2 are given in Fig. 4. A summary of the fraction of rotational to horizontal components for the three accelerometer locations is given in Table 2.

ii) CN Tower, Toronto, Canada. The structural details are presented in Ref. 3 and the mode shapes are shown in Fig. 5, along with the respective tangents to the transducer locations at the tip and below the lower observation platform. The rotational signal as a portion of translation is shown in Table 3.

iii) Reinforced Concrete Chimney, Badenwerk, Karlsruhe, Germany. This chimney is currently the object of a measurement program concerning response due to wind (6). The structural details and mode shapes are given in Fig. 6, showing the proposed locations of the transducers. The ratio of rotation to translational signal is shown in Table 4.

iv) Reinforced Concrete Chimney, Esso Refinery, Karlsruhe, Germany. Its response under wind loads was reported in Ref. 5. Radii of rotation for the only transducer at the tip were determined as described above and the ratios of rotation to translation are shown in Table 5.

v) SWF Television Tower, Hornisgrinde, Germany. Structural properties and behaviour in wind were reported in Ref. 6. Mode shapes (Ref. 4) are shown in Fig. 7, together with radii of rotation for an assumed transducer location at the tip and actual location at the 143 m level. Properties of rotational to translational signals are shown in Table 6.

vi) Mount Isa Stack, Queensland, Australia. From the mode shape presented in Ref. 7, the radius of rotation for the top accelerometer was determined to be 150 m. With a fundamental frequency of 0.263 Hz, the rotation is then 2.4% of the translation at the tip. This agrees well with the tilt correction of 2.5% quoted in Ref. 7.

D. DISCUSSION OF RESULTS

The numerical results indicate that rotations at the location of horizontal transducers can introduce errors in the signals of lateral motion measurements. Of the two governing parameters, the frequency f and the radius of rotation R, the former is the more important one since in Eq. (10) it is squared. Thus, the lower the frequency of the structure, the more important the influence of rotation becomes.

For the towers and chimneys investigated here, the maximum effect of rotations occurs in the fundamental mode. However, this maximum effect is not always associated with the tip transducer, as is shown by the results from the SWF Television Tower, Hornisgrinde, and the chimney at Badenwerk in Karlsruhe in Tables 6 and 4, respectively. The largest correction for rotation for the structures considered here occurs for the fundamental mode of the CN Tower, where the rotational component constitutes 14% of the translational component of the signal from the tip transducer.

When a transducer is located at or near a node of a mode, large values of the signal ratio C will be obtained since the translational component is very small there. This, however, does not occur for any of the structures and transducer locations considered here. A negative value of C obtained from a negative radius of rotation R indicates that the observed signal has to be augmented by the correction factor rather than reduced, as is the case for all positive values of C.

The above consideration of transducer rotation does not include a component of base rotation of the structure. Where such motion is significant, the radius of rotation is similarly determined as the distance on the null line from the transducer location to where the tangent to the deflected shape at the transducer location intersects the null line. This assumes, of course, that the mode shape drawn includes the contribution of the base rotation.

The determination of the radius of rotation by graphical means is of course subject to some uncertainties. First, the accuracy of the graphical representation of the mode shape is limited by the care with which this shape has been determined and drawn. Second, the drawing of the tangent at the transducer location is subject to some personal judgement. Fortunately, the radius R is not a very sensitive parameter, as was pointed out before, so that estimates of rotational signal contributions can still be obtained by this graphical method provided reasonable care is exercised. A preferable method of determining R would be to calculate the slopes associated with the mode shapes during the mode shape determination.

E. CONCLUSIONS

Dynamic lateral motions of slender structures such as towers and chimneys are usually accompanied by flexural rotations about a horizontal axis. Horizontal motion transducers mounted on such structures will give a signal that is partly affected by the rotational component.

