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Stress-Relieving Techniques for Cantilever Beam Tests in an Ice Cover

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RÉSUMÉ

Des essais de poutre en console effectués sur une couverture de glace d'eau douce ont montré que le fait de relâcher les contraintes en forant des trous près de l'encastrement de la poutre augmentait la résistance en flexion déterminée par la théorie des poutres simples dans une proportion de 1/4 à 1/3. L'analyse par la méthode des éléments finis a confirmé l'existence d'une concentration des contraintes au bord de la poutre, lesquelles sont 1,5 plus fortes qu'au centre en présence de trous et 2 fois plus fortes en l'absence de trous. On peut déterminer la résistance en flexion des poutres en consoles en appliquant un facteur de correction de 1,08 lorsque des trous ont été pratiqués près de l'encastrement et de 1,35 dans le cas contraire.

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STRESS-RELIEVING TECHNIQUES FOR CANTILEVER BEAM TESTS IN AN ICE COVER

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ABSTRACT

Cantilever beam tests carried out on a freshwater ice cover showed that introducing stress relief holes at the root of the beam increased flexural strength, as determined from simple beam theory, by 1/4 to 1/3, compared to conventional cantilever beams. Finite element analysis confirmed the existence of stress concentrations at the edge of the beam, 1.5 times the centre stress for the case of a stress relief hole and 2 times in the case of a cantilever beam without stress relief holes. A correction factor of 1.08 has been determined for the flexural strength of cantilever beams with stress relief holes at the root; for those with no special root treatment, the correction factor is 1.35.

INTRODUCTION

The flexural behaviour of ice is important for the prediction of bearing capacity of ice covers and ice loading on structures. It is also a significant factor in natural ice failure processes such as ridge building, ride-up, pile-up and rubble building. Usually the flexural properties of an ice cover are determined from *in situ* cantilever beam tests. The interpretation of the results of such tests is critical for their subsequent application to other cases of flexural behaviour. Flexural testing has the drawback of being an indirect test, i.e., certain assumptions have to be made about material and beam behaviour in order to interpret the results. On the other hand, the test is an analogue of the loading condition which it models.

The cantilever beam test is often used for *in situ* flexure tests on ice covers in the field and in model basins (Frankenstein, 1970; Schwarz, 1975; Vaudrey, 1978). The results of such tests are normally analyzed in terms of simple beam theory, assuming the cantilever is rigidly clamped at the root and ignoring any stress concentration at that location. These shortcomings have been recognized and various analytical and experimental techniques proposed to compensate for them. Määttänen (1975) studied beam and root geometry effects during field tests carried out in the Baltic, and found a decrease in strength and modulus with increasing ratio of beam width to ice thickness. Beam lengths up to ten times ice thickness had no effect on flexural strength, but modulus decreased with decreasing length. Beams with a large radius of curvature at the root had strengths about 30% greater than ones terminating with straight saw cuts at the root. Gow (1977) carried out an extensive series of *in situ* tests on both cantilever and simply supported beams. He found that the flexural strength of simple beams was as much as twice the strength of corresponding cantilever beams. The difference was attributed to the effect of stress concentration. When Svec and Frederking (1981) examined the influence of geometry at the root of a cantilever beam, we found moments in the root area to be 50% greater than those determined from simple beam theory.

To address more fully this problem of performing and interpreting the results of cantilever beam tests, parallel investigations were implemented using photoelasticity, finite element analysis and full-scale field tests. This paper presents the results of *in situ* beam tests in a natural ice cover where special measures

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were taken to reduce the stress concentrations at the root of the beam. These results are compared with predictions of a finite element analysis.

FIELD TESTS

Field tests were carried out in late January and early February 1982 on the ice cover of an outdoor manoeuvering basin (60 X 120 m) at the National Research Council in Ottawa. At that time the ice thickness was 0.35 m, snow depth 0.2 m and freeboard 0.005 m. The ice, as shown in Fig. 1, was made up of a 170 mm thick top part of snow ice (Type T1) (see Michel and Ramseier, 1971) with a grain diameter of 0.5 to 5 mm, and a remaining 180 mm of columnar grain ice with very large grain diameters. Care was taken to select a site with homogeneous ice conditions and without visible cracks. Air temperature during the tests was in the range –5 to –20°C, so there was a temperature gradient through the ice cover. Environmental conditions for the tests are given in Table 1.

