Limit states design: a probabilistic study
Allen, D. E.

Publisher’s version / Version de l'éditeur:

Canadian Journal of Civil Engineering, 2, 1, pp. 36-49, 1975-03

NATIONAL RESEARCH COUNCIL OF CANADA
CONSEIL NATIONAL DE RECHERCHES DU CANADA

LIMIT STATES DESIGN—A PROBABILISTIC STUDY

BY

D.E. ALLEN

Reprinted from
CANADIAN JOURNAL OF CIVIL ENGINEERING
Vol. 2, No. 1, March 1975
14 p.

Research Paper No. 630
OF THE
DIVISION OF BUILDING RESEARCH

Price 25 cents OTTAWA NRCC 14516

Limit States Design—A Probabilistic Study

D. E. ALLEN

Building Structures Section, Division of Building Research, National Research Council of Canada, Ottawa, Canada K1A 0R6

Received October 28, 1974

Accepted December 17, 1974

Canadian structural standards for buildings are moving toward a unified limit states philosophy with common safety and serviceability criteria for all materials and types of construction. Structural steel and cold formed steel will have limit states design rules by 1975 and concrete, masonry, and wood will follow later.

This paper compares the new rules with existing NBC/CSA requirements on the basis of probability of failure calculated by simplified theory. The main emphasis is on load combinations of dead, floor, and wind loads for office and residential buildings where failure occurs by yielding of steel. Other aspects of the new limit states design rules—column formula for structural steel, performance factors for composite structures, the importance factor which reflects the seriousness of failure, and safety factors during construction, are also considered.

The results indicate that the new rules provide more consistent safety than existing rules for different combinations of loads and materials; and that simple rules are sufficiently accurate, keeping in mind the predominating influence of human error on failures and the simplifications used in analyzing complex building structures.

Les normes canadiennes pour le calcul des édifices subissent actuellement une évolution qui les oriente vers une approche unifiée fondée sur les états limites et mettant en jeu les mêmes critères de sécurité et d'utilisation pour tous les matériaux et tous les types de constructions. En 1975 on disposera des règles de calcul aux états limites pour les aciers de structure et les aciers formés à froid, des règles semblables devant être publiées plus tard pour le béton, la maçonnerie et le bois.

Dans cet article, l'auteur compare aux règles existantes (C.N.B.—ACNOR) les nouveaux règlements sous l'angle de la probabilité de ruine calculée par une théorie simplifiée. Il met l'accent sur les combinaisons des charges dues au poids propre, aux surcharges de planchers et au vent, pour les immeubles à bureaux et immeubles d'habitation, constructions dans lesquelles la ruine survient par déformation plastique de l'acier. L'auteur examine également d'autres aspects des nouveaux règlements: la formule des poteaux applicable aux aciers structuraux, les coefficients de performance pour les structures composites, le coefficient d'importance qui reflète la gravité d'une ruine potentielle, et les coefficients de sécurité pour la phase de construction.

Il ressort de cette analyse, d'une part, que les règlements nouveaux fournissent une sécurité plus homogène que les règles présentes pour différentes combinaisons de charges et de matériaux, et d'autre part, que des règles simples se révèlent suffisamment exactes, compte tenu du rôle dominant que jouent les erreurs humaines dans les ruines potentielles, et des simplifications admises dans l'étude des structures complexes. [Traduit par la Revue]

Introduction

Various structural standards in Canada are moving toward a unified 'limit states' philosophy with common safety and serviceability criteria for all materials and types of construction. The common requirements will be contained in Section 4.1 (Structural Loads and Procedures) of the National Building Code of Canada (NBC) 1975, and detailed requirements will be contained in the various material structural standards. Structural steel (Canadian Standards Association, CSA S16.1 – 1974) and cold formed steel (CSA S136 – 1974) will have limit states design in 1975[1]; concrete, wood, and masonry plan to have it by 1980. To avoid abrupt changes in office design practice, the changeover to limit states design will be a gradual one, with existing procedures such as allowable stress design maintained as an alternative, at least until 1980. The changeover for concrete design, however, will not involve any difficulty to designers, since it is already in a form very similar to limit states design. All this activity is being coordinated by a CSA/NBC

[1]For application to structural steel design using CSA S16.1 – 1974, see Kennedy (1974).

Joint Committee, made up of representatives of all structural and foundations codes and standards used for buildings.

Following a brief description of limit states design, this paper presents results of a probabilistic study which compares safety levels of the new limit states design rules with previous standards. The main emphasis is on different load combinations for the basic case of failure of a critical section by yielding of steel. Other aspects of the new limit states design rules, namely, the column formula for structural steel, performance factors for composite structures, the importance factor, which reflects the seriousness of failure, and safety factors during construction, are also investigated.

Limit States Design[2]

All building structures have in common two basic functional requirements, namely, serviceability during the useful life of the building and safety from collapse during the construction and useful life of the building. Limit states define the various types of collapse and unserviceability that are to be avoided. Those concerning safety are called the *ultimate limit states* and include: collapse due to crushing, fracture, buckling, etc.; overturning, sliding; large deformation, flutter. Those concerning unserviceability are called the *serviceability limit states* and include: excessive deflection, vibration, cracking or permanent deformation.

Fundamentally limit states design is not new; it is a redefinition of terms in conformity with basic design requirements. Existing standards contain different 'design methods' – allowable stress design, plastic design, ultimate strength design, etc., each strongly associated with a particular structural theory and a particular limit state. With limit states design, instead of having different 'design methods' all standards will talk the same language; the appropriate structural theory is chosen as a function of the limit state being considered and the behavior of the structure.

Instead of the traditional single factors of safety, limit states design uses partial safety factors. *Load factors*, α, are applied to the loads to take into account the variability of the loads and load patterns and, to some extent, inaccuracy in structural analysis. A *load combination factor*, ψ, is applied to loads other than dead load to take into account the reduced probability of loads from different sources occurring simultaneously. An *importance factor*, γ, is applied to the loads to take into account the consequences of collapse as related to the use and occupancy of the building — *i.e.*, danger to human safety, economic loss. All these factors, which are common to all materials and types of construction, will be contained in Section 4.1 of the NBC. Finally *performance factors*, ϕ, are applied to material or structural resistance to take into account variability of material properties, dimensions, workmanship, and type of failure (*i.e.*, whether it gives warning or not) and uncertainty in the prediction of resistance. The performance factors will be contained in the various structural standards when limit states design is adopted.

Safety and serviceability are controlled not only through the use of partial safety factors, but also by defining specified loads and material properties statistically in terms of probability level (*e.g.*, 5% maximum probability of underrun for material properties) or return period (10 to 100 years for snow, wind, and earthquake loads).