It has been demonstrated that the rotational component in the signal from horizontal motion transducers for towers and chimneys can be obtained graphically from the mode shapes. Quantitative estimates for this effect were obtained for six sample structures. The two governing parameters are the natural frequency and the radius of rotation at the transducer location. It was shown that for low frequencies the rotational effect can be a significant portion of the translational signal.

For the six towers and chimneys considered here, the largest contribution to the transducer signal from rotation occurs for the tip transducer in the CN Tower where for the first mode the rotational component is 14% of the translation. A transducer located at lower levels can, however, reach a larger rotational component ratio than the tip transducer since transducers located near a node have small translational components, while the rotational component can be large.

F. ACKNOWLEDGEMENTS

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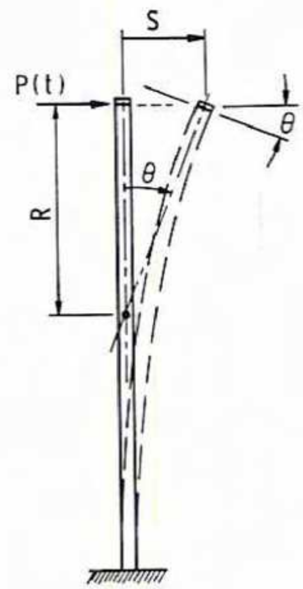

FIGURE 1

GEOMETRIC RELATIONS BETWEEN TRANSLATION AND FLEXURAL ROTATION OF STRUCTURES

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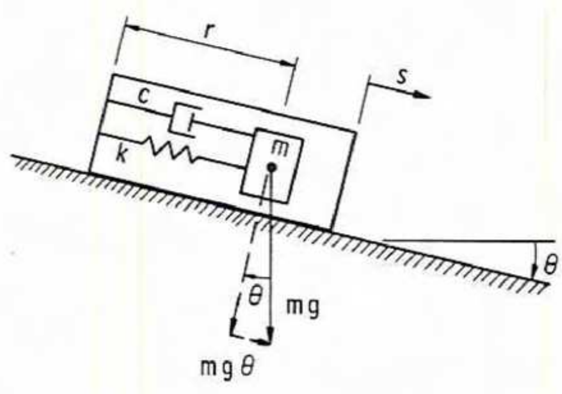

FIGURE 2

TRANSDUCER SHOWING FORCES AND COORDINATE DIRECTIONS

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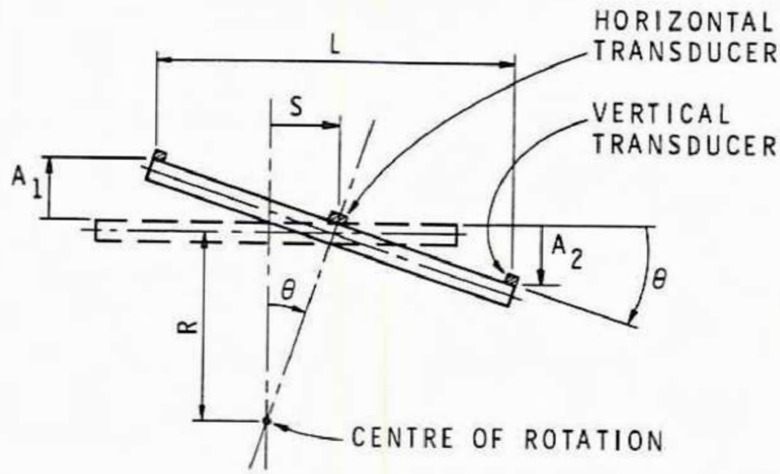