The beams were cut out with a chain saw. Initial cuts were made through about 3/4 of the ice thickness, thus keeping the cut dry. The final part of the cut was done with a handsaw to minimize the amount of surface flooding. Stress relief holes were drilled at the root with ice augers of various diameters. In order not to change the original loading and temperature conditions, the snow cover was left undisturbed on the ice except along the cutting paths. Nevertheless there was a warming of the ice during beam preparation and testing, due mainly to the presence of 0°C water in the saw cut and stress relief holes. Also there was sufficient snowfall between Tests 5 and 6 to depress the ice, causing a negative freeboard of 40 mm.

Following the guidance of photoelastic model studies, stress relief holes were drilled at the root of the beam tangent to the side cuts. A variety of treatments at the beam root were used, including parallel saw cuts, 25, 100, 150 and 250 mm diameter holes, and 45° angled cuts (see Fig. 2). Load was applied to the tip of the beam by a hydraulic cylinder reacting against a loading frame anchored to the ice cover. Load was measured by a load cell between the cylinder and the ice beam. Three displacement

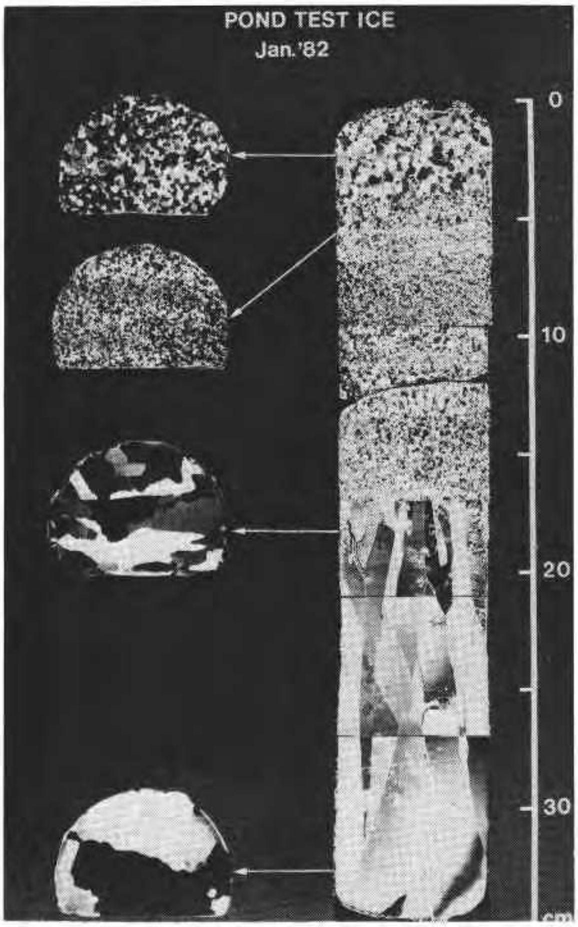

Fig. 1. Thin section of ice tested.

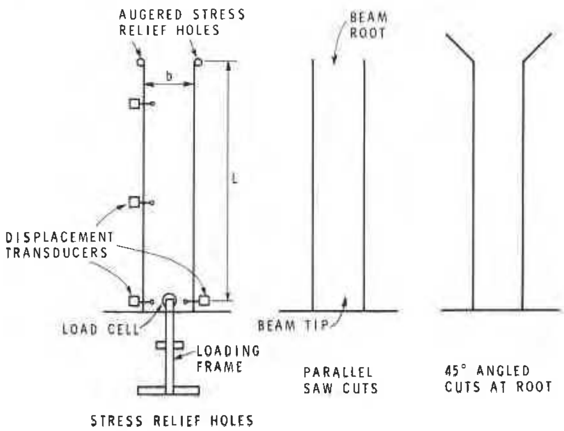

Fig. 2. Schematic of test arrangements for different beam geometries.

transducers were used to measure deflections of the beam. Continuous records of load and deflections versus time were made. A typical record is shown in Fig. 3 (Test 1). Because a hand pump was used to pressurize the cylinder, a step-wise loading resulted. A load relaxation between strokes of the hand pump is apparent. Load was always applied downwards so that the top fibres of the beam were in tension. The load versus deflection results for Test 1, plotted in Fig. 4, show that the load/deflection curve is relatively linear; serrations in the curve are due to the load relaxation mentioned above.

