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COMPARISON OF MODES OF OPERATION FOR GUARDED HOT PLATE APPARATUS WITH EMPHASIS ON TRANSIENT CHARACTERISTICS

BY

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COMPARAISON DES MODES DE FONCTIONNEMENT DES PLAQUES CHAUFFANTES PROTEGEES ET PARTICULIEREMENT DE LEURS CARACTERISTIQUES

SOMMAIRE

Les auteurs établissent des comparaisons entre la méthode à apport constant de chaleur et celle du fonctionnement à température constante des plaques chauffantes protégées (guarded hot plate), et étudient l'existence d'un apport optimal de chaleur. Ils dressent des tables d'équations de réponse en escalier pour les différents modes de fonctionnement et différentes températures de préparation et en donnent une expression graphique. Ils indiquent les températures de préparation optimales, examinent les effets des perturbations et de la capacité de la plaque et comparent les problèmes de régulation et de mesure. Ils proposent un mode d'utilisation composite et examinent l'application des résultats à l'emploi des calorimètres et des appareils de mesure des écoulements transitoires de chaleur. L'utilisation d'une température constante ou d'un mode de travail composite semblent les plus pratiques. Les durées de réponse peuvent être beaucoup plus courtes que lors des essais à apport constant de chaleur. Pour les autres mesures, les auteurs proposent l'utilisation de la plaque chauffante protégé à température constante.

Comparison of Modes of Operation for Guarded Hot Plate Apparatus with Emphasis on Transient Characteristics

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The constant heat flux and constant temperature modes of operation of the guarded hot plate apparatus are compared and the existence of an optimum heat flux boundary condition is examined. The step response equations for the different modes and for various preconditioning temperatures are tabulated and presented in graphical form. Optimum preconditioning temperatures are determined. Effects of disturbances and plate capacity are discussed and control and measurement problems compared. A hybrid mode of operation is suggested. The extension of the results to the use of heat meters and transient heat flow apparatus is discussed. Constant temperature or a hybrid mode of operation is found to be the most practical. The response times can be an order of magnitude less than constant heat flux operation. Use of the guarded hot plate apparatus in the constant temperature mode is suggested for other measurements.

Key Words: Guarded hot plate, heat meters, modes, optimum boundary conditions, response time, thermal conductivity, thermal diffusivity, transient errors, transient heat flow.

1. Introduction

Historically, the guarded hot plate apparatus (1)[1] is a "Standards Lab" type instrument where the emphasis is on accuracy rather than on minimizing the duration of the test. The transient heat flow characteristics and control problems have therefore been neglected. Other types of apparatus have been evolved for short duration tests of lesser accuracy.

It was believed that the standard guarded hot plate apparatus was much more versatile than assumed and that with the proper control system and measurement techniques its use could be extended to rapid, high accuracy testing. The apparatus was thought to offer advantages in the transient heat flow area and for use in conjunction with heat meters. Also, since the guarded hot plate apparatus is being proposed for testing "super" insulations with extremely long response times, it was believed that the control mode that is optimum in terms of speed and accuracy should be determined and the response functions tabulated to allow estimation of the time to reach an adequate steady state condition.

The guard balance is assumed to be controlled automatically in this analysis so that it does not introduce additional disturbances into the system. This is believed to be a reasonable assumption as most laboratories have provided this type of control for their apparatus. The temperature of the cold plate is assumed to be constant.

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[1] Figures in parentheses indicate the literature references at the end of this paper.

2. Modes of Operation and Transient Characteristics

Two modes of control of the hot plate in the guarded hot plate apparatus are in general use. For the simplest and most widely used the plate is supplied with constant power, thereby producing a constant heat flux at the surface of the plate. The second, less widely used, method is to keep the temperature of the hot plate constant by controlling the power with a temperature controller. The latter method was used by Sommers (2) for "high speed" tests, but has not been widely used because of the extra equipment involved and the lack of knowledge of the relative advantages and problems of this mode compared to the constant heat flux mode.

2.1. Constant Heat Flux Operation

The usual sequence of events in performing a test with the guarded hot plate apparatus with constant heat flux from the hot plate is as follows:

(i) The sample is conditioned to the mean temperature of the test

(ii) The cold plates are at the required temperature

(iii) No power is being supplied to the hot plate and it is not at the required test temperature

(iv) The thermal conductivity and, therefore, the power to the test area are estimated

(v) The sample is placed in the plates, the power to the test area increased for an unspecified period of time, to heat up the plate and the sample, then reduced to the estimated power. The conditions then are allowed to reach steady state

(vi) Additional adjustments of the power are usually necessary and disturbances must settle out.

