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# AN ASSESSMENT OF COMMON ASSUMPTIONS IN ESTIMATING COOLING LOADS AND SPACE TEMPERATURES

BY

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VERIFICATION DE CERTAINES HYPOTHESES COURAMMENT UTILISEES POUR EVALUER LA CHARGE FRIGORIQUE DE CLIMATISATION ET LA TEMPERATURE DE L'AIR A L'INTERIEUR DES LOCAUX

## SOMMAIRE

Le présent article expose les résultats d'une étude analytique visant à déterminer les erreurs découlant de l'utilisation d'un modèle mathématique linéaire pour calculer avec une précision suffisante les éléments d'une installation de climatisation; l'étude a également pour but d'évaluer l'effet des caractéristiques architecturales des locaux sur leur charge frigorique. On croit qu'un modèle mathématique linéaire permettrait, à l'aide d'un ordinateur numérique, de prévoir à un coût raisonnable quelle seraient les charges frigoriques de climatisation. Le modèle mathématique linéaire est fondé principalement sur l'hypothèse voulant que la transmission de chaleur par convection et par rayonnement soit directement proportionnelle aux différences de températures respectives. Les calculs révèlent que la charge frigorique n'est pas intimement liée aux coefficients de transmission calorique des surfaces. Les résultats montrent que le modèle mathématique, utilisant un coefficient global de transmission calorique par les surfaces intérieures, ne représente pas exactement le bilan thermique de la pièce; lors du calcul de la charge frigorique, on peut considérer que dans la pièce les éléments similaires constituent un tout, partie de son enceinte; il importe également de tenir compte des éléments constitutifs légers de celle-ci.

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No. 1949

G. P. MITALAS

# An Assessment of Common Assumptions in Estimating Cooling Loads and Space Temperatures

The instantaneous air-conditioning cooling load for a building is not equal to the instantaneous heat gain because of the non-steady nature of the heat gain and the associated heat storage by the building elements. This fact is recognized in the ASHRAE Guide And Data Book,[1] but no adequate method of taking account of heat storage is given in the determination of the cooling load. Another widely known reference for the design of air-conditioning systems[2], presents tables of factors that relate the room heat gain to the room cooling load. Because of the large number of possible combinations of the parameters affecting the heat storage, these tables are incomplete. Some of the parameters, such as the geometry of the room and furniture, are omitted, while others are lumped into a single variable. For instance, the floor, partitions, outside wall and furnishings are accounted for by a single factor—mass of the room envelope per square foot of floor area. Information on the error introduced by these simplifications has not been published.

In the last twenty years, various analogue simulations have been used to provide information on cooling load. Analogue simulation is not usually practical for the actual design problem because the analogue computer is relatively expensive, is not readily available, and requires considerable skill to set up and run the problem. With the advent of electronic digital computers, the possibility exists that cooling load can be predicted at reasonable cost using only digital calculations. It is impossible to set up a mathematical model that will exactly represent the building thermal behavior because the heat transfer in a building is very complex. Various assumptions and simplifications must be made. One of the basic assumptions which makes the all-digital calculation approach practicable is that the building thermal behavior can be described sufficiently accurately for practical design problems, by a linear mathematical model; the components of cooling load from each source (driving function) can then be determined independently and the net effect determined simply by addition.

This paper records an analytical study that was carried out to determine the errors associated with various simplifying assumptions as well as to evaluate the significance of the various room construction features. A system of linear equations was derived to describe the thermal behavior of a typical air conditioned office. These equations took separate account of the radiative and convective heat transfer in the room as well as the heat storage of the room. Appropriate convective heat transfer coefficients were selected for inside and outside surfaces, window air space, and suspended ceiling air space to define a "standard model." Using this model, cooling load and surface temperatures were determined with a coupled digital-analogue computer. The convection coefficients were varied, and different simplifications were introduced, one at a time; the results were compared with those for the standard model, the difference being the error attributable to the simplification.

The following parameters were checked:

(1) The inside surface convection heat transfer coefficient.
(2) The combined inside surface heat transfer coefficient.
(3) The outside heat transfer coefficient.
(4) The subdivision of the room enclosure for modelling.
(5) The room envelope construction and the presence of lightweight objects in the room.

The room selected for these calculations was an office module, 20 x 20 x 10 ft, with 160 sq ft of glass in the outside wall. The weather conditions selected were essentially the same as given in the Guide And Data Book for 40 deg N latitude and August 1. Internal load due to lights was taken into

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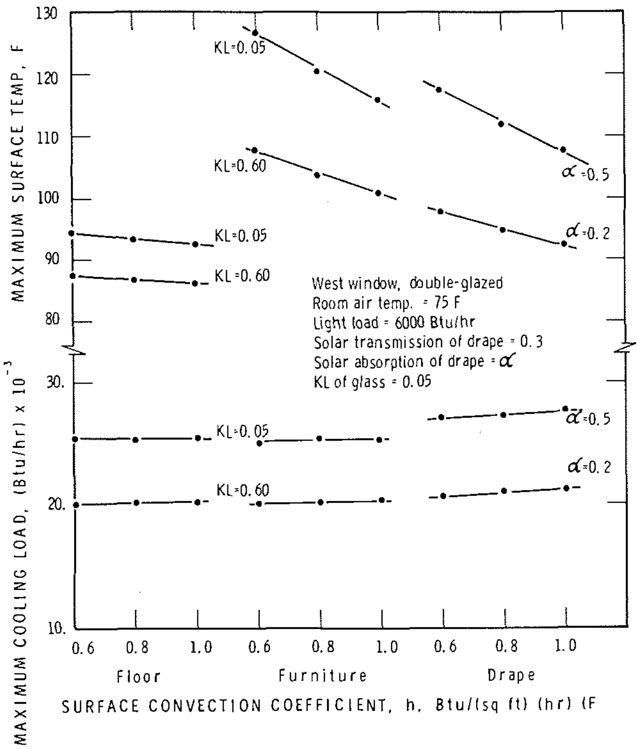