Why introduce partial safety factors? First, they result in more consistent safety for different combinations of loads and different combinations of materials. This will be demonstrated later. Second, it is easier to assess safety factors for new types of construction, for unusual situations, and for design by load tests. This is because most of the partial factors for different loads and materials will be well established. Finally, by having common load factors and load combination rules, conflicts and confusions when switching from one material to another are avoided.

The limit states design criteria can be expressed as follows:

factored resistance $\geqslant$ effect of factored loads

[1] $$\phi R \geqslant \text{effect of } \gamma \left[\alpha_D D + \psi\,(\alpha_L L + \alpha_Q Q + \alpha_T T)\right]$$

where D, L, Q, and T refer to dead, live, wind

[2]See Commentary F of Supplement No. 4 of the NBC 1975 for a more detailed explanation.

TABLE 1. Partial safety factors—ultimate limit state

Load Factors	
Dead load	$\alpha_D = 1.25$ or 0.85*
Live load	$\alpha_L = 1.5$ or 0
Wind or earthquake	$\alpha_Q = 1.5$ or 0
Imposed deformation	$\alpha_T = 1.25$ or 0
Load Combination Factor	
$\psi = 1.0$ when one of L, Q, or T acts	
$\psi = 0.7$ when two of L, Q, or T act	
$\psi = 0.6$ when all of L, Q, or T act	
Importance Factor	
$\gamma = 0.8$ for storage type buildings of low human occupancy†	
$\gamma = 1.0$ for all other buildings	
Performance Factor for Yielding of Structural Steel	
$\phi = 0.9$	

*In cases of overturning, uplift and stress reversal.
†See NBC 4.1.4 for precise definition.

(or earthquake) loads and imposed deformation (temperature, etc.) respectively, and ϕR is the factored resistance, *i.e.*, the calculated resistance, including performance factors. For serviceability limit states, ϕR represents a criterion such as an allowable deflection, acceleration, stress, or crack width.

Partial safety factors for the ultimate limit states are given in Table 1. Partial safety factors for serviceability limit states are generally 1.0 with the exception that ψ is the same as for the ultimate limit states.

Probabilistic Study

The only real measure of safety or serviceability is the rate of failure of structures in service. Satisfactory failure rates for different limit states correspond to a trade-off between human safety or serviceability on the one hand, and economy, including expected losses due to failures, on the other hand. In practice satisfactory failure rates are achieved through competent structural engineering, manufacture, and erection, and by the use of safety and serviceability criteria such as Eq. [1] in the design calculations.

In this study, the new criteria (Eq. [1]) are compared with previous rules on the basis of a calculated safety index, β, used as a measure of the probability of failure. Kuipers (1968) has done such a comparative study of Dutch design rules for steel, concrete and wood. The safety index is derived as follows.[3]

Let S be the load effect and R the resistance at a critical section; both R and S are random variables. Failure occurs when $R < S$ or when

$$[2] \qquad \ln R - \ln S \equiv u < 0$$

If the random variables, $\ln R$ and $\ln S$, are assumed to be statistically independent, the average of u is given by[4]

$$\bar{u} = \overline{\ln R} - \overline{\ln S}$$

and the standard deviation by[4]

$$\sigma_u = \sqrt{(\sigma_{\ln R})^2 + (\sigma_{\ln S})^2}$$

From Eq. [2] the probability of failure corresponds to the area of the probability distribution curve of u in the tail $u < 0$, as shown in Fig. 1. For any given distribution curve, this area is a function only of the number of standard deviations between $\bar{u}$ and 0. This number is adopted as the definition of the safety index, β, *i.e.*,

$$[3] \qquad \beta = \frac{\overline{\ln R} - \overline{\ln S}}{\sqrt{(\sigma_{\ln R})^2 + (\sigma_{\ln S})^2}}$$

[3]The theory is the simplified first-order second-moment probabilistic theory developed by Rzhanitzyn (1957), Cornell (1969), and others.

[4]For the algebra of random variables, see Benjamin and Cornell (1970).

The safety index β therefore indicates consistent safety provided the shape of the distribution curve for u does not change. To calculate the probability of failure requires knowledge of the distribution curve. If R and S are assumed to be log-normal,[5] the calculated probability of failure, P_F, is the area under the normal curve beyond β standard deviations from the mean. For the log-normal case Eq. [3] for β can be written[6]

$$[4] \qquad \beta = \frac{\ln\left[\frac{\bar{R}}{\bar{S}}\sqrt{\frac{1+V_S^2}{1+V_R^2}}\right]}{\sqrt{\ln\left[(1+V_S^2)(1+V_R^2)\right]}}$$

where V refers to the coefficient of variation (cov), and $\bar{R}/\bar{S}$, the ratio of mean resistance to mean load effect, is determined from the design safety criterion, *e.g.* Eq. [1], as follows:

$$[5] \qquad \bar{R}/\bar{S} = k_R \cdot FS \cdot (1/k_S)$$

where k refers to the ratio of mean value to the specified value and FS is the over-all design factor of safety.

Even if the correct distribution were known, calculated failure probabilities implied by the safety index β cannot be directly related to actual failure rates in service. Two major reasons for this are: simplifications in the assumptions made, particularly in the structural analysis of complex indeterminate building structures, and, as discussed later, the fact that the theory does not include failures due to gross human errors. Therefore β can be considered only as a relative measure of safety; it is nevertheless useful for comparing design rules for different load combinations, different materials and their combinations and different types of failure.

Load Combinations

In this study, combinations of dead, floor, and wind load for office and residential buildings are considered. These combinations are of practical importance for most tall buildings in Canada. Load effects vary in time, and S in this study refers to the maximum load in 30 years. Failure is assumed to occur by over-all yield of steel in a critical section of a statically determinate structure. The load combinations are treated probabilistically as follows.

The load effect, S, can be expressed as follows:

$$S = E(D + L + Q) = E\,T$$

where D, L, and Q are random variables referring to the effect of dead, floor, and wind load respectively, and E is a random variable with mean 1.0 representing errors in structural analysis. It can be assumed that D, L, Q, and E are statistically independent; therefore[4]

$$\bar{S} = \bar{T} = \bar{D} + \bar{L} + \bar{Q} = k_D D + k_L L + k_Q Q$$

$$[6] \qquad \sigma_T^2 = \sigma_D^2 + \sigma_L^2 + \sigma_Q^2$$

$$V_S^2 = V_E^2 + V_T^2$$

where D, L, and Q now and in the following refer to the specified values of the loads, and k refers to the ratio of the mean value of the specified value.

The following load combinations are considered:

(a) Dead + Floor Load

(b) Dead + Floor + Wind Load

Since maximum floor loads and wind loads do not in general occur simultaneously two cases are considered: (i) combination of the maximum 30-year wind load (Q) with the instantaneous floor load at any time (L_i); (ii) combination of the maximum 30-year floor load (L) with the maximum daily wind load (Q_d). Calculations were made for both cases and the lower value of β was chosen.