There has been no easy way to estimate the time required to reach the steady state condition using this procedure.

2.2. Constant Temperature Operation

The usual sequence of operations for use with the hot plate at a controlled temperature is as follows:

(i) The sample is conditioned to the mean temperature of the test

(ii) Both the hot and cold plates are at the required temperatures

(iii) The sample is placed in the apparatus and the test area power increases automatically to supply the required heat and then drops to the steady state value.

It should be noted that no guessing or second adjustments are required.

2.3. Optimum Boundary Conditions

Constant temperature operation is easier than constant heat flux operation, and probably faster, but the question remains as to whether there is a boundary condition that will give even faster response.

Assuming there is a better, nonconstant, heat flux boundary condition, $F(t)$, where t is time, and the appropriate unit step response of the sample is $V(x, t)$, the surface temperature of the hot plate can be written as given in ref. (3).

$$\theta(x,t) = \int_0^t F(t-t') \, V_t \, (x,t') \, dt'$$

where t' is the dummy time variable, and subscript t indicates the time derivative.

By normal response factor methods, F(t) can be found for any desired response. Since the answer depends, however on the step response of the sample, V(x, t), which is a function of the unknown thermal properties, there is no practical way to find this optimum condition for thermal conductivity apparatus. The most practical boundary conditions would appear to be the constant temperature boundary condition, since it is the only boundary condition that can be specified without regard to sample properties.

2. 4. Response Times

The theoretical solutions to the heat transfer problems corresponding to the constant temperature and constant heat flux modes of operation are given in Appendix I. The solutions have been derived in nondimensional form to make them general. The temperature is nondimensionalized as shown in Appendix I to be zero at the cold plate and 1 at the hot plate under steady state conditions. The non-dimensional heat flux is also 1 under steady conditions. Time zero is referred to as the time at which the boundary conditions are applied, that is, the test is started, and is nondimensionalized by the thermal diffusivity "a" and the thickness, L.

The solutions are given in general form and also for specific locations in the sample at which the measurements are to be made for determining the thermal conductivity, since it is these values that must approach steady state before the measurements can be made accurately.

The response time of a sample is defined here to mean the time required for the uncontrolled, or unspecified, variable at the hot plate to reach an adequate steady state so as to give an error in the measurement of thermal conductivity of less than the specified value. A curve of error in the measurement versus time will result for each mode. The equations for error are given in Appendix II.

Response times for the constant temperature and constant heat flux modes with the initial temperature equal to the steady mean temperature, W = 0. 5, are given in figure 1. The response time for constant temperature mode is seen to be almost an order of magnitude less than for the constant heat flux mode when the errors are small.

The response times for specimens at various initial temperatures, W, are shown in figures 2 and 3, for values of W = 0, the cold plate temperature, to W = 1, the hot plate temperature. In each mode there is an optimum value of W at which the response time is least. The error reverses sign, that is, overshoots, when the initial temperature is greater than 0. 5 for the constant temperature mode and 0. 635 for the constant heat flux mode, and the negative portion of the curve can govern the response time of a test. The optimum values of the initial temperature, W, are therefore approximately 0. 52 and 0. 64, respectively, for the two modes. The response times for the two modes at the optimum W are approximately the same. The response time for the constant flux mode at optimum conditioning is an order of magnitude less than that for conditioning to the mean temperature; this can be useful in cases where the thermal conductivity of a specimen is being checked and the steady state heat flux can be determined beforehand. If the thermal conductivity is not known then the final hot plate temperature and therefore the optimum W cannot be determined.

The equations and figures do not take into account the heat storage capacity of the plates, the three-dimensional nature of the heat flow, or the temperature dependence of the properties. It should therefore be recognized that the equations and figures are only a guide to the actual response time and that checks should be made on the applicability of the figures to any particular apparatus by testing samples of known thermal diffusivity.