*Fig. 1 Maximum cooling load and maximum surface temperature vs surface convection coefficients*

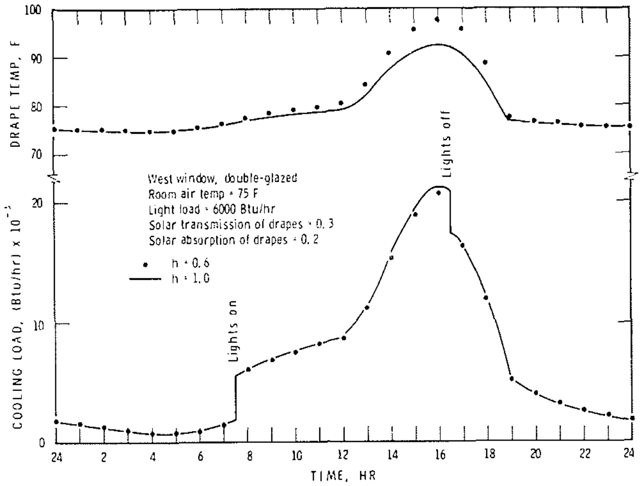



*Fig. 2 Cooling load and drape temperature vs time of day for drape surface convection coefficient h = 0.6 and h = 1.0 Btu per (sq ft) (hr) (F)*

account in some of the calculations. The room details and the method of setting up the mathematical model are given in Appendix A.

The main unverified assumptions involved in the standard model are:

(1) the effect of corners on the heat balance in the room,

(2) the air circulation and the temperature gradients inside the enclosure.

The calculations were made using a hybrid computer, i.e. an electronic analogue computer coupled with an electronic digital computer. The circuit representing the room (or the circuit to solve the set of equations representing the thermal behavior of the room) was set up on the analogue computer. The digital computer was used as the driving function generator as well as an output device for the analogue computer. The various analogue voltages representing the room temperatures and heat flows were sampled at regular intervals and these values were stored in the digital computer for listing at the end of the run.

## THE INSIDE SURFACE CONVECTION HEAT TRANSFER COEFFICIENTS

To set up a linear mathematical model, it is assumed that the convection and radiation heat transfer from the surface to the air and the other surroundings is directly proportional to the respective temperature differences. In an actual case, the radiant heat interchange between the surfaces is a function of the difference of the fourth power of the absolute surface temperatures. The heat interchange by convection depends on the surface and air temperature difference to a power $\gamma$, which is usually not equal to one. The power $\gamma$ depends on the contributions of natural and forced convection in the room as well as on the surface orientation.

The effect of the surface convection heat transfer coefficient, h, on the cooling required to maintain a constant room air temperature was evaluated by considering the following values of h for one of the surfaces and h = 0.8 Btu per (sq ft) (hr) (F) for the other surfaces (i.e. h of the standard model):

(a) Floor slab surface h = 0.6, 0.8 and 1.0 Btu per (sq ft) (hr) (F)
(b) Furniture surface h = 0.6, 0.8 and 1.0 Btu per (sq ft) (hr) (F)
(c) Inside shade surface h = 0.6, 0.8 and 1.0 Btu per (sq ft) (hr) (F).

The values of h = 0.6 and 1.0 Btu per (sq ft) (hr) (F) were selected for these calculations because they bracketed the value of 0.8 Btu per (sq ft) (hr) (F) which is recommended by the ASHRAE Guide And Data Book for cooling load calculations.

These three surfaces were selected because of their distinctly different influence on cooling load: the floor slab surface (a surface of a massive slab); the furniture surface (a surface of a lightweight object inside the room); and the inside shade surface (a surface of a light-weight object close to the room boundary).

For (a) and (b), the cooling load was evaluated at two different solar inputs: one that would occur with clear window glass (KL = .05)°, and the other with heat absorbing window glass (KL = 0.6). The results are given in Figs. 1 and 2. In Fig. 1, the daily maximum cooling load and the daily maximum surface temperature are plotted versus h; in Fig. 2, the daily cycle of the cooling load and the surface temperatures are given for inside drape surface coefficients of 0.6 and 1.0 Btu per (sq ft) (hr) (F).

The importance of radiant heat interchange between the surfaces that enclose the room was evalu-

° K = radiation extinction coefficient for the glass
L = thickness of the glass sheet