(c) Wind − Dead − Floor Load

For combinations where wind is counteracted by dead plus floor load, the stabilizing dead plus instantaneous floor load combines with the resistance to prevent failure. In this case the load tending to cause failure is wind only (Q) and the total resistance, R', including strength plus stabilizing loads effect, is calculated as follows:

$$[7] \qquad \begin{aligned} \bar{R'} &= \bar{R} + \bar{D} + \bar{L}_i \\ \sigma_{R'}^2 &= \sigma_R^2 + \sigma_D^2 + \sigma_{L_i}^2 \end{aligned}$$

[5]This assumption is often reasonable because of control of material properties and positive skewness of known load distribution curves.

[6]See p. 266 of Benjamin and Cornell (1970).

TABLE 2. Probabilistic assumptions

		Mean/Specified	Coefficient of variation V
Structural analysis	E	1.00	0.07
Dead load	D	1.00	0.07
Floor load			
Maximum 30-year	L	0.70	0.30
At any time	L_1	$0.21/(0.3 + 10\sqrt{A^*})$	$0.30 + 4/\sqrt{A^*}$
Wind load			
Maximum 30-year	Q	0.80	0.25
Maximum daily	Q_d	0.08	1.00
Resistance R			
Yield of structural steel		1.10	0.13
Yield under wind load		1.17	0.13

*A is the tributary area in sq. ft. If A is in sq. m, replace A by $10.8A$.

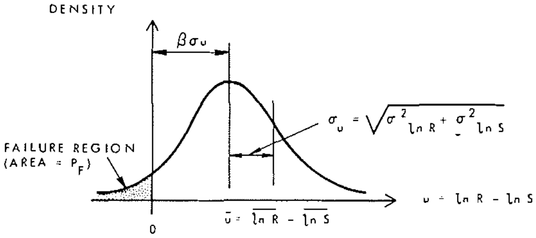


FIG. 1. Definition of safety index (β).

(d) Floor – Dead Load

The same reasoning applies as in case (*c*).

Assumptions regarding the parameters entering into Eqs. [4] to [7] for Canadian office and residential buildings are contained in Table 2 and derived in the Appendix. The assumptions are 'Bayesian' in the sense that they combine statistical information with the author's judgment, which in turn is drawn from the literature, from questioning experts, and from a rough knowledge of actual failure rates.

The results are given in Figs. 2 to 9, in which limit states design, CSA S16.1-1974, is compared with CSA S16-1969 and with the 1969 AISC (American Institute for Steel Construction) Specification for Steel Structures assuming Canadian loading rules and conditions.

Figure 2 shows the results for dead plus floor load, a combination which applies to floor systems. Because of statistical independence of dead and floor loads, β is higher in the middle than at the ends of the L/D range; β can, however, be approximated by a straight line in the regions of practical interest, $(1/5) < (L/D) < 5$. The circled point at full live load corresponds to the calibration point, where the new rules give the same section, hence the same safety level, as CSA S16-1969. Figure 2 shows that limit states design gives more consistent safety for all combinations of

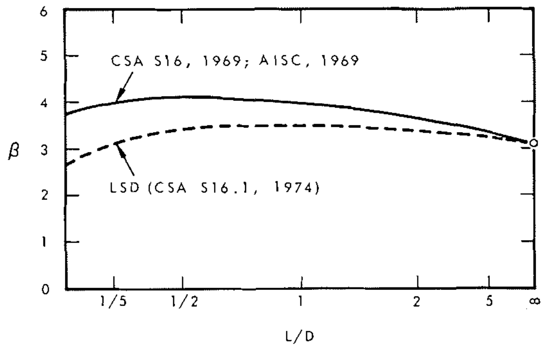


FIG. 2. Safety index (β) for load combination dead load + live load ($D + L$).

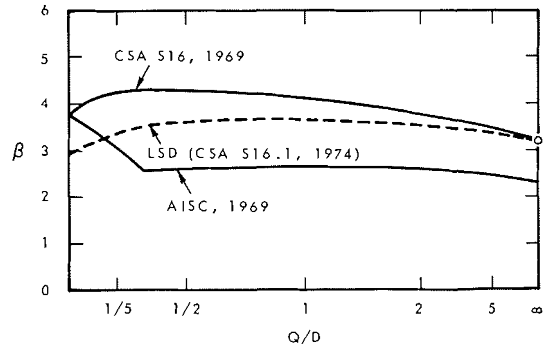
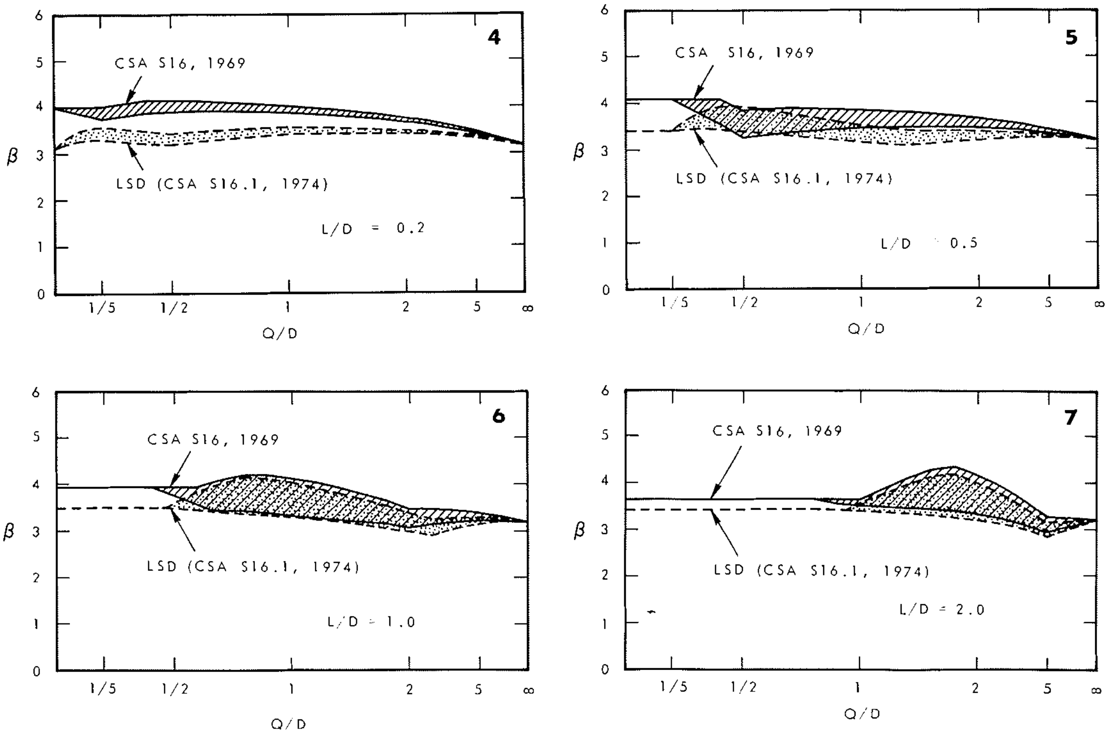


FIG. 3. Safety index (β) for load combination dead load + wind load ($D + Q$).