A test was made on a foam polystyrene sample, 2. 5 cm thick with a 20- by 20-cm guarded hot plate to compare the theoretical response times to the actual values. For constant heat flux operation, the response time to $\frac{1}{2}$ per cent error was less than 15 sec, when the hot plate was preheated to the steady state temperature, and over 10 hr when the plate was not preheated. In each case the exact constant heat flux was used to obtain the desired hot plate temperature. The theoretical value was approximately 40 sec for the specimen. In constant temperature mode the measured response time was approximately 70 sec. The response curves are, therefore, better estimations of the actual response times for the constant temperature mode than the constant heat flux mode.

2.5. Measurement Problems

Each mode of operation has inherent measuring problems. The constant heat flux mode of operation has such long response times that the true steady state value can be difficult to recognize. Measurements must be made over an extended period to ensure that it has been reached. The constant temperature mode of operation requires that the temperature of the hot plate be controlled, by controlling the power to the heater. The steady state power to the heater will, by definition, vary a small amount with time. It is necessary to average the power over a sufficient length of time to obtain the mean values. The averaging time will depend on the controller characteristics and the temperature sensor. The problem of averaging can be eased considerably by using a strip chart recorder, automatic data logger, or a signal averaging circuit.

2.6. Hybrid Mode

The best of both methods can be obtained by starting the test in the constant temperature mode and switching later to the constant heat flux mode. This hybrid mode of operation requires a signal averager and is accomplished by starting in the constant temperature mode, monitoring the average power supply voltage, and when it becomes sufficiently constant, setting the output of the controller to that value, and switching the controller to manual mode, thereby supplying constant heat flux to the hot plate. Depending on the success of the transfer to constant heat flux operation, the measurement can be made with a short settling time, and with normal measuring instruments.

2.7. Disturbances and Readjustments

If the temperature or the heat flux of the hot plate must be readjusted so as to conform to a standard test condition, or if it is disturbed, the time for the transient to settle can be estimated from the response time curves. A change in a parameter at one surface corresponds to the case of a change at the surface and zero initial temperature. The error that the disturbance produces in the steady state value should be used in determining the response time. For example, a 10 per cent change need only settle to 10 per cent of its final value to produce a 1 per cent error in the measurement.

2.8. Plate Capacitance

The hot plate assembly can have more than 100 times the thermal capacitance of the sample and can therefore be the major factor in determining response time (3). Failure to preheat the hot-plate assembly to the desired temperature can greatly extend the test, especially for the constant heat flux mode where the small source of heat must raise the temperature of the large thermal mass. The increase in response time, or lag, decreases as the conductance of samples increases, and can vary from a few seconds to many hours. For constant temperature operation, the hot plate is normally at the required temperature and the thermal mass can be used to advantage to supplement the heater capacity in heating the specimen. In the hybrid mode, the thermal capacitance is helpful during the constant temperature interval and detrimental in the constant heat flux interval.

2.9. Characteristics of Control Systems for Hot Plates

It has been found that the response time for the normal design of hot plates and normal samples is so great that reset is required, i.e. integral action with "anti-windup" feature along with the proportional action in the controller. The sudden changes in power output required when high capacitance samples are placed in the apparatus make it desirable to have rate, i.e. derivative action, as well. Proper design of the hot plate and placement of the temperature sensor can decrease the lags in the control system and ease these requirements.

2.10. Further Refinement

If the shortest possible test time is required, there is no reason why the samples cannot be preconditioned to the proper temperature gradient in a large, simplified pair of temperature controlled plates. The samples could then be tested rapidly in an apparatus operated in the constant temperature mode.

3. Application to Heat Meter Apparatus

The guarded hot plate apparatus, when operated in the constant temperature mode, forms the basic configuration required for measuring thermal conductivity with a heat meter; the advantage of this heat meter method is the simplicity of the measurements.

The solutions in Appendix I can be applied to the heat meter apparatus as well, as long as the heat meter is thin and has a low thermal capacitance. Equations (5), (6), and (7) of Appendix I show that there are three locations in a sample where optimum response without overshoot can be attained. The locations and relevant conditions are listed below.

Test Conditions

(i) the sample is conditioned to a uniform temperature

(ii) the temperature of the hot and cold plates are controlled.

Locations and Initial Temperatures

(i) at the hot surface - sample at the mean temperature

(ii) at the cold surface - sample at the mean temperature

(iii) at the center of the sample - any uniform temperature.