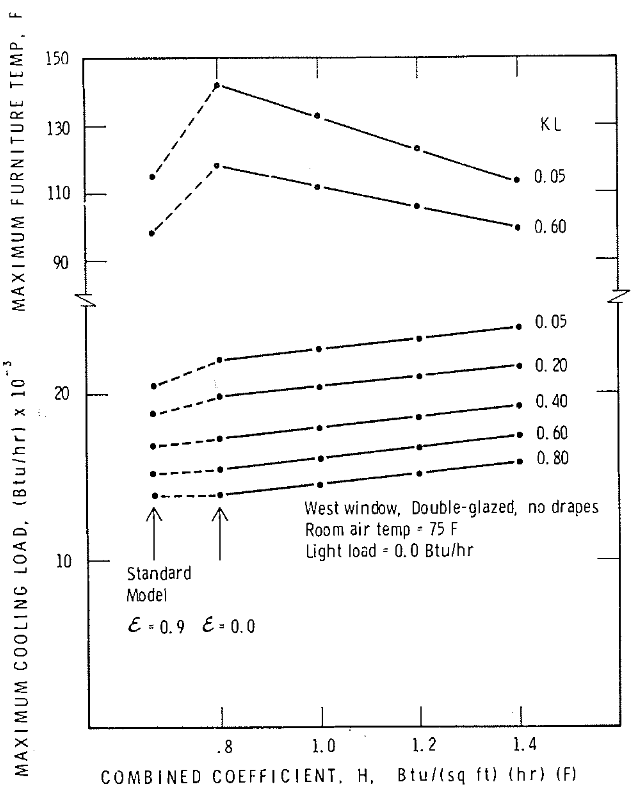



*Fig. 3 Maximum cooling load and maximum furniture temperature vs combined coefficient, H*

ated by determining the cooling load and surface temperatures for room surfaces with zero emissivity and comparing the results with those of the standard model (emissivities of 0.9). The results of these calculations are given in Fig. 3.

The results given in Figs. 1 and 2 indicate that the cooling load required to maintain the room air temperature constant is quite insensitive to changes in the inside surface convection coefficients. The change in the convection coefficient for the drape has the greatest effect, but even in this case a change in h of 0.4 Btu per (sq ft) (hr) (F) only changes the daily maximum cooling load by approximately 2%. The change in the drape convection coefficient has the greatest effect since this change affects the fraction of solar radiation that is absorbed by the drape and dissipated to the outside, and also, the effect is greatest when the solar input is at the maximum.

The results given in Fig. 3 for the emissivities $\epsilon = 0.9$ and $\epsilon = 0.0$ indicate that the heat interchange by radiation between the inside room envelope surfaces is not a major factor affecting the cooling load. The cooling load with zero radiant interchange may be 10 per cent higher than that with an emissivity of 0.9 for the room-envelope inside surfaces. It is concluded, therefore, that the model for the calculations of cooling load can be set up, assuming that the surface convection coefficient has a constant value and that the radiant heat interchange can be represented by a linear relation.

The surface temperatures are sensitive to the change in the convection coefficient, h, particularly where convection is one of the major heat transfer mechanisms (e.g., drapes, furniture). Thus, in calculations of comfort conditions, where the mean radiant temperature is taken into consideration, the surface convection coefficient, h, must be known and accounted for accurately. For example, in a room with drapes, the mean radiant temperature near the drapes changes approximately by 5 F for a change in h of the drapes from 0.6 to 1.0 Btu per (sq ft) (hr) (F) (from the results given in Fig. 1).

## COMBINED INSIDE SURFACE HEAT TRANSFER COEFFICIENT

Eq. (A-1) can be simplified by the use of a combined heat transfer coefficient H. The heat transferred from a surface to the room air by convection and to the rest of the enclosure by long-wave radiation is represented by the following:

$$q_s = H_s A_s (\theta_s - \theta_R) \tag{1}$$

where

$q_s$ = heat gain or loss by the surface by convection and radiation
$\theta_s$ = temperature of the surface
$\theta_R$ = room air temperature
$A_s$ = area of the surface

Eq. (1) cannot represent the surface heat transfer under the following conditions:

(a) when the surface and the room air temperatures are equal and the temperature of the remaining surface is not equal to the room air temperature,

(b) when the solar radiation is intercepted by the light-weight room elements (e.g., drapes, slat-type shade, furniture).

In situation (a), the heat gain by or loss from the surface according to Eq. (1) is zero, but, in an actual case, heat interchange takes place between the surface and the rest of the enclosure. In situation (b), according to Eq. (1), all the solar energy absorbed by the surface appears as a cooling load without delay (assuming negligible heat storage by light-weight elements). In an actual situation, however, a fraction of the absorbed solar energy is lost by long-wave radiation to the rest of enclosure where it may be stored and appear as a cooling load with some delay.

To evaluate the error due to this simplification, Eq. (A-1) relating room enclosure surface temperatures, room air temperature and the heat input to the surface was modified according to Eq. (1). The calculations were performed using values of H for all surfaces of 1.4, 1.2, 1.0 and 0.8 Btu per (sq ft) (hr) (F) for a range of KL values of the window glass. The results are given in Fig. 3 where the daily maximum cooling load is plotted versus the combined coefficient H.

The results show that it is not possible to select a particular value of the combined heat transfer coefficient H which would approximate the radiant heat interchange by the inside room surfaces equally well in all situations. For this particular room, H = 0.8 Btu per (sq ft) (hr) (F), gives a fairly good approximation for the calculation of the cooling load