FIGURE 3

COUPLED TRANSLATIONAL AND ROTATIONAL DISPLACEMENT OF STRUCTURAL MEMBER

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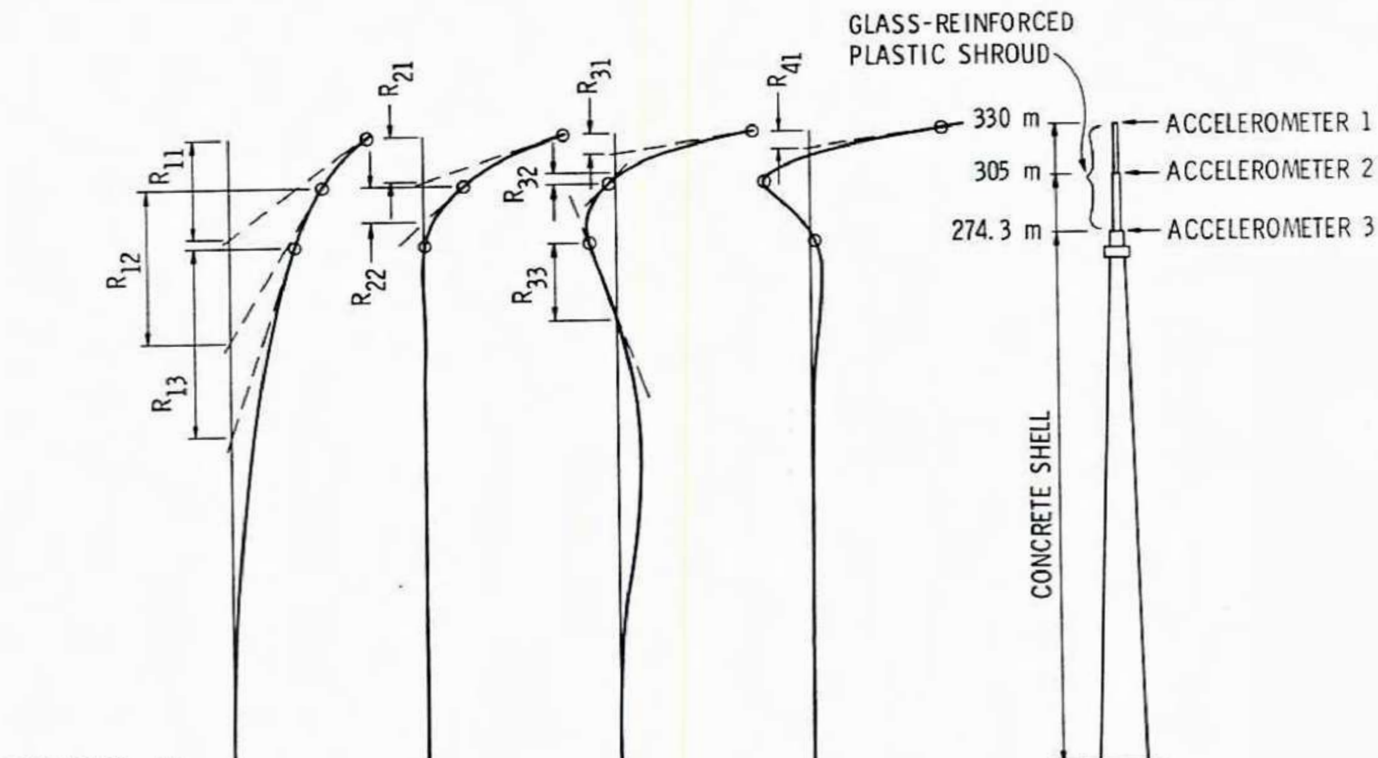

FIGURE 4

MODE SHAPES AND FREQUENCIES FOR EMLEY MOOR TELEVISION TOWER (ADAPTED FROM FIG. 11 IN REF. 2)

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TABLE 1

Signal Ratio of Rotational To Translational Component, $C = \frac{g/R}{(2\pi f)^2}$, For Various Values of Frequency and Radii of Rotation

Frequency f (Hz)	Radius of Rotation, R (m)						
	∞	200	100	50	33.3	10	5
0.05	0	0.50	1.00	2.00	3.00	10.0	20.0
0.10	0	0.125	0.25	0.50	0.75	2.5	5.00
0.20	0	0.0313	0.0625	0.125	0.1875	0.625	1.25
0.30	0	0.0183	0.0277	0.0555	0.0831	0.277	0.554
0.40	0	0.0078	0.0156	0.312	0.0468	0.156	0.312
0.50	0	0.0050	0.0100	0.0200	0.0300	0.100	0.200
1.00	0	0.00125	0.0025	0.0050	0.0075	0.025	0.050

Note: g was taken as 10 m/s^2.

