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Inlet Plenum Pressure Drop Calculation for a Cross-Flow Module†

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Outlined here is a method for applying a model of discharge flow through a series of sharp-edged orifices from an inlet plenum, to accurately predict the total pressure drop across the flow distribution geometry. This particular case relates to a cross-flow filtration membrane module, but is applicable for any flow distribution system of a similar format.

On décrit ici une méthode d'application d'un modèle d'écoulement de refoulement dans une série d'orifices à arêtes vives d'une chambre d'admission, afin de prédire la perte de charge totale pour la géométrie de distribution d'écoulement. Ce cas particulier s'adresse à un module de membranes de filtration à écoulement transversal mais est applicable à tout système de distribution d'écoulement de format similaire.

Keywords: cross flow module, flow distributor, pressure drop calculation.

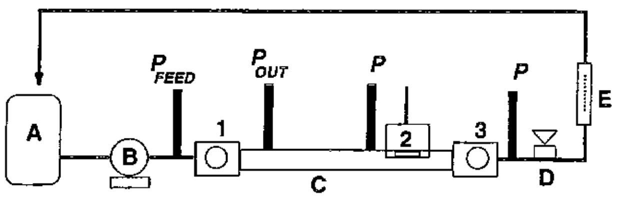

Figure 1 — Schematic diagram of module flow circuit. A – reservoir. B – pump. C – membrane module. 1. inlet plenum. 2. membrane housing and permeate exit. 3. outlet plenum. D – pressure regulator. E – flow meter. Four pressure measurement ports are also shown.

For the performance characterization of separation membranes, it is desirable to have uniform flow and fluid properties over the permeation area. This facilitates decoupling the intrinsic membrane-feed properties from the overall observed properties which are system dependent. Analysis, such as the boundary layer model or the osmotic pressure model, can be applied in a straightforward manner with a (near) constant flow field (Kleinstreuer and Belfort, 1984). Otherwise, the diffusion models must be coupled with flow models and the level of complexity can make analytical resolution intractable (Yoshikawa, 1994). A module design approach was undertaken to create an environment of uniform hydrodynamic conditions over the permeation area, and thus allow application of the above models with less inherent error (Darcovich et al., 1997). These criteria were met in a system with a perforated-pipe type cylindrical header chamber for pressure equilibration, positioned perpendicular to the feed tube and outlets to a cross flow channel (Figures 1 and 2). The pressure drop across this inlet plenum is substantial and is an important system design consideration. The module was designed to operate between pressures of 10 and 200 psig, and with cross flow velocities up to 5 m/s. For high velocity cases, especially when coupled with low pressure conditions, the pressure drop across the plenum could be over 15 times the pressure drop through the rest of the system. Thus, the pressure rating required for construction of the flow circuit would have to be about double the high end operating pressure inside the module.

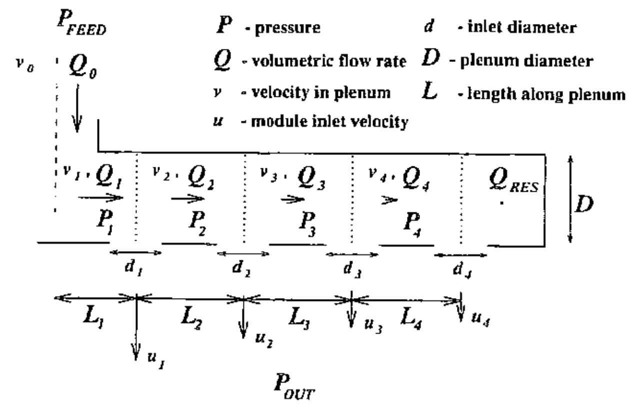

Figure 2 — Specification of the problem for solving the trans-plenum pressure drop.

The objective of this report is to outline a calculation devised to accurately predict the pressure drops across a perpendicularly positioned flow distributing plenum, especially in view of the limited amount of practical information available in the membrane literature on such topics.