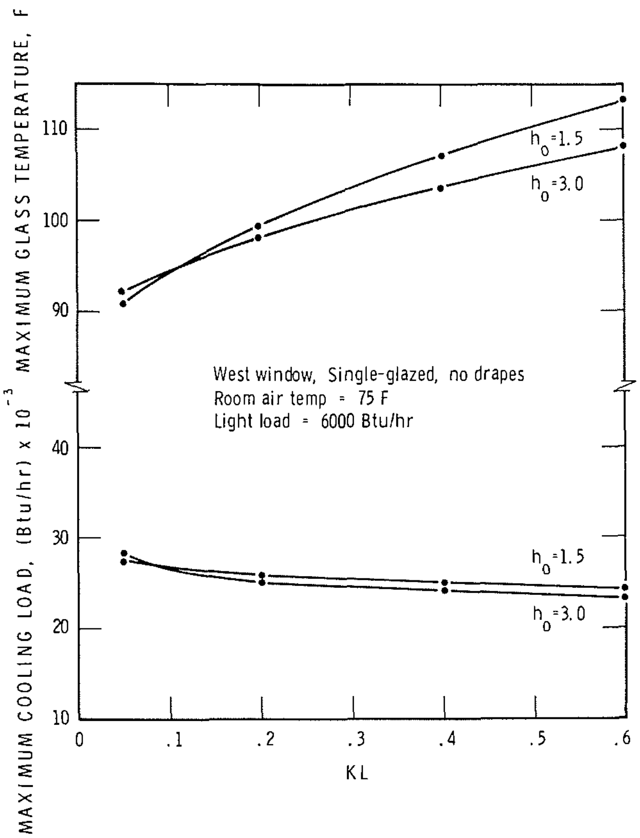



*Fig. 4 Maximum cooling load and maximum glass temperature vs KL of window glass for outside heat transfer coefficient $h_o$ = 1.5 and $h_o$ = 3.0 Btu per (sq ft) (hr) (F)*

when the solar radiation input to the room is relatively small (KL = 0.8). As the solar input increases, even the value of H = 0.8 Btu per (sq ft) (hr) (F) is too high. The value of H = 1.65 Btu per (sq ft) (hr) (F), recommended by the ASHRAE Guide And Data Book, would give a considerably higher maximum cooling load than the standard model.

The other disadvantage of the combined coefficient H is that the value of H, which gives a good approximation of the cooling load, introduces large errors in the calculation of surface temperatures.

## THE OUTSIDE HEAT TRANSFER COEFFICIENT $h_o$

To evaluate the effect of the outside surface heat transfer coefficient $h_o$ on the cooling load, the sol-air temperatures were calculated for $h_o$ = 1.5 and 3.0 Btu per (sq ft) (hr) (F) and the model was set up for these values of $h_o$. A single-glazed rather than a double-glazed window was used because the change in $h_o$ makes a greater change in the over-all thermal resistance of the former.

The calculations were performed for a range of KL values. In Fig. 4 the daily maximum cooling load and the daily maximum glass temperatures for $h_o$ = 1.5 and 3.0 Btu per (sq ft) (hr) (F) are plotted versus KL of the window glass.

The value of the outside surface heat transfer coefficient, $h_o$, has a relatively small effect on the cooling load as indicated by Fig. 4. The effect increases with increasing solar radiation absorption by the glass. When KL = 0.6, a difference of 1.5 Btu per (sq ft) (hr) (F) in $h_o$ changes the daily maximum cooling load by approximately 5 per cent.

The heat gain by the room from the window can be considered to consist of two parts:

(1) heat transfer across the window due to the difference between outside air and inside temperature conditions,

(2) heat gain due to absorbed radiation by the window glass (mainly solar radiation).

The heat transfer across the window increases as the $h_o$ value is increased since the over-all window resistance is reduced. The fraction of the solar radiation absorbed by the glass that is transferred to the room is reduced when the $h_o$ value is increased since the resistance between the glass to the outside surroundings is reduced relative to the resistance between the glass to the room air.

Consequently, for estimations of maximum cooling load and glass temperature, the $h_o$ value should be selected according to the KL value of the glass, and the intensity of the solar radiation incident on the window. For example, with heat absorbing glass in direct sunlight, realistically low values of $h_o$ should be used; for clear glass or any glass in shade, a high value of $h_o$ should be used.

## SUBDIVISION OF THE ROOM ENCLOSURE

To reduce the number of equations needed to describe the room thermal performance, the number of elements representing the room enclosure, e.g., floor-ceiling slab, partitions, window, and furniture, is kept to a minimum and each is assumed to be at a uniform temperature. This mathematical model, particularly where the direct solar radiation input is substantial, cannot exactly represent the actual situation. The direction of the solar beam transmitted through the window changes as the relative position of the sun changes and, therefore, the location on the room surfaces where this beam is intercepted changes during the day. Consequently, the solar input to any one of the room elements is not uniformly distributed over its surface and thus the surface is not at a uniform temperature. The assumption of uniform distribution of solar radiation over the surface introduces no error in the calculated cooling load when there is no heat storage in the element. When significant heat storage is involved, however, it is necessary that the ratio of the flow into the element to the total heat input to the surface is the same for the model element and actual building element. The superposition theorem indicates that, when the boundary effects are negligible (e.g., slab of large surface area) and the surface conductances are constant, this ratio is the same for the following conditions:

(1) the solar radiation is unformly distributed over all the slab surface resulting in one-dimensional temperature field in the slab (as assumed in the model),