TABLE 2

Rotational to Translational Signal Ratio C For
Emley Moor Television Tower

Mode No.	Frequency (Hz)	Transducer 1 at 330 m	Transducer 2 at 305 m	Transducer 3 at 774.3 m
Radius of Rotation, R (m) (See Fig. 4)				
1	0.26	50	80	96
2	0.53	23	18	-
3	0.96	10	-5	41
4	1.54	10	∞	-
Signal Ratio C				
1	0.26	0.074	0.046	0.038
2	0.53	0.039	0.050	-
3	0.96	0.027	-0.054	0.007
4	1.54	0.011	0	-

TABLE 3

Rotational to Translational Signal Ratio C for CN Tower

		Radius of Rotation, R (m) (See Fig. 5)		Signal Ratio C	
Mode No.	Frequency (Hz)	Transducer 1 at tip	Transducer 2 at lower platform	Transducer 1	Transducer 2
1	0.124	114	133	0.142	0.121
2	0.276	44	∞	0.074	0
3	0.488	24	-30	0.043	-0.035
4	0.834	17	∞	0.021	0
5	1.034	13	∞	0.018	0
6	1.841	9.6	0 (node)	0.008	∞

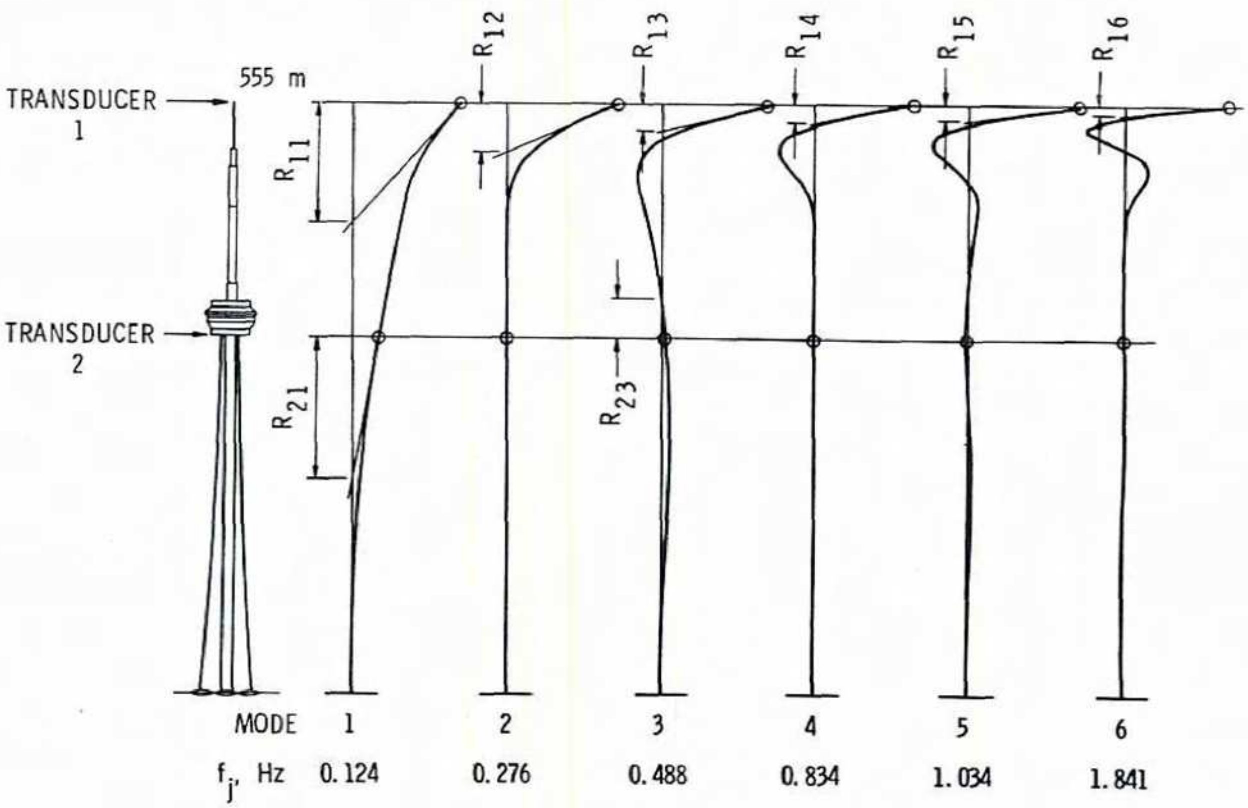

FIGURE 5

CALCULATED VIBRATION MODES OF CN TOWER, TORONTO (ADAPTED FROM REF. 3)

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TABLE 4

Rotational to Translational Signal Ratio C For Chimney at Badenwerk, Karlsruhe

		Radius of Rotation, R (m) (See Fig. 6)		Signal Ratio C	
Mode No.	Frequency (Hz)	Transducer 1 at 227 m	Transducer 2 at 136 m	Transducer 1	Transducer 2
1	0.20	114	53	0.055	0.117
2	0.65	32	∞	0.024	0
3	1.96	16	2	0.004	0.032

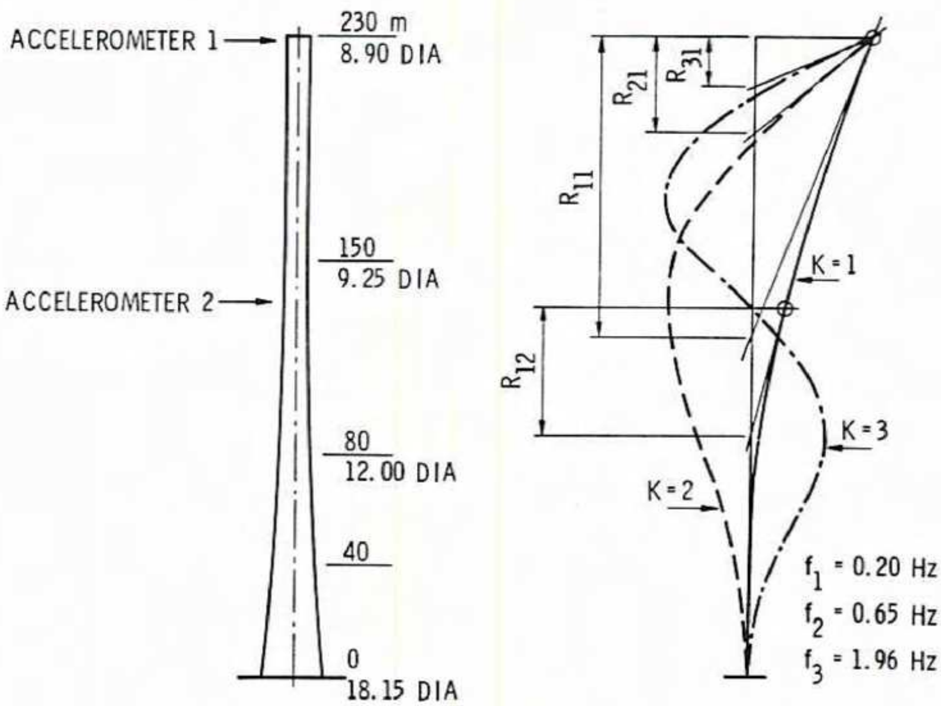

FIGURE 6

STRUCTURAL PROPERTIES AND RADII OF ROTATIONS FOR ACCELEROMETERS OF BADENWERK CHIMNEY, KARLSRUHE, GERMANY (ADAPTED FROM REF. 6)