The third location, at the center of the sample, is not only ideal in terms of response but also provides some isolation from any small independent disturbances at the plates and, ideally, does not require the sample to be conditioned to any specific temperature. The initial temperature must be uniform or at least symmetric about the center point. If there is the possibility that thermal conductivity is a function of temperature, the center position affords the proper location to indicate if the temperature has deviated from the mean value. The center position requires two identical samples. Lang (4) has developed an apparatus with the heat meter in this location but it is not clear if all the advantages were recognized. Norris and Fitzroy (5) gave the theoretical justification for the method.

4. Extension to Transient Measurement Conditions

Beck (6) has determined the optimum configuration for transient heat flow apparatus for simultaneous determination of thermal conductivity and specific heat. When the thermal conductivity is low, the optimum configuration is a large thermal mass "cold plate," preconditioned but not continuously cooled, and a constant temperature warm side with heat flux metered. The cold plate is insulated on all but the active surface. The thermal properties are inferred from the cold plate temperature and the hot side heat flux, using a "nonlinear estimation" procedure. The guarded hot plate apparatus with temperature controlled hot plate can be modified to attain the required configuration; this allows the extension of the apparatus to measurement of both thermal conductivity and specific heat.

With samples of high thermal conductivity, the optimum configuration at the cold plate remains the same while that of the hot side condition is zero heat flux. Thermal properties are inferred from temperatures measured on the two sides by the same procedure. By removing the hot plate from the guarded hot plate apparatus, this condition can be accomplished.

Further work should be considered on the adaptation of the guarded hot plate apparatus to this type of measurement, and a study of the sources of error. Modification in design of the hot and cold plates might be required to make them "universal," or interchangeable plates might be considered.

5. Conclusion

The constant temperature and hybrid modes of operation have been shown to be the most practical for the guarded hot plate apparatus. The response times can be an order of magnitude better than for constant heat flux operation. The figures presented can be used as a guide to estimate the time required to reach an adequate steady state.

Guarded hot plate apparatus with temperature-controlled plates can be used for rapid measurements, either alone or in conjunction with a heat meter. The apparatus also might be used for the simultaneous determination of thermal conductivity and thermal diffusivity.

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6. References

(1) Method of test for thermal conductivity of materials by means of the guarded hot plate (C177-63. Book of ASTM Standards, 1963, Part 3.

(2) Somers, E. V., High-speed guarded hot plate apparatus for thermal conductivity of thermal insulation. Trans., ASME, 78, August 1956, p. 1191-6.

(3) Churchill, R. V., Operational mathematics, Second Edition, McGraw-Hill Book Co. Inc., New York, 1958.

(4) Lang, D. L., A quick thermal conductivity test on insulating materials, ASTM Bulletin No. 216, September 1965, p. 58-60.

(5) Norris, R. H. and Fitzroy, (Mrs.) Nancy D., A quick thermal conductivity test on insulating materials, Materials Research and Standards, VI, September 1961, p. 727-729.

(6) Beck, J. V., The optimum analytical design of transient experiments for simultaneous determinations of thermal conductivity and specific heat. Michigan State University, Ph.D., 1964 Engineering, Mechanical.

Appendix I

Response Times for an Infinite Slab

1. Basic Equation

The nondimensionalized heat equation for an infinite slab with uniform boundary conditions is

$$\frac{\partial^2 \theta}{\partial X^2} = \frac{\partial \theta}{\partial \tau}$$

where $\theta = \left[T(x,t) - T(0,\infty)\right] / \left[T(L,\infty) - T(0,\infty)\right]$

$X = x/L$, $\tau = a\,t/L^2$, L = the thickness of the slab, x = the space variable, running from 0 to L, t = time, a = the thermal diffusivity.

The nondimensionalized heat flux is written as

$$\frac{q}{q_1} = \frac{\partial \theta}{\partial X}$$

where q = heat flux, q_1 = the steady state heat flux $\left(-\lambda\left[T(L,\infty) - T(0,\infty)\right]/L\right)$, λ = the thermal conductivity, ∞ = indicates the steady state value.

1.1. Boundary Conditions (B.C.)[2]

Constant temperature at $X = 0$

$$\theta(0, \tau) = 0.$$

Two separate boundary conditions at $X = 1$ will be considered:

(i) Constant temperature: $\theta(1, \tau) = 1$, and

(ii) Constant heat flux: $\left.\frac{\partial \theta}{\partial X}\right)_{1,\tau} = 1.$

In each case the steady state temperature difference and steady state heat flux are equal to unity.