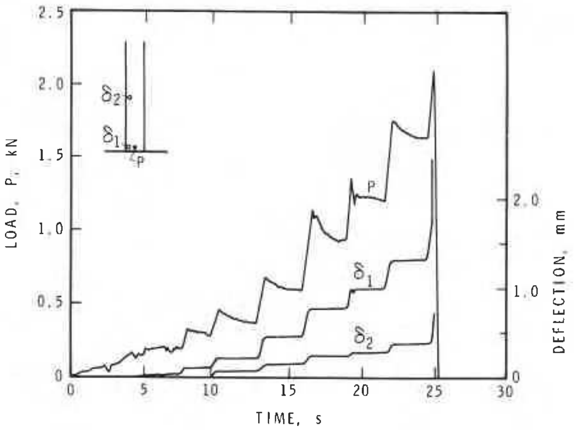

Fig. 3. Load (P) and deflection at tip (δ_1) and mid-point of beam (δ_2) versus time, Test No. 1.

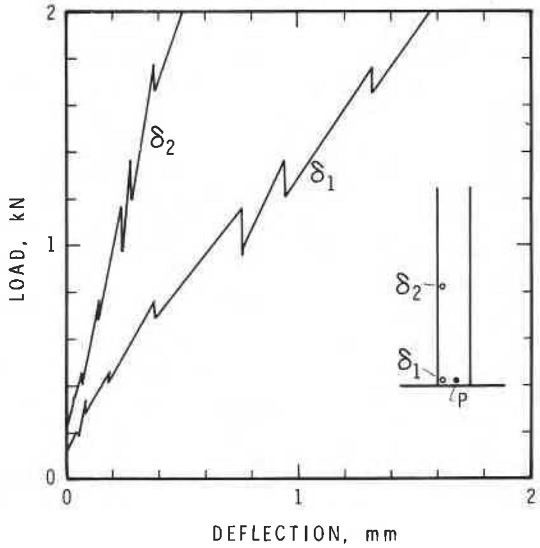

Fig. 4. Load (P) versus deflection at tip (δ_1) and at mid-point of beam (δ_2), Test No. 1.

TEST RESULTS

The results of all the tests are summarized in Table 2. Flexural strength (σ_{sb}) and elastic modulus (E_{sb}) were calculated from simple elastic beam theory using the following equations

$$\sigma_{sb} = \frac{6P'L}{bh^2} \tag{1}$$

TABLE 1

Environmental conditions during cantilever-beam tests

Test	Date	Air temp. °C	Ice surface temp. before °C	Ice surface temp. after °C	Sky	Notes
1	Jan. 27 pm	−15	−4	−0.5	clear	
2	Jan. 28 pm	−10	−2	0.0	snow flurries	
3	Jan. 28 pm	−8	−0.1	−0.2	snow flurries	Slush between snow cover and ice surface
4	Jan. 29 am	−10	−0.1	−0.1	clear	,,
5	Jan. 29 pm	−8	−0.2	0.0	clear	,,
6	Feb. 01	−8	−1.9	0.0	clear	Snowed overnight, snow depth 37 cm, 4 cm of water on ice surface
7	Feb. 01	−7	−1.0	0.0	clear	,,

TABLE 2

Geometry of beams and test results

Test No.	Root	P' (kN)	L (m)	b (m)	h (m)	σ_{sb} (kPa)	t_f (s)	P/δ (MN/m)	E_{sb} (GPa)
1	saw cuts	2.05	2.41	0.40	0.35	610	25.0	1.44	4.7
2	15 cm ϕ	2.55	2.34	0.405	0.35	720	13.7	1.51	4.4
3	2.5 cm ϕ	1.85	2.42	0.415	0.35	530	7.3	1.40	4.5
4	45° angle	2.80	2.12	0.465	0.34	660	8.5	–	–
5	25 cm ϕ	2.10	2.34	0.38	0.34	680	6.9	–	–
6	10 cm ϕ	2.65	2.45	0.405	0.35	780	6.0	–	–
7	saw cuts	1.55	2.35	0.39	0.35	460	6.3	1.77	5.4

$$E_{sb} = \frac{4}{b}\left(\frac{L}{h}\right)^3 \frac{P}{\delta} \tag{2}$$

where P' is breaking load, L is beam length, b is beam width, h is ice thickness and P/δ is the slope of the load-deflection curve established by drawing a line from the origin to 50% of failure load. There is always a considerable variability in strength obtained from cantilever beam tests; a standard deviation of 20% is common. However, the results here indicate that beams with stress relief holes at the root (Tests 2, 3, 5 and 6) had strengths 25% greater than beams ending with parallel saw cuts at the root (Tests 1 and 7). The next section will treat in more detil the stress concentration factor for the geometry of the beams tested here.