Model formulation

A view of the flow across the flow distributing plenum is given in Figure 2. The system variables are itemized here as well. As less rigorous preliminary calculation was presented where the plenum outlets were treated as points, rather than gaps of real width (Darcovich et al., 1997). A stream-splitting calculation devised by Strakey et al. (1997) for the treatment for a single manifold discharge, was adapted for a series of consecutive outlets, such as considered here.

Figure 2 illustrates conceptually how the flow is treated for a branched outlet where some of the plenum flux continues past the opening. The fluid under consideration is

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water at 25°C, with a density of 997.1 kg/m^3 and a viscosity of 8.94 × 10^{-4} Pa·s. The present treatment is based on Bernoulli's energy equation given below. An interative procedure was developed to adapt this energy balance to the flow exiting via a number of split streams as shown in Figure 2.

$$\frac{P_1}{\rho}+\frac{1}{2}\overline{V}_1^2 - E_{\text{turb}} - E_{\text{fric}} = \frac{P_{OUT}}{\rho}+\frac{1}{2}U_1^2 \quad \ldots\ldots\ldots (1)$$

E_{fric} represents the turbulent friction energy loss, and is defined as:

$$E_{\text{fric}} = f\frac{e}{d}\frac{U_1^2}{2} \quad \ldots\ldots\ldots (2)$$

where e is the plenum distributor plate thickness. The Darcy coefficient, f in Equation (2) is taken as 0.04, suitable for turbulent flow over a smooth tube wall. System dimensions are: D = 30 mm, e = 5 mm, d = 3 mm and the overall perpendicular length of the plenum is 90 mm.

The turbulent loss E_{turb} comes from the constriction through an outlet orifice, where the diameter and the direction of flow both change. Let V_c be the fluid velocity through the outlet. Thus,

$$E_{\text{turb}} = \frac{V_c^2}{2}\left(1-\frac{d^2}{D^2}\right)^2 \quad \ldots\ldots\ldots (3)$$

Combining Equations (1) through (3), an expression for the plenum outlet velocity can be obtained:

$$U_1 = \sqrt{\frac{\frac{P_1 - P_{OUT}}{\rho}+\frac{\overline{V}_1^2}{2}}{\frac{1}{2}\left(\frac{D^2}{d^2}-1\right)^2+\frac{1}{2}+\frac{f}{2}\frac{e}{d}}} \quad \ldots\ldots\ldots (4)$$

Above, $\overline{V}_1$ is the average velocity of the portion of the fluid that splits off from the main flow in the plenum and exits via the outlet. The main plenum flow was estimated with the turbulent-$\frac{1}{7}$ th model. The Reynolds number in the plenum varied between about 7.9 × 10^3 and 3.15 × 10^4. Although this is a suitable regime for the $\frac{1}{7}$ th model, the length of the plenum is only 1.5 diameters. The flow would not be fully developed over this distance, and this represents a source of error in the calculations. However, for an entry region where the flow is decelerating from a fully turbulent profile, as would exist in the inlet tube, the entry length will at least be shorter than when the fluid must accelerate (Kays and Crawford, 1980).

Figures 2 and 3 show schematically the parameter representation for the flow splitting calculation. The whole calculation is iterative and encompasses the entire system starting with an estimate of P_1. A sub-iteration is also begun by estimating a value for H_1, the split level in the plenum stream as first proposed by Escobar (1982). H_1 corresponds to the area A_1, shown in Figure 3.