1.2. Initial Conditions

The following uniform initial condition is assumed:

$$\theta(X, 0) = W$$

where W is a finite constant.

1.3. Solutions to Basic Problem

Solutions have been determined using basic solutions and the "Superposition Principle" as given by Churchill (3).

The solutions are given in terms of the variable not specified at $X = 1$, that is, in terms of heat flux for constant temperature boundary condition, and in terms of temperature for the constant heat flux boundary condition. The solutions are first given in general form, then for special cases with X, or both X and W, specified. Tables of the functions may be obtained by writing the authors directly.

(1) General Form of Solutions

(a) Constant Temperature B.C.

$$\left.\frac{\partial \theta}{\partial X}\right)_{1,\tau} = 1 + 2 \sum_{n=1}^{\infty} (-1)^n \cdot \cos(A_n X) \cdot \exp(-A_n^2 \tau) + 4W \sum_{n=1}^{\infty} \cos(B_n X) \cdot \exp(-B_n^2 \tau) \qquad (1)$$

where $A_n = n\pi$, $B_n = (2n-1)\pi$, $\exp(\ )$ is the exponential function, $e^{(\ )}$

(b) Constant Heat Flux B.C.

$$\theta(X, \tau) = X - 8 \sum_{n=1}^{\infty} \left[(-1)^{n-1}/B_n^2\right] \cdot \sin(B_n X/2) \cdot \exp(-B_n^2 \tau/4)$$

$$+ 4W \sum_{n=1}^{\infty} (1/B_n) \cdot \sin(B_n X/2) \cdot \exp(-B_n^2 \tau/4) \qquad (2)$$

[2] Boundary conditions will be abbreviated as B.C.

(2) Special Cases

(a) Constant Temperature B.C., with X = 1

$$\left.\frac{\partial\theta}{\partial X}\right)_{1,\tau} = 1 + 2\sum_{n=1}^{\infty} \exp(-A_n^2\tau) - 4W \sum_{n=1}^{\infty} \exp(-B_n^2\tau) \quad (3)$$

(b) Constant Heat Flux B.C., with X = 1

$$\theta(1,\tau) = 1 - 8\sum_{n=1}^{\infty} (1/B_n^2) \cdot \exp(-B_n^2\tau/4) + 4W\sum_{n=1}^{\infty} \left[(-1)^{n-1}/B_n\right] \cdot \exp(-B_n^2\tau/4) \quad (4)$$

(c) Constant Temperature B.C., with X = 0.5, W finite

$$\left.\frac{\partial\theta}{\partial X}\right)_{0.5,\tau} = 1 + 2\sum_{n=1}^{\infty} \exp(-4A_n^2\tau) \quad (5)$$

(d) Constant Temperature B.C., with X = 1, W = 0.5

$$\left.\frac{\partial\theta}{\partial X}\right)_{0.5,\tau} = \text{right-hand side of eq (5)} \quad (6)$$

(e) Constant Temperature B.C., with X = 0, W = 0.5

$$\left.\frac{\partial\theta}{\partial X}\right)_{0,\tau} = \text{right-hand side of eq (5)} \quad (7)$$

(f) Constant Temperature, with X = 1, W = 1

$$\left.\frac{\partial\theta}{\partial X}\right)_{1,\tau} = 1 + 2\sum_{n=1}^{\infty} (-1)^n \cdot \exp(-A_n^2\tau) \quad (8)$$

2. Calculation of Errors

Errors in determination of thermal conductivity, if measurements are made before steady state conditions are attained, are derived below.

2.1. Constant Temperature B.C.

Error at time τ and position X.

$$\text{Error} = \frac{q(X,\tau) - q(X,\infty)}{q(X,\infty)}$$

$$= \left.\frac{\partial\theta}{\partial X}\right)_{X,\tau} - 1$$

since $\frac{q(X,\tau)}{q(X,\infty)} = \left.\frac{\partial\theta}{\partial X}\right)_{X,\tau}$, and $\left.\frac{\partial\theta}{\partial X}\right)_{X,\infty} = 1$

2.2. Constant Heat Flux B.C.

Error at time τ and at position X

$$\text{Error} = \frac{\left[\theta(X,\infty) - \theta(0,\infty)\right]}{\left[\theta(X,\tau) - \theta(0,\tau)\right]} - 1$$

$$= \frac{1}{\theta(X,\tau)} - 1$$

$$= \frac{1}{\theta(1,\tau)} - 1 \quad \text{at} \quad X = 1$$

since $\theta(0,\tau) = 0$, all τ, and $\theta(X,\infty) = X$.