In all cases where stress relief holes were drilled, failure occurred at, or slightly behind (away from the beam tip) the point of minimum width between the holes. Test 4, with the 45° angled cuts, failed across the intersection of the angled cuts with the parallel side cuts of the beams. The failure plane in all cases was essentially vertical and plane.

There were problems in the experiments with both the deflection transcucers and the recorders, so that a complete record of the deflections of the beams could not be obtained in all cases. When elastic modulus values could be calculated using eqn. (2), remarkably consistent results were obtained.

FINITE ELEMENT ANALYSIS

The results described above were compared to a numerical analysis using a finite element computer program. This program, based on thin plate bending theory and originally developed by Svec and McNeice (1972), was subsequently modified to include other features (Svec et al., 1985). The reader is referred to these publications for more information on mathematical and finite element backgrounds of this program.

Full-scale tests were modelled as closely as possible. Care was taken to duplicate exact test geometry as well as boundary and loading conditions. The infinite plate to which the cantilever beam was attached was simulated by a rectangular plate extend-

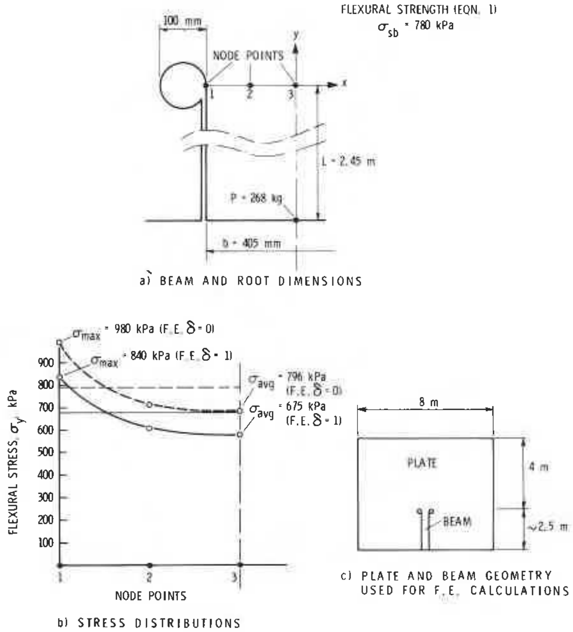

Fig. 5. Finite element analysis of test beam with relief hole, Test 6.

ing 4 m to either side of the beam, 4 m behind the beam and about 2.5 m in front of the root (see Fig. 5c). There are many other factors influencing actual field results which were not duplicated, e.g., temperature gradients in the beam; nonapparent cracks in the ice; and roughness of ice surfaces cut by the chain saw acting as crack-initiating notches.

Two specific cases were examined using finite element analysis: Test 6 with 10 cm diameter stress relief holes and Test 7 with saw-cut slots at the root. The ice was taken to have elastic modulus E = 9.5 GPa and Poisson's ratio of 0.3. The Young's modulus value was used in the calculations since it represents rate-independent pure elastic behaviour of the ice. The measured elastic moduli (E_{sb} in Table 2), which are secant values including time-dependent and plastic deformations, are about half of Young's modulus. The distribution of bending stress across the root of the beam and the average bending stress were calculated for an ice plate hydrostatically supported by water (δ = 1) and for an ice plate which is simply supported along its outside boundaries (δ = 0). In both cases the plate and load geometry of Fig. 5c was used. The exact dimensions (Figs. 5a and 6a) and corresponding breaking loads (P') of the two tests were used in the finite element analysis to calculate stress distributions at the root, as shown in Figs. 5b and 6b.