(2) the solar radiation is concentrated in one spot

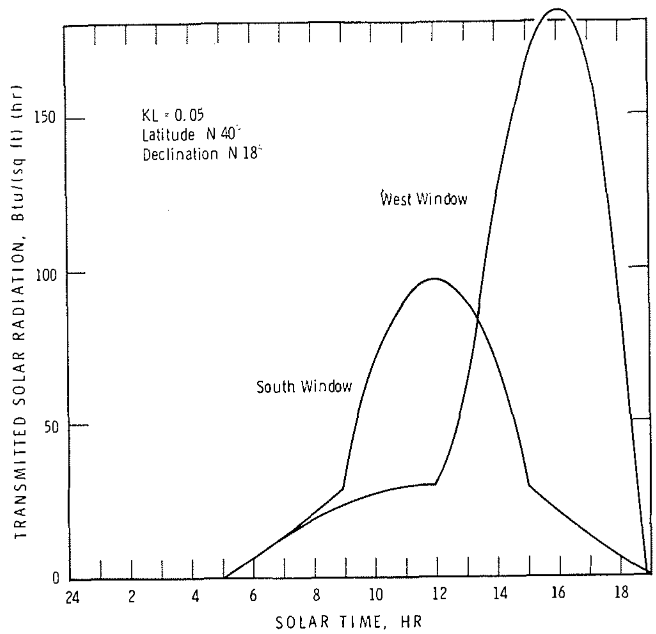



*Fig. 5 Transmitted solar radiation through double-glazed window without drapes*

which changes position on the slab surface with time, resulting in a three-dimensional moving temperature field (as actually happens).

The surface conductance is the sum of the convective and radiative conductances. The radiative conductance is a function of position on the surface (i.e. the over-all view factor from the spot to the rest of the enclosure surfaces) and of the absolute surface temperatures. To check the effect of a change in the radiative conductance, the floor slab was divided in two parts—one part adjacent to the window which received all the direct solar radiation transmitted through the window, and the other part adjacent to the inside wall. The cooling load values obtained using this model were essentially the same as the values using the model in which the floor slab was represented as one element. The maximum difference was less than half of one per cent.

The change in the surface convection conductance as well as the radiative conductance due to a change in the surface temperature will be relatively small since the temperature changes of surfaces of the massive room elements are moderate even with large solar inputs. Thus the model in which the main elements of the room are individually represented as a single unit, and where it is assumed that the radiation input is unformly distributed over the whole surface, will represent the actual elements with adequate accuracy for cooling load and average surface temperature calculations.

## ROOM ENVELOPE MASS

Air-conditioning load calculations would be considerably simplified if the heat storage factor (i.e. ratio of the daily maximum cooling load over the daily maximum instantaneous heat gain) could be related to the mass of the room envelope by a simple expression. Calculations were made, therefore, to determine the effect on the heat storage factor of the mass of the room envelope, furniture, inside shading, interpane shading, solar radiation absorption characteristics of the window glass (i.e. KL of window glass), and orientation.

To evaluate the effect of the room envelope mass on the storage factor, the floor slab and the partition thickness were varied from 3 to 12 in. in 3-in. increments. To check the effect of orientation, south and west orientations were considered; the two curves of solar radiation transmitted through south- and west-facing windows are given in Fig. 5, to show the difference of the major heat gain component for the two orientations. The results of these calculations are given in Figs. 6 to 9 where the heat storage factor is plotted versus the room envelope construction.

The effect on the cooling load of several combinations and types of inside shade, interpane shade, KL of the window glass, and furniture was evaluated and the results given in Table I.

The results indicate that it is not possible to relate the heat storage factor to a parameter indicating room mass alone, since the storage factor is also a function of the shape of the solar radiation curve, the type of window construction, the furnishings, and other lightweight objects inside the room.

The most active layer of the room envelope with respect to the heat storage is the one closest to the surface. As the thickness of a room envelope element is increased, the effective heat storage per unit mass is decreased. For the particular room under consideration, extra concrete beyond 12-in. thickness has a small effect on the heat storage factor as indicated by the results given in Figs. 6, 8 and 9.

The results for the south and west orientations indicate a significant difference in heat storage factor; thus, in estimating this factor, the orientation should

**Table I Maximum Cooling Load for a Room With and Without Furniture**

**(Double-glazed window, west, room air temperature = 75 F, light load = 0.0 Btu/hr)**

| Window and Shade Characteristics | With Furniture: Max. Cooling Load $\times 10^{-3}$ Btu/hr | With Furniture: Max. Cooling Load/ Max. Instan. Heat Gain | Without Furniture: Max. Cooling Load $\times 10^{-3}$ Btu/hr | Without Furniture: Max. Cooling Load/ Max. Instan. Heat Gain |
|---|---|---|---|---|
| KL = .05 | 25.1 | 0.67 | 20.7 | 0.55 |
| KL = .60 | 20.2 | 0.72 | 17.4 | 0.62 |
| With Inside Drape: | | | | |
| KL = .05, $\tau$ = .5, $\alpha$ = .2 | 18.9 | 0.75 | 18.0 | 0.72 |
| KL = .05, $\tau$ = .2, $\alpha$ = .5 | 19.8 | 0.82 | 19.4 | 0.80 |
| With Interpane Drape: | | | | |
| KL = .05, $\tau$ = .5, $\alpha$ = .3 | 15.5 | 0.75 | | |
| KL = .05, $\tau$ = .3, $\alpha$ = .5 | 14.3 | 0.77 | | |