An initial value for $\overline{V}_1$ is obtained by integrating the main plenum flow from the inside boundary to the distance H_1. With $\overline{V}_1$ known, U_1 can be determined from Equation (4). Using this value of U_1, the flow rate through the first outlet, q_1, can be determined. If H_1 has been correctly estimated, the quantity q_1 should match the flow rate integrated over

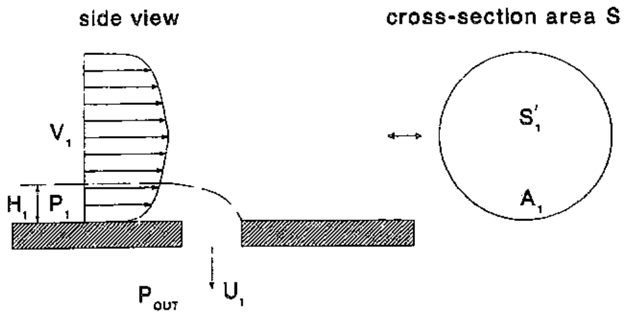

Figure 3 — Parameter definitions used in the flow-splitting calculation.

the area A_1 in the plenum: that is, $Q_1 = \overline{V}_1 A_1$. Thus if $q_1 > Q_1$, then the area A_1 must be increased, or decreased in the opposite case. An iterative calculation to estimate H_1 (and thus A_1) is done to equate q_1 and Q_1. When this is achieved, the flow characteristics in the plenum beyond the first outlet may be treated. If Q is the overall flow rate into the system, then, $Q_2 = Q - q_1$. From this, $\overline{V}_2$, q_2 and U_2 can all be determined as outlined above.

The next step is to calculate the pressure P_2 in the plenum ahead of the second outlet. The pressure change from P_1 and P_2 can be attributed to three things: a sudden enlargement for the fluid in the plenum that does not exit via the first outlet, a friction term along the plenum walls, and the point to point velocity change.

The expansion term is determined by:

$$E_{\text{expansion}} = \frac{1}{2}\rho\overline{V_1'}^2\left(1-\frac{S'_1}{S}\right)^2 \quad \ldots\ldots\ldots (5)$$

where $\overline{V_1'}$ is the average velocity of the fluid in the first part of the plenum (area = S'_1) above the split level which continues into the second section of the plenum.

The wall friction term is:

$$E_{\text{wall}} = \frac{1}{2}\psi\frac{L}{D}\rho V_2^2 \quad \ldots\ldots\ldots (6)$$

with $\psi = 0.184\,Re^{-0.2}$ for the turbulent regime (Geankoplis, 1983).

Including the velocity change, the expression for P_2 is:

$$P_2 = P_1 + \frac{1}{2}\rho\left(V_1^2 - V_2^2\right) - E_{\text{wall}} - E_{\text{expansion}} \quad \ldots\ldots\ldots (7)$$

Now with values for V_2 and P_2, the same iteration as performed for the first outlet can be done for the second outlet and so on, until the last outlet.

In our case, there are four outlets. Once the flow rate in the last outlet is calculated, a residual flux Q_{RES} will remain for comparison to a tolerance value Q_{TOL}, set to 10^{-6} × Q. If the condition is met that $Q_{RES} < Q_{TOL}$, then the calculation can be considered complete. Otherwise an adjustment is made to P_1 to reduce the absolute value of Q_{RES} and another iteration is begun.

Experimental results

The view of the system in Figure 1 shows the locations of the pressure measuring ports. A few additional terms must be added to calculate a value of P_{FEED}, which can be directly