There is a significant difference in the magnitude of the stress depending upon whether the beam/plate combination is simply supported (δ = 0) or hydrostatically supported (δ = 1). Using the secant modulus (E_{sb}) rather than Young's modulus would make the stress on a hydrostatically supported (δ = 1) plate even lower. In contrast, if the problem was analysed as a simple cantilever beam, which assumes the beam is rigidly clamped at the root, the influence of the hydrostatic support of the water on loading stress is negligible (Frederking and Haüsler, 1978, or Tatinclaux and Hirayama, 1982). This points up the importance of selecting the correct boundary conditions in analysing the problem. The maximum bending stress at the edge of the hole (Test 6, Fig. 5b) for a hydrostatically supported beam (δ = 1) is 840 kPa, compared to a value of 780 kPa calculated from simple beam theory. For Test 7 (Fig. 6b), a beam with saw-cut slots at the root, the maximum bending stress at the edge of the slot is 620 kPa,

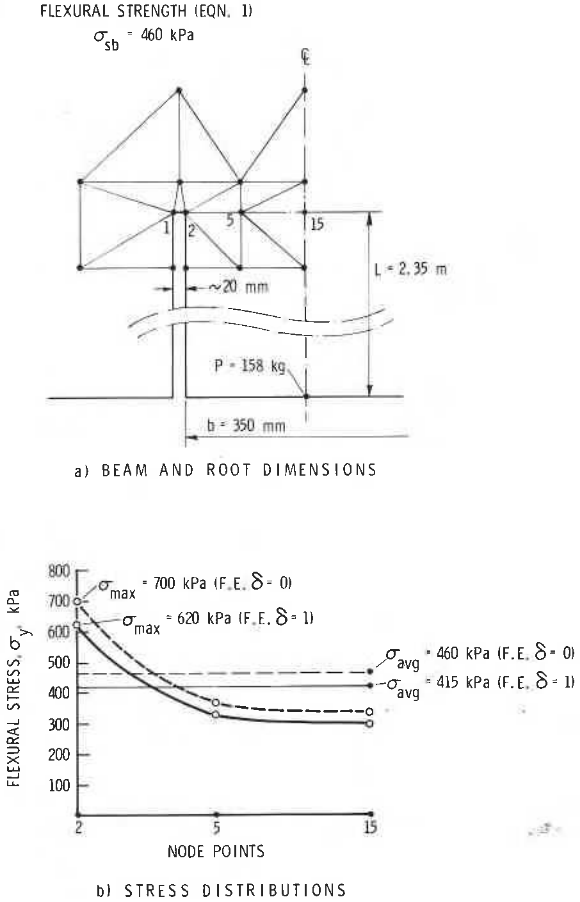

Fig. 6. Finite element analysis of test beam created by parallel saw cuts, Test 7.

compared to 460 kPa calculated from simple beam theory. Ideally the maximum bending stress (flexural strength) as calculated by the finite element method should agree for the two cases. The two values, 840 kPa and 620 kPa, and the difference between them, however, are typical of the variation found for beam tests in ice. Note that the experimental flexural stress as calculated from simple beam theory, is the average stress and can be compared to the finite element average stress.

It is interesting to note the agreement between the "average" flexural strength calculated from simple beam theory (Eqn. (1)) and the average as determined from the finite element analysis for the case of the simply supported plate/beam combination (δ = 0), i.e., 780 kPa and 796 kPa, respectively, for Test 6 and 460 kPa and 460 kPa, respectively, for Test 7. This is not surprising since, without any buoyancy force on the beam or beam/plate combination, equilibrium conditions require that the resisting moment at the root be equivalent to the moment resulting from the load applied at the beam tip. Therefore, this agreement is a verification of accuracy of the calculations.

The bending stress distribution across the beam root with the stress relief hole can be compared with the distribution in the beam with saw-cut slots. For the case of the beam with stress relief holes, the stress at the edge of the hole is about 50% greater than the stress at the centre of the beam (Fig. 5b) while, in the case of the saw-cut slot, the stress at the edge of the beam is about twice that at the centre (Fig. 6b). This shows that the relief hole reduces the stress gradient across the beam root. It also provides a means of accounting for the stress concentration effect at the root of cantilever beams. In the case of the beam with a stress relief hole, the stress at the edge of the beam was 840 kPa, compared to 780 kPa calculated from simple beam theory, giving a ratio of 1.08. The beam with saw-cut slots to the root had a maximum stress at the edge of the beam of 620 kPa, compared to 460 kPa calculated from simple beam theory, giving a ratio of 1.35. When these correction factors are applied to the test results in Table 2, the average flexural strengths from saw-cut beams (Tests 1 and 7) and stress-relieved beams (Tests 2, 5 and 6) are 720 kPa and 780 kPa, respectively.