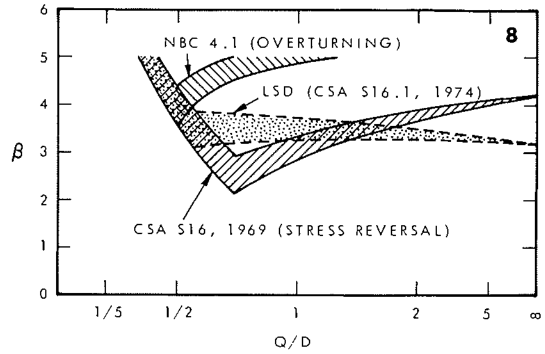
FIGS. 4, 5, 6, 7. Safety index (β) for load combination dead load + live load + wind load ($D + L + Q$): (4) $L/D = 0.2$; (5) $L/D = 0.5$; (6) $L/D = 1.0$; and (7) $L/D = 2.0$.

L/D. Savings are obtained when there is significant dead load.

Figure 3 shows similar results for the combination of dead load plus wind only. The AISC rules, which allow a one-third increase in allowable stress for this combination, give considerably lower safety levels for wind combinations than for dead plus floor load. Limit states design gives safety levels consistent with those in Fig. 2, and with some saving compared with existing Canadian rules when there is significant dead load.

Figures 4–7 show results for the combination of dead, floor, and wind loads—a combination of practical interest for building columns and for girders of unbraced structures. Instead of single lines, bands of β are obtained. This arises because the probability of load combinations, Eq. [6], depends on the ratio of instantaneous to maximum floor load ($\bar{L}_i/\bar{L}$), which in turn depends on the tributary area (A)—the greater the tributary area the higher is L_i/L and the smaller is β. Two tributary areas, 200 ft^2 (18.6 m^2) and 10 000 ft^2 (929 m^2), were selected as extremes. The load combination factor, $\psi = 0.7$, was chosen by trial and error to give safety levels for dead plus floor plus wind load comparable to that for dead plus floor load in Fig. 2. Existing rules (probability factor of 0.75 applied to dead plus live plus wind) also give comparable safety levels. Some economy is obtained for limit states design compared with existing rules when the dead load is high or when the floor load is small.

Another load combination which has given trouble in the past (Allen 1969), occurs in cases of overturning, uplift, and stress reversal, where dead plus floor load at any time acts as a stabilizing influence. Figure 8 shows results for the combination $Q - (D + L_i)$ and Fig. 9 for the combination L–D, for example a web member in a roof truss subjected to non-uniform snow load. The curves in Figs. 8 and 9 are made up of two intersecting lines, one a stability line, which gives high safety for low ratios of Q/D or L/D (heavy structure), the other due to stability plus strength required for



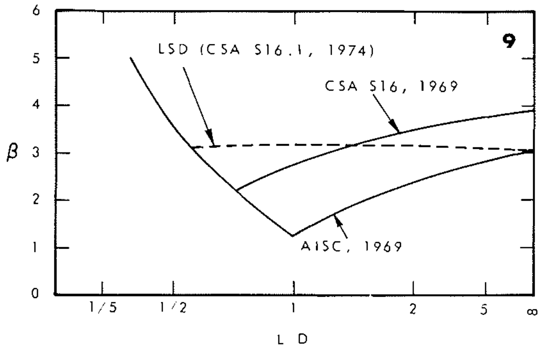


FIG. 8. Safety index (β) for wind (Q) counteracted by dead load + instantaneous live load ($D + L_i$).

FIG. 9. Safety index (β) for live load (L) counteracted by dead load (D).

high ratios of Q/D or L/D (light structure). In Fig. 8 there is a range of β's depending on the amount of floor load acting—the greater the floor load acting, the safer the structure, since stabilizing floor load is neglected in the design rules. Two extreme cases are chosen—no floor load ($L_i = 0$), and L_i for $L/D = 0.5$ and 10 000 ft^2 (929 m^2) tributary area. Some previous rules give quite inconsistent safety levels, sometimes very low levels when $Q \simeq D + L_i$ or $L \simeq D$. Limit states design rules, on the other hand, give consistent safety levels similar to those obtained in Figs. 2 to 7. This is essentially because limit states design uses load factors less than 1.0 for stabilizing loads (Table 1). The change in rules for counteracting loads will not appreciably affect material consumption.

The results of Figs. 2 to 9 are summarized in Fig. 10, which shows how β varies for all load combinations considered. Limit states design gives a significantly narrower range of

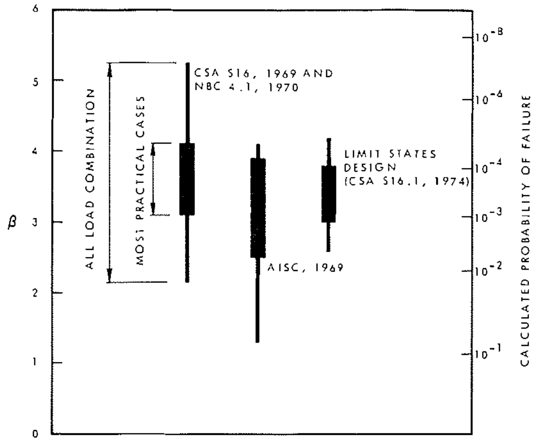


FIG. 10. Range of safety index (β) for different load combinations.

safety levels than do previous rules. Thus the same safety is maintained with some saving in material for previously over-designed structural members.

Column Formula for Structural Steel

Failure by yield applies to steel tension members and short compression members. As compression members get longer, however, they fail by inelastic or elastic buckling, and the design rules should provide a safety level at least equal to that provided by failure by yield. Galambos and Ravindra (1973) have studied this problem for a pin-ended column and their results, given in Fig. 11, show that the existing

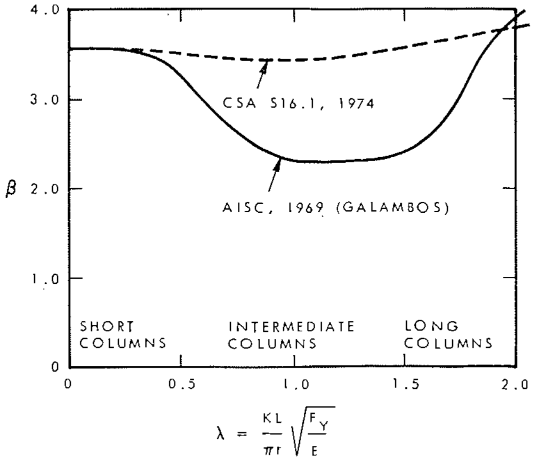


FIG. 11. Safety index (β) for different column design curves.