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TABLE 5

Rotational to Translational Signal Ratio C For Esso Refinery Chimney, Karlsruhe (As Determined From Ref. 5)

		Radius of Rotation, R (m)	Signal Ratio C
Mode No.	Frequency (Hz)	Transducer 1 at 180 m	Transducer 1
1	0.26	92	0.040
2	0.99	37	0.007
3	2.34	22	0.002

TABLE 6

Rotational to Translational Signal Ratio C For
SWF Television Tower, Hornisgrinde

		Radius of Rotation, R (m) (See Fig. 7)		Signal Ratio C	
Mode No.	Frequency (Hz)	Assumed Transducer 1 at tip	Transducer 2 at 143 m	Transducer 1	Transducer 2
1	0.37	36.2	30.0	0.050	0.060
2	0.61	22.5	∞	0.030	0
3	1.47	8.1	-9.3	0.014	-0.012

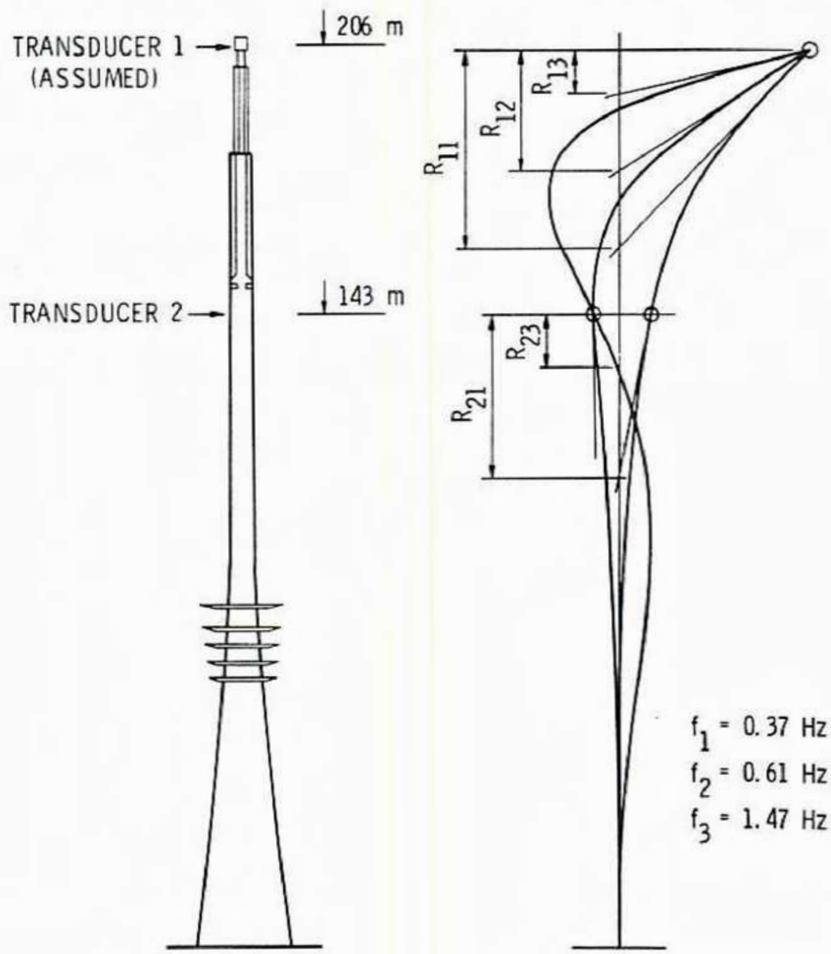

FIGURE 7

MODE SHAPES OF TRANSMISSION TOWER OF SWF HORNISGRINDE, GERMANY (ADAPTED FROM REF. 6)

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