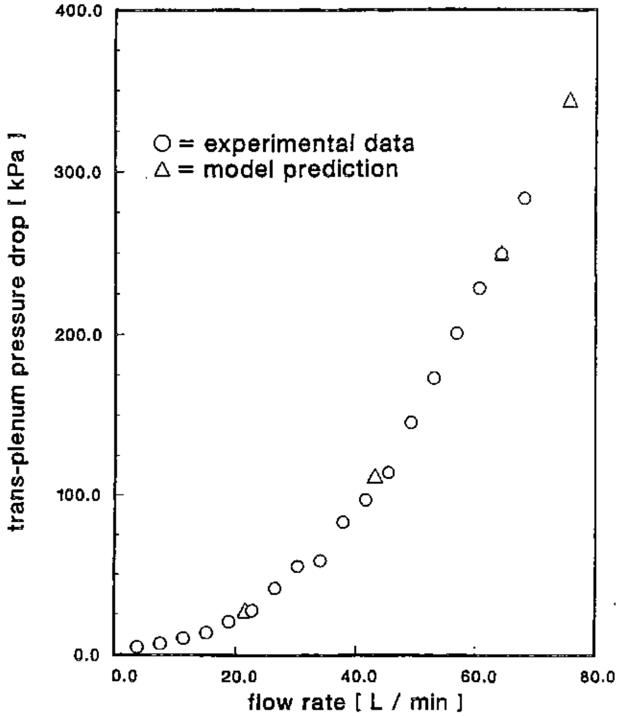

Figure 4 — Comparison of predicted and measured data.

compared to the pressure measured at the port in front of the distributing plenum.

Equation (1) can be reexpressed for the pressure drop between the inlet line manometer and P_1 inside the plenum:

$$P_{FEED} = P_1 + \frac{1}{2}\rho\left(U_1^2 - U_{FEED}^2\right) + E_{\text{elbow}} + E_{\text{fric}} \quad \ldots\ldots (8)$$

Above, E_{fric} is determined as in Equation (2) for flow in a half-inch steel pipe. E_{elbow} is calculated as:

$$E_{\text{elbow}} = \frac{1}{2}\rho U_1^2\left(1 - \frac{A_{\text{tube}}}{S}\right) \quad \ldots\ldots (9)$$

The physical equipment consisted of a Hydra-Cell D25 positive displacement pump with a Toshiba Tosvert 130G2+ controller, which drove the flow through the circuit depicted in Figure 1. For these tests, the system was operated without any backpressure control, so that the module pressures increased with the flowrate. After a brief start-up period of a few minutes, pressure readings were taken at 5 min intervals for 1 h. Averages were then calculated for each condition, with variances across the entire range being less than 1.0%.

The measurements and calculations were made for this system with flow rates from about 3 to 75 L/min. The results are shown in Figure 4. The estimates are nearly exact.

Conclusions

The stream splitting method of applying Bernoulli's equation incorporated into the iterative algorithm presented here is a sufficiently accurate means of calculating pressure drops across a multi-outlet, perpendicularly oriented flow distribution plenum. The conditions tested here were all in the fully turbulent regime where energy loss functions are well characterized.

Nomenclature

- A_i = plenum cross-section area contributing to flow through the ith exit, (m^2)
- d = plenum outlet diameter, (m)
- D = plenum diameter, (m)
- e = plenum distributor plate thickness, (m)
- E = energy loss terms in modified Bernoulli's equation, (m^2/s^2)
- f = Darcy friction coefficient
- H_i = distance from inside wall of plenum to fluid split point for the ith exit, (m)
- P = pressure, (Pa)
- q_i = flow rate through ith plenum outlet, (m^3/s)
- Q = overall feed flow rate into the plenum, (m^3/s)
- Q_i = flow rate of portion of fluid in plenum which exits via the ith outlet, (m^3/s)
- S = cross-sectional area of the plenum, (m^2)
- S_i' = cross-sectional area in ith section of the plenum whose fluid continues along into the next section of the plenum, (m^2)
- U = velocity through plenum outlet, (m/s)
- V = axial velocity in plenum, (m/s)
- $\overline{V}_i$ = mean velocity of portion of fluid in plenum which exits via the ith outlet, (m/s)
- $\overline{V}_1'$ = mean velocity of portion of fluid in plenum which continues on beyond the ith section, (m/s)

Greek letters

- μ = viscosity, (Pa·s)
- ρ = density, (kg/m^3)
- ψ = wall friction coefficient

Subscripts and Superscripts

- i = ith section of the plenum
- $FEED$ = upstream of plenum
- OUT = downstream of plenum, in module channel

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