TABLE 3. β study of bending resistance of reinforced concrete

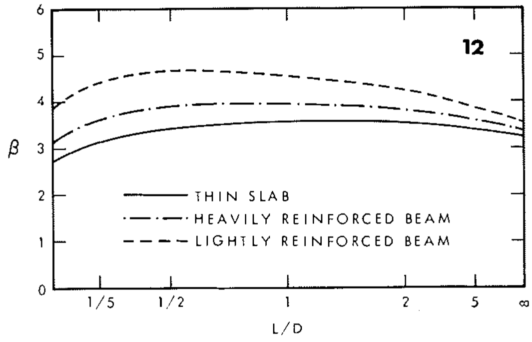
Case	Depth to reinf. d in. (cm)	Percent reinf. p/p_m*	Bending resistance $\bar{M}/M_{CSA}$	V_M	Overall ϕ factor		β for $L = D$	
					CSA	LSD†	CSA	LSD†
Thin slab	2¼ (6.4)	0.72	1.1	0.17	0.8	0.76	3.6	3.8
Heavily reinforced beam	10 (25)	1.00*	1.1	0.14	0.8	0.81	4.0	3.9
Deep beam	20 (51)	0.36	1.1	0.10	0.8	0.87	4.6	4.1

*p_m maximum allowed by CSA A23.3-1973.
†Based on separate ϕ factors (0.9 F_y, d-¼″ or 6.4 mm, 0.85 × 0.7 f'_c).

AISC column formula (similar to that used in CSA S16-1969) gives less safety for intermediate columns than for short or very long columns. For this reason, CSA S16.1-1974 has introduced a new column formula, which provides a consistent safety level for short and intermediate columns, and a little more safety for very long columns (see Fig. 11).

Composite Structures

Sometimes it is preferable to apply performance factors separately to the main resistance parameters than to the resistance of a member as a whole. An example is the bending resistance of reinforced concrete beams and slabs, which depends on the yield point of reinforcing steel, the depth to reinforcement and, for heavily reinforced sections, the crushing strength of concrete. Table 3 contains probabilistic assumptions for bending resistance for three extreme cases (thin slab, heavily reinforced beam, and a deep, lightly reinforced beam) based on a detailed probabilistic study of the above parameters (Allen 1970). Figure 12 shows results of β for these three cases designed for dead plus floor load according to CSA A23.3-1973 (where $\phi = 0.80$ when $\alpha_D = 1.25$ and $\alpha_L = 1.5$). Figure 13 shows similar results when the single performance factor 0.80 is replaced by separate performance factors—0.9 for yield strength, $(d - 0.25$ in. $(6.4$ mm$))/d$ for depth to reinforcing and 0.7×0.85 for concrete strength. Because of the better uniformity of β, savings can be made for deep or lightly reinforced beams, as seen in Table 3 by comparing the over-all ϕ factor with 0.8 for CSA A23.3-1973.



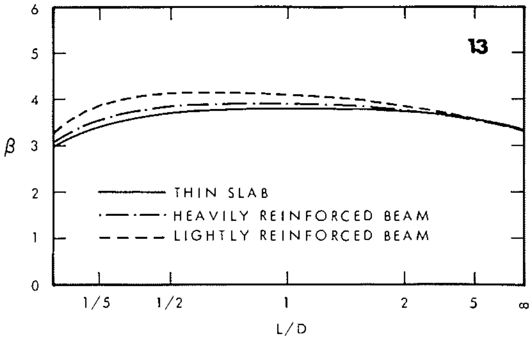


FIG. 12. Safety index (β) for bending resistance of reinforced concrete: single performance factor (CSA A23.3-1973).

FIG. 13. Safety index (β) for bending resistance of reinforced concrete: separated performance factors.

Importance Factor

Present safety factors for building structures of normal human occupancy ($\gamma = 1.0$) are based mainly on human safety requirements—*i.e.*, protection against loss of life or injury. Evidence for this is the reluctance of the Associate Committee on the National Building Code to reduce safety factors to a more economic level—*i.e.*, one which minimizes the initial cost plus the expected losses due to failure. The basic measure of human safety is risk of death or injury per year of our lives; this risk must not exceed an acceptable level, regardless of whether one is in an arena, a 100-storey building, or a temporary shack. The present death rate in Canada due to structural collapse is very small, about 0.2×10^{-6} per year. A death risk of 10^{-6} per year, or 0.01% during the life of a human, is practically insignificant compared with deaths from all other accidents —approximately 5×10^{-4} per year or 4% during the life of a human.

On this basis a decrease in the importance factor for certain classes of structures can only be justified if failure is much less likely to result in death or injury than is usually the case. Very light structures whose collapse would not endanger people inside, fail-safe structures, and structures used for storage purposes only, fall into this category. Here a higher failure risk is not only justified, but also desirable in view of economic considerations. At the other end of the spectrum are those structures whose collapse may involve increased danger to a large number of people or increased economic loss. The collapse of a structure housing services essential to survival during an earthquake, hurricane, or tornado, would jeopardize the safety of a large population; the collapse of a power station would result in widespread economic loss. The importance factor is therefore used to set the design safety levels for different kinds of buildings as a function of the consequences of failure.

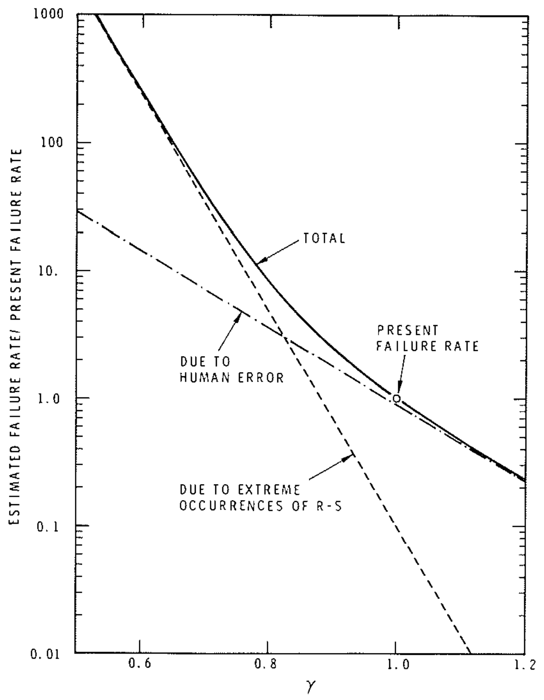


FIG. 14. Estimated failure rates for steel structures.