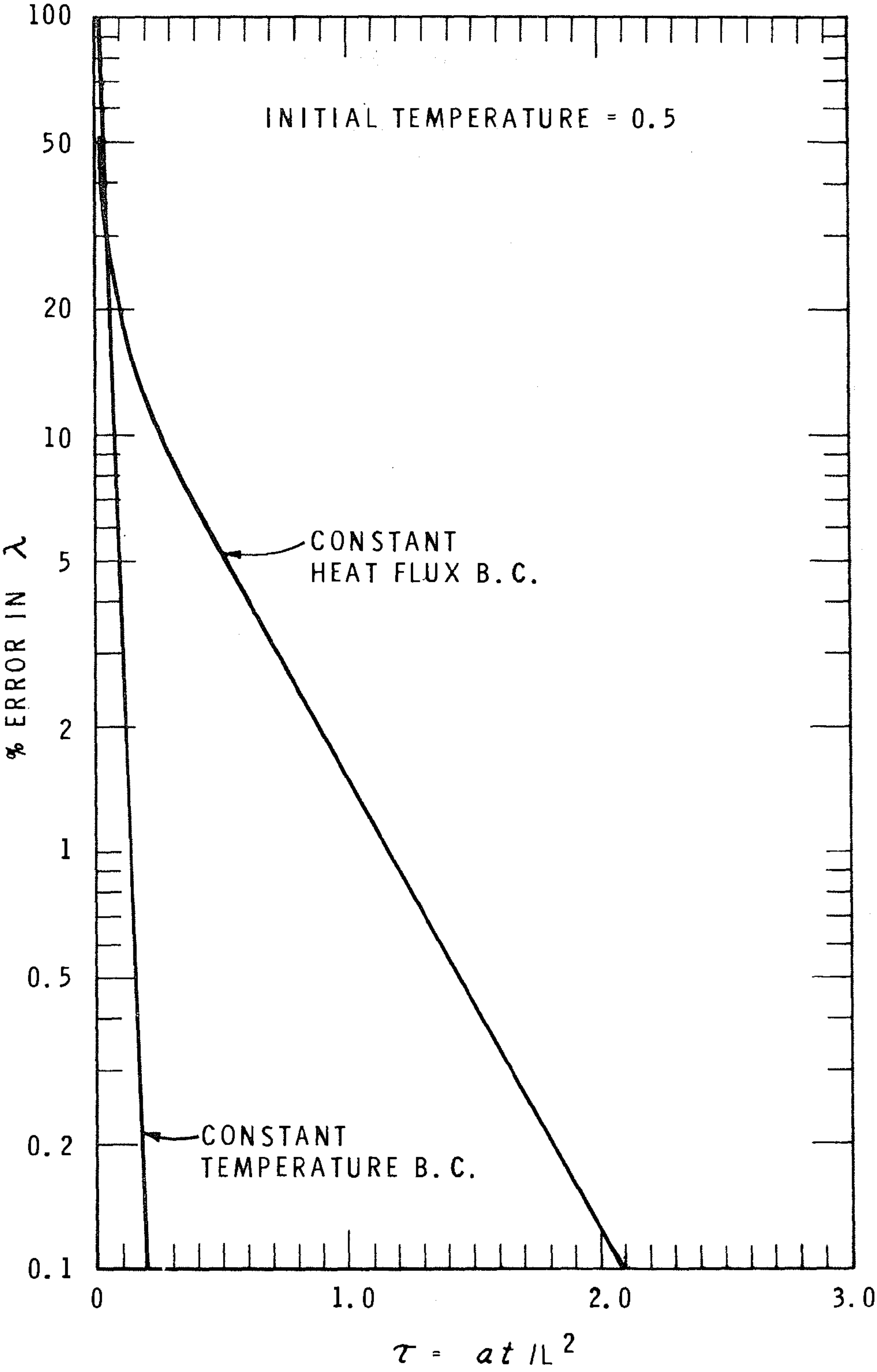

Figure 1 Response Times for Measuring Thermal Conductivity
- Error at Time τ After Start of Test
- Constant Temp. and Constant Heat Flux Modes
- Initial Temp., W = 0.5