$\tau$ = Transmission of the shade
$\alpha$ = Absorption of the shade

KL = KL of outer glass sheet, the KL of inner pane = 0.05 in all cases

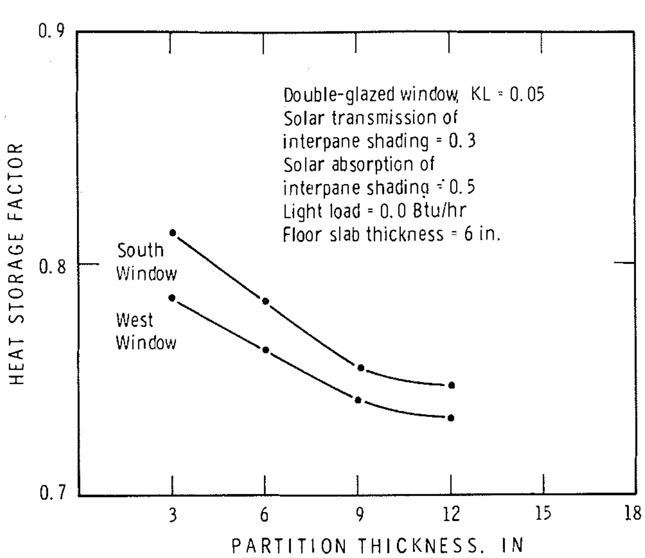



*Fig. 6 Heat storage factor vs partition thickness for a room with a shade*

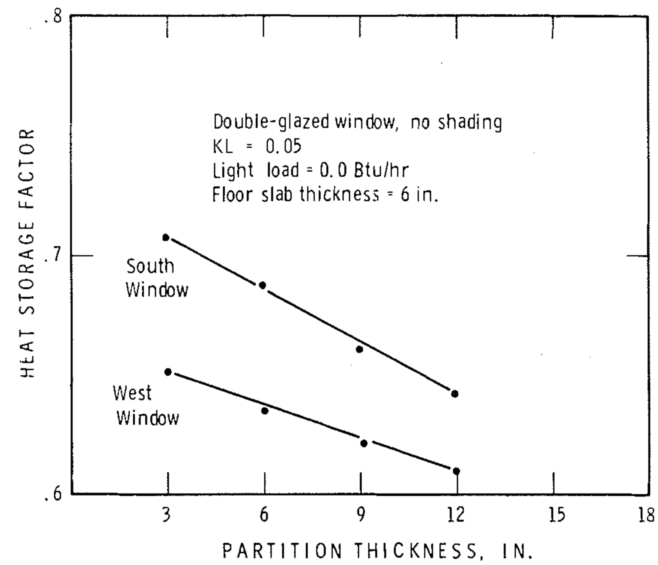



*Fig. 7 Heat storage factor vs partition thickness for a room without a shade*

be taken into consideration. From this it also follows that the heat storage factor is a function of the time of the year since the shape of the curve of solar irradiation versus time of day changes during the year.

The lightweight objects in the room significantly increase the heat storage factor as is indicated by the results given in Table I. The radiant energy absorbed by a lightweight object increases its surface temperature rapidly and this energy is lost to the surrounding surfaces by radiation and to the room air by convection with little lag. With a massive slab, the radiant energy absorbed by its surface is partly conducted into the slab and thus the fraction lost instantaneously to the room air is reduced.

In general, shading reduces the cooling load although the heat storage factor is increased. In some situations with inside shading, or heat absorbing glass, it is possible that the reduction in the heat storage could be greater than the reduction in the solar heat gain and thus the maximum cooling load could be greater with an inside shading device than without one.

It should be noted that the furniture representation in the mathematical model was very much idealized, as described in Appendix A. The results, therefore, should be regarded as qualitative rather than as quantitative.

## SUMMARY

The thermal behavior of a room can be represented sufficiently accurately for cooling load calculations by a linear mathematical model; the calculations can thus be carried out conveniently on electronic digital computers, which are widely available.

A mathematical model based on a combined heat transfer coefficient for inside surfaces does not accurately represent the room thermal response. The

*Fig. 8 Heat storage factor vs floor slab thickness for a room with a shade*

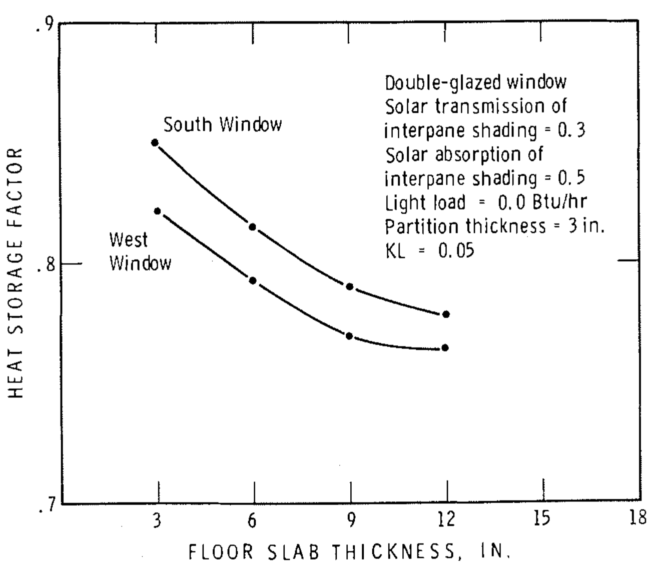
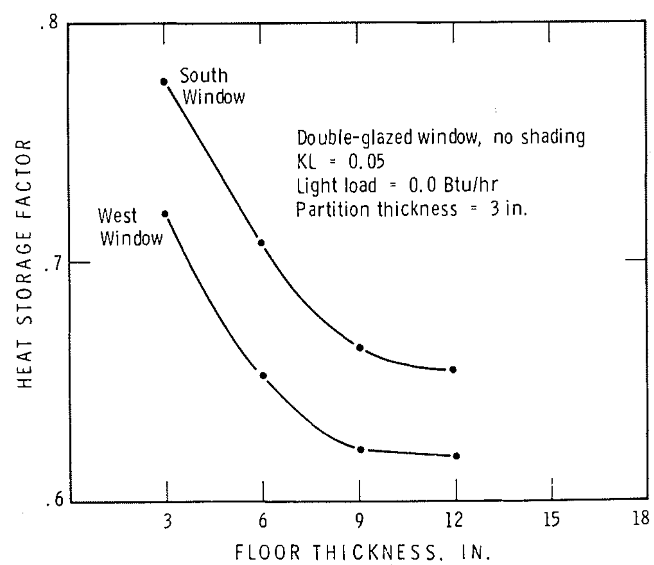



*Fig. 9 Heat storage factor vs floor slab thickness for a room without a shade*



accuracy of modeling the room thermal behavior is improved by accounting separately for the heat interchange inside the room enclosure by long-wave radiation and the convective heat interchange between the room envelope surfaces and the room air.