Figure 14 indicates how a change in the importance factor affects the failure rate for engineered structures, taking into account failures due to human error as well as failures due to extreme occurrences of *R–S*. Present failure rates ($\gamma \doteq 1.0$) in Canada and elsewhere (Rusch and Rackwitz 1972?) indicate that no more than about 10% of the collapses are due to extreme occurrences of *R–S*. Based on the assumptions previously used for steel structures ($V_R \simeq 0.13$), a 10% decrease in γ corresponds approximately to a 7-fold increase in calculated failure probability. Failures due to human error are not nearly as sensitive to changes in safety factors, and for Fig. 14 it is assumed that a 10% decrease in γ will no more than double the number of such failures. From Fig. 14 the following estimated failure risks are obtained for steel structures, assuming there is no change in the incidence of human error in the future.

γ	Estimated Failure Rate
1.1	$\frac{1}{2}$ × present failure rate
1.0 (normal structures)	1 × present failure rate
0.9	$2\frac{1}{2}$ × present failure rate
0.8 (storage sheds)	8 × present failure rate
0.7	40 × present failure rate

For more variable materials such as wood ($V_R \simeq 0.3$) a 10% decrease in γ corresponds only to approximately a 4-fold increase in calculated failure probability. However, since most failures are due to human error, the effect of a change in γ on estimated failure rates will be about the same, except when γ is less than about 0.8.

Present levels of safety for finished structures indicate that no increase in γ is needed other than what is already done for post-disaster structures through increased wind and earthquake loads, but that some decrease is justified for buildings for which collapse is unlikely to endanger its occupants. Furthermore, because of the predominating influence of human error, increased safety is most effectively obtained by greater care in design and construction.

The same reasoning regarding consequences of collapse applies also to the design of members within a structure. Certain key members whose failure is sudden and produces widespread collapse should be more reliable than members whose failure is gradual and leads to only localized collapse. In previous standards this has been explicitly considered in the design of steel tension members with large holes and in reinforced concrete design to tied, as compared to spirally reinforced, concrete columns. For limit states design, changes in safety levels depending on type of member or type of failure are taken into account by the ϕ factors. Because of the predominating influence of human error, however, increased safety for key members is better obtained by greater care in design and construction, by alternate paths of support to avoid progressive collapse, etc., than by decreasing ϕ.

Load Factors During Construction

In contrast to the finished structure, present safety levels during construction are not high. The risk of death for construction workers due to structural collapse in Ontario is 30×10^{-6} per year, approximately, compared to 0.2×10^{-6} for the user. The main reason for this difference is the lack of engineering during erection, in particular for temporary supports; steps have been taken in the National Building Code and elsewhere to require engineered design of temporary supports.

Figure 2 shows that for dead load only, with a load factor of 1.25, $\beta \simeq 2.7$, which corresponds to a calculated failure probability of about 10^{-3}. Since the dead load is applied during construction this represents a danger to the construction worker. If the load factor were reduced to 1.0, say by applying an importance factor of 0.8, then the calculated failure probability is in the order of 10% ($\beta = 1.2$). Such a risk is not acceptable in view of the hazard to construction workers and, therefore, minimum load factors of not less than 1.25, or greater if the construction load is more uncertain than the dead load, are recommended. In applying the load factors during construction, the importance factor should be taken equal to 1.0. Also the factored resistance should be based on a realistic assessment of the material properties at the time of load application. Exceptions should only be allowed if, during critical times of construction, the collapse will definitely not affect anybody.

Conclusions

1. This probabilistic study indicates that limit states design partial safety factors give more consistent safety for various load combinations and various combinations of materials than do existing rules. With more consistent safety it is possible to make some material savings in cases where existing rules are overly safe (*e.g.*, high dead load stresses, deep reinforced concrete beams), and to make significant savings for buildings in which danger to human safety is significantly reduced (*e.g.*, storage sheds). As safety factors are reduced, however, serviceability considerations will become more important in design.

2. The probabilistic study indicates that a simple rule for load combinations such as the 'load combination factor $\psi = 0.7$' is in general sufficiently accurate for office and residential building construction subject to dead, live, and wind loads. Other cases neglected in this study require further examination, in particular, load combinations involving earthquakes or other exceptional loads, and load combinations for industrial buildings and other special occupancies.

3. Because of the predominating influence of human error on failure rates, increased safety for key buildings or key members of buildings is better obtained by greater care in design and construction than by increasing safety factors.

4. A minimum load factor of not less than 1.25 with importance factor equal to 1.0, together with a realistic assessment of factored resistance, is recommended to obtain adequate safety during construction. A higher load factor is needed if the construction loads are known less accurately than the dead load.

5. The main improvements to be obtained from a change to limit states design are the unification of structural standards, and the wider applicability of common safety and serviceability criteria to new and different structures.

Acknowledgments

The author wishes to acknowledge the advice and encouragement of many people, in particular the members of the Building Structures Section of the Division of Building Research, National Research Council of Canada, of CSA Committee S16 Steel Structures for Buildings, and of the Task Group on Safety Criteria for Limit States Design. This paper is a contribution of the Division of Building Research, National Research Council of Canada, and is published with the approval of the Director of the Division.

Appendix A. Probabilistic Assumptions for Table 2

Structural Analysis

For statically determinate structures with well defined support conditions member forces and moments can be obtained accurately from statics. On the other hand, building structures are highly indeterminate, and member forces corresponding to the ultimate limit state can only be obtained approximately; in general, however, structural analysis is conservative. This study is modelled on the simplified statically determinate case, so only a small dispersion ($V_E = 0.07$) is taken into account for structural analysis.

Dead Load

Except in cases where lower parts of the building have to be designed before the upper part is well defined, dead loads are known accurately in comparison with other loads. The biggest variations appear to be slab thickness and masonry weights. A coefficient of variation of 0.07 is representative of actual variations for normal buildings.

Floor Loads

Although all types of occupancies are included in the new limit states design rules, the main emphasis in this study is on offices and residential buildings—the most prevalent type of occupancies, particularly for buildings over a few storeys high. When studying combinations of floor and wind load, storage occupancies will be more critical because of a higher expected live load, L_i, during extreme winds; however, wind load combinations do not generally play a significant role in design of structures for low-rise warehouses.