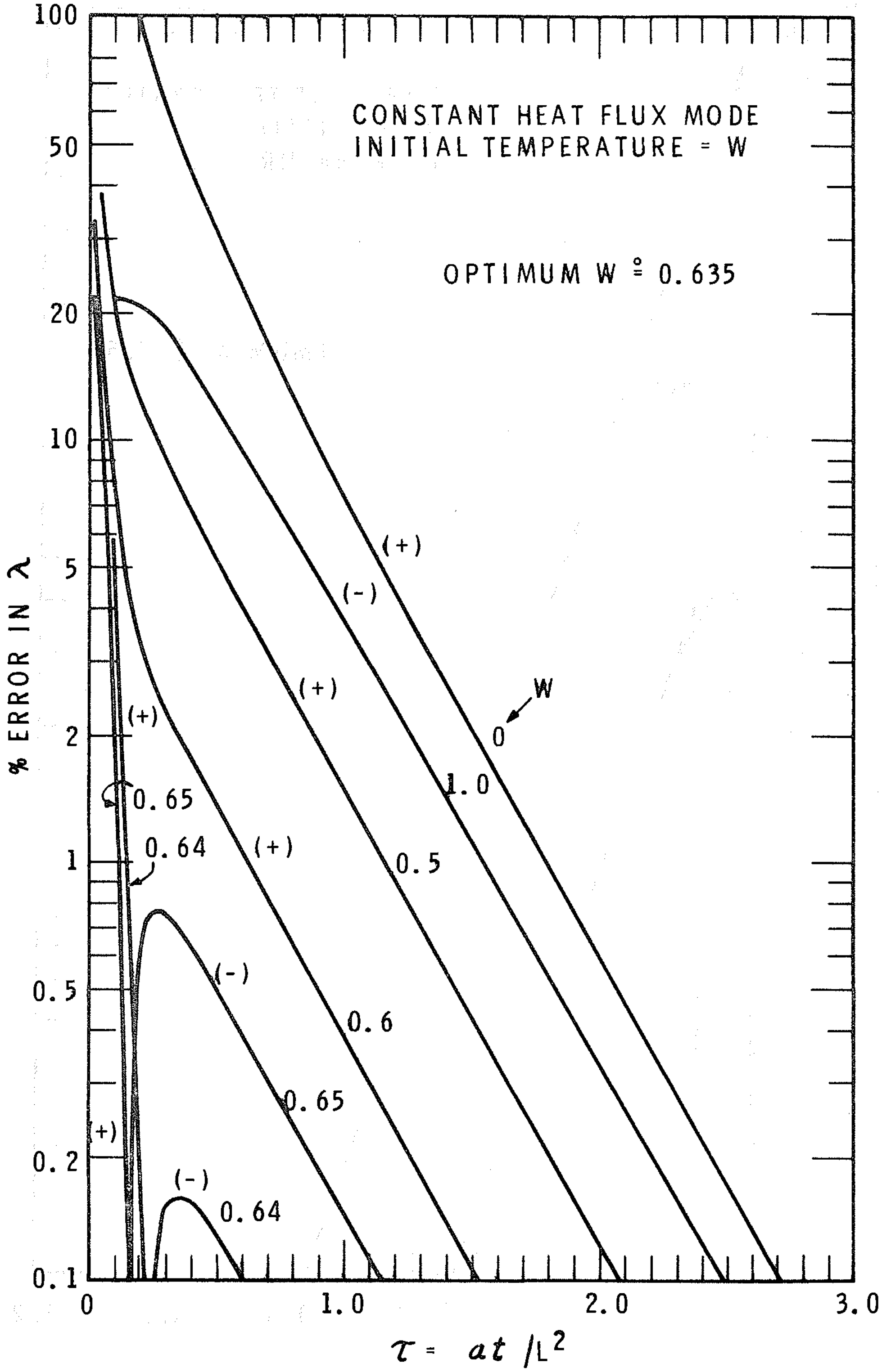

Figure 2 Response Times for Measuring Thermal Conductivity
- Error at Time τ After Start of Test
- Constant Heat Flux Mode
- Initial Temp., W = 0 to 1.0