A room element, e.g., floor-ceiling slab, partitions, of uniform construction can be represented in the mathematical model as a single unit, and it can be assumed that the surface temperature of any one element is uniform without introduction of large errors. Lightweight room elements such as drapes, blinds and furnishings, as well as the heavy room elements, must be accounted for in the mathematical model.

The heat storage factor cannot be related simply to the mass of the room envelope since such factors as the presence of lightweight elements in the room, orientation, and shading have a significant effect. The temperature of a particular spot on the room envelope cannot be calculated with a high degree of certainty using this simple model.

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## APPENDIX A

### Details of the Room

An office module with inside dimensions of 20 x 20 x 10 ft with 160 sq ft of glass in the outside wall was selected for this analytic investigation. It was assumed that this room was surrounded by similar rooms on the remaining three sides so that the heat transfer through the partitions and the sum of the daily average heat transfer through the floor and ceiling surfaces are zero. It was also assumed that at all times sufficient cooling was available to maintain the room air temperature of 75 F. The thermal properties of the room elements are listed in Table A-1. The emissivities and the surface convection coefficients for all inside surfaces of the room envelope were assumed to be constant and equal to $\epsilon = 0.9$ and $h = 0.8$ Btu per (sq ft) (hr) (F), respectively.

### Mathematical Model

The equations used to describe the room thermal characteristics were as follows:

(1) Surface temperature

$$G \sum_{n=1}^{m}{}' \, {}_sF_n\theta_n + h\theta_R + q_n - \left( G \sum_{n=1}^{m}{}' \, {}_sF_n + h \right)\theta_s = -I_s \qquad \text{(A-1)}$$

where

$G = 4\,\sigma\,T^3\,av$

($T$ av $= 540$ R — assumed time average of all the inside surface temperatures

$\sigma$ = Stefan-Boltzman constant)

$\sum_{n=1}^{m}{}'$ = the sum without the term, $n = s$

${}_sF_n$ = over-all interchange factor from surface s to surface n

$\theta_n$ = temperature of the surface n, which forms part of the room enclosure

h = surface convection coefficient

$\theta_R$ = room air temperature

$q_n$ = heat flow through the slab surface

$\theta_s$ = temperature of the surface where $n = s$

$I_s$ = short-wave radiation and long-wave radiation from lights absorbed by the surface.

(2) Temperatures at points within the wall or floor. These can be calculated from the following, taken from Reference 3:

**Table A-1 Construction of Room Envelope Elements**

| | |
|---|---|
| Floor and ceiling slab | 6-in. heavyweight concrete slab with false ceiling<br>Thermal diffusivity = 0.046 sq ft per hr<br>Thermal conductivity = 1.00 Btu per (ft) (hr) (F)<br>False ceiling air space, over-all heat transfer coefficient = 1.6 Btu per (sq ft) (hr) (F) |
| Partitions | 3-in. lightweight concrete<br>Thermal diffusivity = 0.0154 sq ft per hr<br>Thermal conductivity = 0.095 Btu per (ft) (hr) (F) |
| Opaque outside wall | 2-in. heavyweight concrete on the outside<br>4-in. lightweight concrete on the inside |
| Window | Single- or double-glazed with or without inside or interpane shading<br>Negligible heat storage capacity |
| Furniture | The furniture representation was idealized by assuming thin slabs covering half of the floor area<br>The slab thermal resistance was assumed to be very great so that the heat flow in or out of the slabs was negligible |

$$\dot{\theta}_n = \frac{\alpha N^2}{L^2} ( \theta_{n-1} + \theta_{n-1} - 2\theta_n ) \qquad \text{(A-2)}$$

where

$\alpha$ = thermal diffusivity of the slab material
L = slab thickness
N = number of slices used in the approximation of the slab thermal behavior

(3) Outside surface temperature

$$( \theta_{s/a} - \theta_{s/o} ) h_o + q_{no} = 0 \qquad \text{(A-3)}$$

where

$\theta_{s/a}$ = sol-air temperature
$\theta_{s/o}$ = surface temperature
$h_o$ = surface heat transfer coefficient.
$q_{no}$ = heat flow through the outside surface of the wall

Fundamentally, the mathematical model based on Eqs. (A-1), (A-2) and (A-3) is the same as the model to describe the room thermal performance in Refs 5, 6, 7, 8, and 9.

## DRIVING FUNCTION

The time-dependent conditions used for these calculations were:

(1) Window, outside pane, sol-air temperature
(2) Opaque outside wall, sol-air temperature
(3) Direct solar radiation transmitted through the window
(4) Diffuse solar radiation transmitted through the window
(5) Solar radiation absorbed by the inner pane of the double-glazed window
(6) Solar radiation absorbed by the interpane shading or inside blind
(7) Heat generated by electric lighting.

The solar radiation absorbed by the pane of the single-glazed window or by the outer pane of a double-glazed window were taken into account by the sol-air temperature of the glass.