The probabilistic assumptions given in Table 2 are based mainly on survey data of Mitchell and Woodgate (1971) and Karman (1969). For an office building floor designed for 50 lb/ft^2 (2400 N/m^2) the expected maximum live load (not including partitions) in 30 years would be about 35 lb/ft^2 (1700 N/m^2). Recent studies (*e.g.*, McGuire and Cornell (1974)) have indicated that the NBC formula for reduction of floor load with tributary area, $0.3 + 10/\sqrt{A^*}$, is fairly consistent with calculated maximum lifetime loads based on measurements. Therefore, the ratio of expected 30-year load to NBC design load will be assumed to be 0.7, independent of tributary area. Load survey results of Mitchell and Woodgate (1971), Karman (1969), and Bryson and Gross (1967) indicate that the coefficient of variation of maximum floor load is about 0.3 and is unchanged with increasing area. Based on these assumptions the specified floor load corresponds to an exceedence probability of 8–9%, depending on the distribution assumed.

For combination of dead, live, and wind loads, the load at any time, L_i, is required. For office and residential buildings, the expected load at any time is approximately equal to the 30-year load for an infinite area. For offices this corresponds to $0.7 \times 0.3 \times 50 = 10.5$ lb/ft^2 (500 N/m^2), a value confirmed by survey results (Bryson and Gross 1967). On the other hand, the cov of load at any time increases with a decrease in area. The following is based on Table 7 of Mitchell and Woodgate (1971):

$$V_{L_i} = 0.3 + 4/\sqrt{A^*}$$

Wind Loads

Probabilistic information for both the maximum 30-year wind loads and the maximum daily wind loads is needed for calculating β for combinations of dead, live, and wind loads.

Wind loads p are determined as follows (NBC Commentary on Wind Loads):

$$p = q\, C_e\, C_g\, C_p$$

where q is the reference velocity pressure obtained from climatic data, C_e is the exposure factor which takes into account ground roughness, C_g the gust effect factor, and C_p the pressure coefficient for the surface.

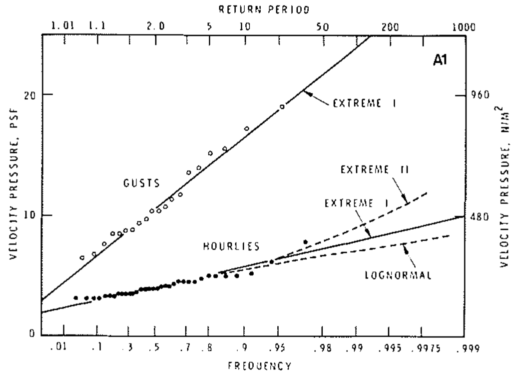
The main source of wind load uncertainty is in the measured wind speeds, *i.e.*, the reference velocity pressure q, equal to $0.0027\ V^2$

*See footnote to Table 2.

TABLE A1. Wind statistics (approx. 30 years data)

	Maximum hourly pressures PSF*						
	Daily		Yearly		30-Yearly (derived)		
Place	$\bar{q}_D$	V_D	$\bar{q}_y$	V_y	$\bar{q}_{30}$	V_{30}	$\frac{\bar{q}_{30}}{q_{NBC}}$
Victoria	1.3	0.95	7.8	0.26	13.1	0.15	1.07
Ottawa	0.7	0.80	4.2	0.28	7.3	0.16	1.08
Edmonton			4.2	0.34	8.0	0.18	1.08
Winnipeg			5.4	0.29	9.5	0.16	1.08
Sydney			7.1	0.25	11.8	0.15	1.07

*To get N/m² multiply psf by 47.9.



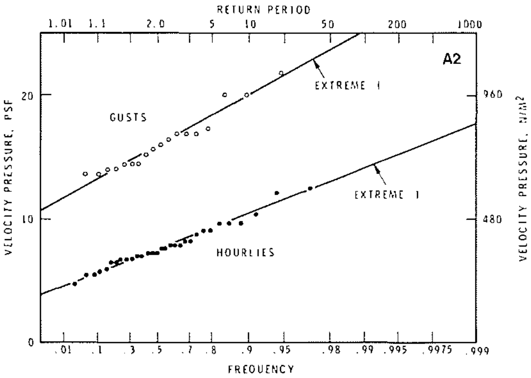


FIG. A1. Annual maximum winds—Ottawa (Uplands).

FIG. A2. Annual maximum winds—Victoria (Gonzales).

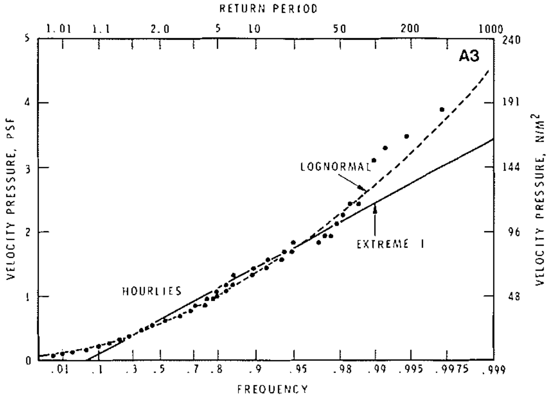
(0.050 V^2) where V is the maximum annual wind speed in mph (km/h). Figures A1 and A2 show distributions, plotted on Extreme Value Type I paper, of the annual maximum velocity pressures at two Canadian locations (Anonymous 1946–1974); the main statistical parameters are given in Table A1. Coefficients of variation (cov) of annual maximum velocity pressures (hourlies) vary from 0.25 to 0.34, with an average 0.28. If the Extreme Value Type I distribution is assumed for winds, by transformation the maximum 30-year velocity pressures have an average cov of 0.16. If the Extreme Value Type II distribution is assumed, by transformation the cov of maximum 30-year velocity pressures is 0.28, *i.e.*, is unchanged. Which distribution is correct in the extreme upper tail cannot be determined theoretically. Climatic data on wind loads in Canada (Anonymous 1946–1974) indicate that the cov decreases somewhat with the length of period. A cov of 0.20 is therefore assumed for variability in 30-year maximum wind pressures.

The other source of wind load uncertainty is in the estimation of the parameters C_e, C_g, and C_p. The greatest uncertainty is in estimation of the ground roughness. An increase in ground roughness increases C_g, but decreases C_e, so the two should be considered together, *i.e.*, $C_e \times C_g$. For a typical building, 100 × 50 × 200 ft (30 × 15 × 60 m) high, three exposure categories are given in the NBC—open, urban, and city center; for these three categories, $C_e \times C_g$ is calculated to be in the ratio of 0.75/1.00/1.48. This indicates that uncertainty in exposure can be represented by a cov of approximately 0.13, which compares reasonably well with a cov of 0.1 for C_g determined by Vickery (1970). Estimation uncertainty for all other parameters, including C_p, is assumed to be represented by a cov of 0.10. This gives a total uncertainty cov for 30-year wind loads, including actual wind variability and modelling uncertainty, of $\sqrt{.20^2 + .13^2 + .1^2} = 0.26$. A value of 0.25 is assumed for this study.