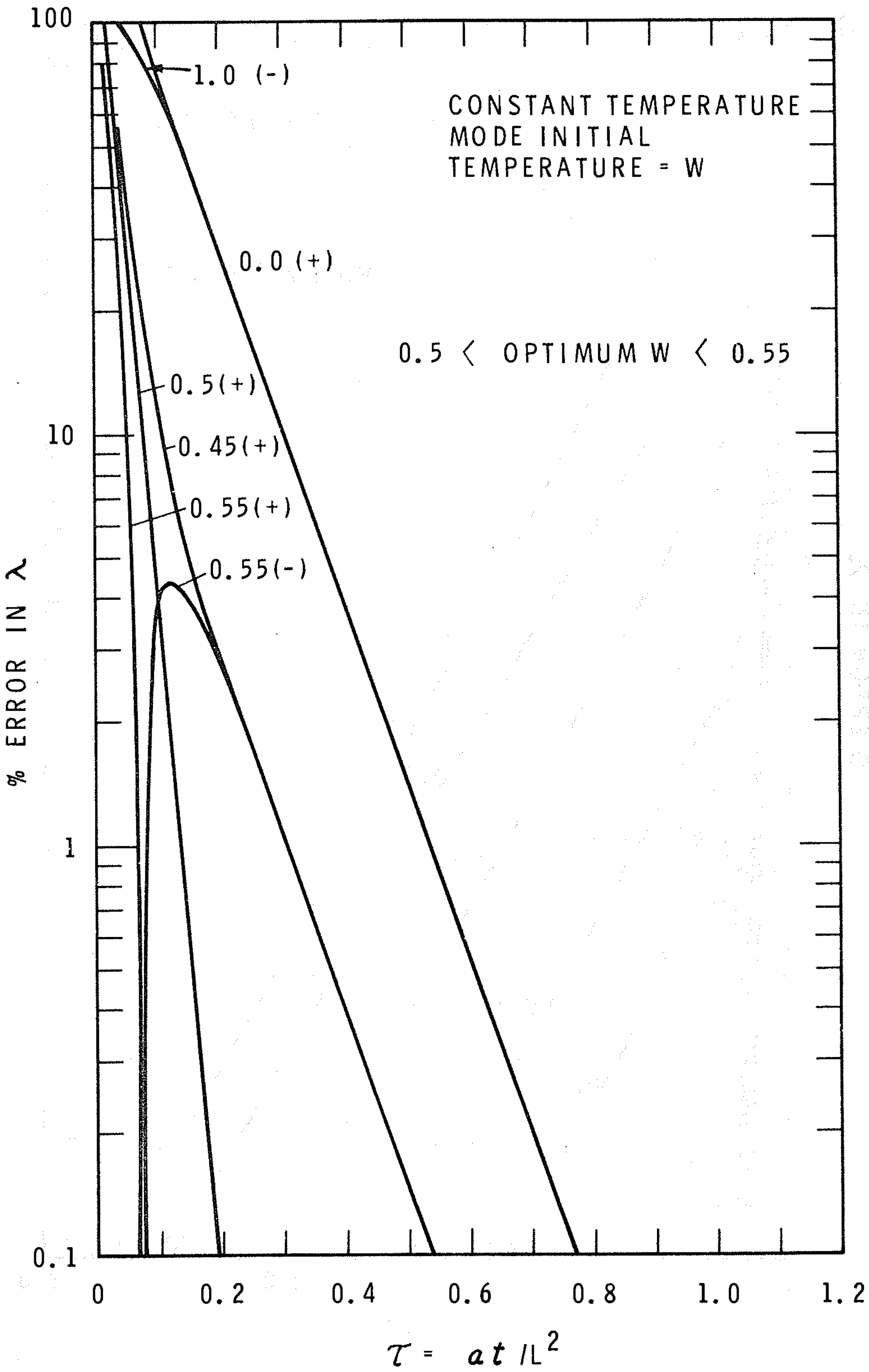

Figure 3 Response Times for Measuring Thermal Conductivity
- Error at Time τ After Start of Test
- Constant Temp. Mode
- Initial Temp., W = 0 to 1.0