The outside air temperature cycle given in the ASHRAE Guide And Data Book for cooling load calculations was used as a basis for the calculations of the sol-air temperatures as well as the sky and ground long-wave radiations. These were calculated using Brunt's formula.[4] The dewpoint temperature for the sky long-wave radiation calculations was assumed to be 50 F.

The direct solar radiation incident on the outside wall is given by the following equation, taken from Ref. 10:

$$I = I_o^{-a/\cos\gamma} \cdot \cos\theta \qquad \text{(A-4)}$$

where

$I_o$ = 368 Btu per (sq ft) (hr) apparent solar constant
a = 0.223, extinction coefficient for the atmosphere
$\gamma$ = solar Zenith angle
$\theta$ = incident angle.

The solar radiation values calculated by Eq. (A-4) are equal to the corresponding values given in the ASHRAE Guide And Data Book.

The diffuse solar radiation incident on the outside wall was calculated by

$$I_D = I_n/X \, [F(\cos\theta) + F_\rho(X \cos\gamma + 1)] \qquad \text{(A-5)}$$

where

X = 10 = the ratio of the direct solar radiation incident on a surface normal to the solar beam, i.e. $\cos\theta = 1$ in Eq (A-4), to the diffuse solar radiation incident on a horizontal surface. Since the seasonal variation of X may be between 1/.07 and 1/.12, 10 is used and is constant throughout a clear day.
$F(\cos\theta)$ = functional relation between $\cos\theta$ and $F(\cos\theta)$ is given in Ref 11, p. 330
$F_\rho = 0.1$ = product of the geometric view factor from the surface to the ground, and the ground reflection factor for the solar radiation.

The transmission and absorption factors for various window arrangements were taken from Ref. 12. It was assumed that 90% of the direct solar radiation transmitted through the window was absorbed by the floor and the furniture in equal portions and the remaining 10% was absorbed by the other surfaces according to the view factor from the floor and the furniture.

The transmitted diffuse radiation was distributed over the surfaces according to the geometric view factor from the window.

Lights were the only heat source within the room taken into account by the model. The lights were on from 0730 to 1630 hr. One-half of the heat generated by the lights dissipated to the room air by convection and the other half was dissipated to the room inside surfaces by radiation according to the geometric view factor from the ceiling to the other room inside surfaces. In some of the calculations, the lighting load was omitted due to a mechanical failure.

## DISCUSSION

**C. W. PENNINGTON,** Gainesville, Fla.: An $h_o$ value of 3 was used throughout. Is this typical usage in Canada? The ASHRAE Guide And Data Book normally uses $h_o$4 for summer conditions and 6 for winter conditions.

The temperatures both for the glass and the draperies shown are quite low compared with our measured typical values. What was the source of the drapery and glass temperatures?

**AUTHOR MITALAS:** The value of $h_o$ = 3.0 Btu/ sq ft, hr, F was selected arbitrarily.

The temperatures of the glass and draperies were calculated from the conditions of well mixed room air at 75F. Since these conditions probably are considerably different from those in your test, the calculated and measured glass and drapery temperatures cannot be compared.

**L. F. SCHUTRUM,** Creighton, Pa. (Written): The room envelope mass and its effect on the heat storage factor are given in Figs. 6 and 8 for interpane shading, and in Figs. 7 and 9 for double glazed windows without shading. The heat storage factor is influenced by the solar radiation passing through the window to a different degree than the heat gains are influenced by conduction from the outdoor air. The greater amount of transmitted solar energy for the west orientation than for the south one, as shown in Fig. 5 (disregarding the difference in time period), indicates that a greater percentage of total load for the west orientation is due to the sun. If the heat storage factor could be subdivided into a solar component and an outdoor air temperature component, the curves for these individual components for the south window and west window might be closer together than when these two load inputs, solar radiation and outdoor air temperature, are combined.

Would the heat storage factor for interior partitions be different from that for the floor, which intercepts the transmitted solar radiation?

It would be interesting to attempt to correlate the overall heat storage factor for the double glass data of Figs. 7 and 9 with the four component factors: solar radiation, outdoor air temperature, mass of the floor and furniture, and mass of the partition.

Were the floor above the room under consideration and the ceiling below this room assumed to be exposed to the same thermal environment as the floor and ceiling of the room under consideration? The transmission and absorption factors for various window arrangements were taken from Ref. 12. What heat transfer coefficients were assumed for the various parts of these fenestrations?

**AUTHOR MITALAS,** (Written); The heat storage factors given in Figs. 8 and 9 are basically for the solar heat gain component, since the conduction component is only 8 to 14% of the total heat gain (the lower percentage for clear glass with no shading, and the higher percentage for inside shading). Also, the heat storage factor for the conduction component will be close to one, since approximately half of the heat gain by conduction through window will appear as cooling load instantaneously (convection loss from glass to room air), and only the other half will be affected by the room envelope mass.

The heat storage factor would be different if the solar radiation were absorbed by the interior partitions, rather than by the floor and furniture.

The floor above and the ceiling below the room under consideration were assumed to be exposed to the same thermal environmental conditions as the floor and ceiling of this room itself.

Fenestration heat transfer coefficients were:

1. Inside convection coefficient = 0.8 Btu/ sq ft, hr, F
2. Outside heat transfer coefficient = 3.0 Btu/ sq ft, hr, F
3. The heat transfer coefficient = 1.32 Btu/ sq ft, hr, F for interpane space of double glazed window and for the space between glass and drape when the drape was inside double glazed window interpane space.