A similar approach is required to estimate the ratio of expected 30-year wind load ($\bar{p}_{30}$)

to NBC design wind load (p_{NBC} — the 30-year return wind). The statistical results in Table 1 show that the ratio $\bar{q}_{30}/q_{NBC}$ is close to 1.1 for Extreme Value Type I distribution; a similar result is obtained for Extreme Value Type II. Modelling of wind loads in the NBC tends to be conservative for the following reasons: (1) wind loads are assumed to occur in the most unfavorable direction; (2) estimation of modelling parameters—ground roughness and pressure coefficients C_p tend to be chosen conservatively on the average. The following reduction coefficients are assumed for these effects: reduced probability of wind blowing in the most unfavorable direction 0.85; ground roughness, pressure coefficient, and all other modelling assumptions 0.85. This gives a ratio of expected 30-year wind load to the NBC design wind load of $1.1 \times 0.85 \times 0.85 \simeq 0.8$.

For maximum daily wind loads Figs. A3 and A4 show distributions of maximum daily velocity pressures (hourlies) at two locations in Canada (Anonymous 1946–74)—Ottawa, a location of moderate winds, and Victoria Gonzales, a consistently windy location. The results are given for one year of records only, but a check with records of other years showed very little difference in the results. Figures A3 and A4 show that maximum daily velocity pressures follow the lognormal distribution more closely than the Extreme Value Type I distribution. Table A1 contains the main statistical parameters. The ratio of expected maximum daily to expected 30-year pressure is close to 0.1 in both cases, so that the ratio of expected maximum daily wind pressure to NBC design load would be $0.8 \times 0.1 = 0.08$. The cov, considerably higher for maximum daily wind pressure, will be taken as 1 for calculation.



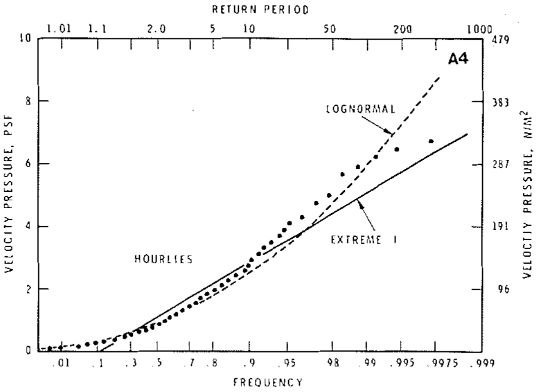


FIG. A3. Daily maximum winds—Ottawa (Uplands).

FIG. A4. Daily maximum winds—Victoria (Gonzales).

Resistance to Yield

The resistance of a structural member to general yielding can be expressed as

$$R = M\,F\,P$$

where M is a material property such as the yield point of steel, F is a geometric property such as area or plastic section modulus, and P represents the resistance formula. These parameters are subject to uncertainty. Based on existing information (Galambos and Ravindra 1973), the following probabilistic assumptions are made under static loading conditions:

	Expected/Specified	COV
Yield point (static)	1.05	0.10
Geometric	1.0	0.05
Resistance formula	1.05	0.07

From this the expected resistance is $1.05 \times 1.05 = 1.1$ times the specified resistance and the cov is $\sqrt{0.1^2 + 0.05^2 + 0.07^2} = 0.13$. For wind loading, rate effect increases the yield stress by 6% approximately (Rao *et al.* 1966), so that the expected resistance is 1.17 times the specified resistance.

ALLEN, D. E. 1969. Safety factors for stress reversal. Int. Assoc. Bridge Struct. Eng. Publ., **29-11**, pp. 19–27.
——— 1970. Probabilistic study of reinforced concrete in bending. Nat. Res. Counc. Can., Div. Bldg. Res. Tech. Paper No. 311 (NRCC 11139), Ottawa.

ANONYMOUS. 1946–74. Monthly records of meteorological observations in Canada. Environment Canada, Atmospheric Environment Service, 4905 Dufferin St., Downsview, Ont.

BENJAMIN, J. R., and CORNELL, C. A. 1970. Probability, statistics and decision for civil engineers. McGraw-Hill. Book Co., New York.

BRYSON, J. O., and GROSS, D. 1967. Techniques for the survey and evaluation of live floor loads and fire loads in modern office buildings. Nat. Bur. Stand., Bldg. Sci. Ser. 16, Washington, D.C.

CORNELL, C. A. 1969. A probability-based structural code. J. Am. Concrete Inst., **66**, No. 12, pp. 974–985.

GALAMBOS, T. V., and RAVINDRA, M. K. 1973. Tentative load and resistance factor design criteria for steel buildings. Struct. Div., Civ. Environ. Eng. Dept., School of Engineering and Appl. Sci., Washington Univ., St. Louis, Mo., Res. Rep. No. 18.

KARMAN, T. 1969. Statistical investigations on live load on floors. Unpubl. rept., presented to Committee W23 of the Conseil International du Bâtiment (International Council for Building Research, Studies and Documentation). Copy on file, Library, Div. Bldg. Res., Nat. Res. Counc. Can., Ottawa.

KENNEDY, D. J. L. 1974. Limit states design—an innovation in design standards for steel structures. Can. J. Civ. Eng., **1**, No. 1, pp. 1–13.

KUIPERS, J. 1968. Structural safety. Heron, No. 5, pp. 1–41.

MCGUIRE, R. K., and CORNELL, C. A. 1974. Live load effects in office buildings. J. Am. Soc. Civ. Eng., Struct. Div., **100**, ST 7, pp. 1351–1366.

MITCHELL, G. R., and WOODGATE, R. W. 1971. Floor loadings in offices—the results of a survey. Dept. Environ., Bldg. Res. Station, Garston, England, Current Paper 3/71.

RAO, N. R. N., LOHRMANN, M., and TALL, L. 1966. Effect of strain rate on the yield stress of structural steels. J. Materials (Am. Soc. Test. Mater.), **1**, No. 1, pp. 241–262.

RUSCH, H., and RACKWITZ, R. 1972? The significance of the concept of probability of failure as applied to the theory of structural safety. Special print from the commemorative publication entitled "Development —Design—Construction" on the occasion of the hundredth anniversary of Held und Francke Bauaktiengesellschaft, Munich, West Germany.

RZHANITZYN, A. R. 1957. It is necessary to improve the standards of design of building structures. Stroitel'naya Promyshlennost' No. 8. (English translation *in* Tech. Translation 1368, Nat. Res. Counc. Can., 1969).

VICKERY, B. J. 1970. On the reliability of gust loading factors. Proc. Tech. Meet. Concerning Wind Loads on Buildings and Structures. Nat. Bur. Stand., Bldg. Sci. Ser. 30, pp. 93–104.