Gow (1977), conducting beam tests on temperate lake ice, found that the ratio of flexural strength as determined from simple beam tests and cantilever beam tests varied from 1:1 at cantilever beam strengths of 400 kPa, to 2:1 at cantilever beam strengths of 800 kPa. The effect was attributed to stress concentrations at the root of the cantilever beam, which become less significant for warmer (weaker) ice. Määttänen (1975), in tests on brackish ice, found a stress concentration ratio of 1.2:1 between simple and cantilever beams, and 1.5:1 between cantilever beams with 1.5 m radius arc at the root and beams with no relief holes. These test results are similar to the experiments and analysis of this investigation.

Recently Timco (1985) has examined the relation between flexural strength as measured from cantilever beam and simple beam tests for a number of ice types and found that, in the case of sea ice and urea model ice, the results were equal for both tests. This is attributed to the presence of brine and air pockets in the ice, which relieve stress concentrations through plastic flow. The case of sea ice at very low temperatures, say of the order of –20°C, is not known, but stress concentration could again become relevant.

CONCLUSIONS

(1) The flexural strength of cantilever beams with stress relief holes at the root was about 25% greater than for those with no relief holes.
(2) Finite element analysis, using the dimensions and loading conditions of two test cases, showed that the apparent strength as calculated from simple beam theory should be 25% greater for beams with stress relief holes.
(3) Average flexural strength as determined by finite element analysis and simple beam theory agreed, verifying the calculation methods.
(4) True flexural strength of a freshwater ice cover can be determined by applying a correction factor of 1.08 in the case of a cantilever beam with a stress relief hole at the root, or 1.35 for a cantilever beam with no stress relief, to the strength value calculated from simple beam theory.
(5) Stress relief corrections, as indicated above, should not be necessary for sea ice covers at moderate temperatures (> -10°C).

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REFERENCES

Frankenstein, G.E. (1970). The Flexural Strength of Sea Ice as Determined from Salinity and Temperature Profiles. National Research Council of Canada, Associate Committee on Geotechnical Research, TM 98, pp. 66–73.

Frederking, R.M.W. and Haüsler, F.U. (1978). The flexural behaviour of ice from *in situ* cantilever beam tests. In: Proc. International Association for Hydraulic Research (IAHR), Symposium on ice problems, Luleå, Sweden, Part 1, pp. 197–215.

Gow, A.J. (1977). Flexural strength of ice on temperate lake. J. Glaciol., 19(81): 247–256.

Määttänen, M. (1975). On the flexural strength of brackish water ice by in situ tests. Proc. Third International Conference on Port and Ocean Engineering under Arctic Conditions, Fairbanks, pp. 349–359.

Michel, B. and Ramseier, R.O. (1971). Classification of river and lake ice. Can. Geotech. J., 8: 35–45.

Schwarz, J. (1975). On the flexural strength and elasticity of saline ice. In: G.E. Frankenstein (Ed.), Proc. 3rd International Symposium on Ice Problems, Hanover, N.H., pp. 373–386.

Svec, O.J. and Frederking, R.M.W. (1981). Cantilever beam tests in an ice cover: Influence of plate effects at the root. Cold Regions Sci. Technol., 4: 93–101.

Svec, O.J., Thompson, J.C. and Frederking, R.M.W. (1985). Stress concentrations in the root of an ice cover cantilever: model tests and theory. Cold Regions Sci. Technol., 11: 63–73.

Svec, O.J. and McNeice, G.M. (1972). Finite element analysis of finite sized plates bonded to an elastic half space. Comp. Meth. Appl. Mech. Eng., 1(3): 265–277.

Tatinclaux, J.-C. and Hirayama, K.-I. (1982). Determination of the flexural strength and elastic modulus of ice from in situ cantilever beam tests. Cold Regions Sci. Technol., 6(1): 37–47.

Timco, G.W. (1985). Flexural strength and fracture toughness of urea model ice. Proc. 4th Offshore Mechanics and Arctic Engineering Symposium, Dallas, Texas, Vol. II, pp. 199–208.

Vaudrey, K.D. (1978). Determination of mechanical sea-ice properties by large scale field beam experiments. Proc. 4th Int. Conf. on Port and Ocean Engineering under Arctic Conditions, Sept. 1977, St. John's, Newfoundland, pp. 529–543.