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Modelling the Dynamics of Heat Addition from Urban Stormwater Runoff into a Receiving Water Body

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ABSTRACT
A set of equations is derived to model the dynamics of the thermal advection/diffusion processes that are caused by the discharge of stormwater effluent into a mixing zone of a water body. The mixing zone is divided into two regions, ambient and heated, which are separated by a moving boundary that is an isotherm defined by a threshold temperature limit for the water body. The motion of the isotherm is governed by the net heat flux at the boundary surface. This is similar to a Stefan condition in heat conduction problems where the fusion/melting isotherm is tracked to delineate the liquid and solid regions of a material that is undergoing a phase transition due to heating or cooling. For the application described in this paper, the Stefan condition is extended to include the heat fluxes at the boundary due to advection. This model will be used to predict the size, temperature, and duration of the region of a water body that exceeds a temperature limit due to the discharge of urban stormwater runoff.

KEYWORDS
Heated surface discharge, Stefan condition

INTRODUCTION
The focus of this paper is to present a mathematical model that describes the dynamics of a heat plume that could result from the discharge of urban stormwater runoff into a receiving water body, such as a creek or river. The need for such a model arises from observations (James, 1999; Kieser et al., 2004; Li and James, 2004; Van Buren, 1999; Verspagen, 1995) that during a rainfall event over an urban watershed the stormwater runoff can transport enough heat from the impervious surfaces, such as roads and parking lots, to warm the water of a receiving body above temperatures that can harm biota living within it. Several models exist for heated surface discharges into water bodies (Harleman and Stolzenbach, 1972; Jirka, 2007; Jones, 1990; McGuirk and Rodi, 1976; Stefan at al., 1971; Stolzenbach and Harleman, 1971; Stolzenbach and Harleman, 1973) but the focus of their development is for discharges from sources with constant flow rates and temperatures, such as from thermal power plants, and steady-state formulations provide effective solutions for these plume spreads. However, the duration and thermal intensity of heat plumes that result from stormwater discharges depend on the time-varying flow rates and temperatures of the discharge, which depend on the time-varying rainfall rates and antecedent temperatures of the impervious surfaces (other factors such as meteorological conditions can also influence the plume spread). Steady-state models can be used to obtain bounding solutions for these types of problems and if such solutions showed no adverse temperature increases then further analyses would not be needed. However, if the bounding solutions showed that the biota could be adversely impacted they would not provide insight into the dynamic relationships between the physical processes occurring and the size and duration of the plume.

In the following sections of this paper, a set of equations is derived to model the dynamics of the thermal advection/diffusion processes that are caused by the discharge of stormwater effluent into a mixing zone of a water body. It is assumed that a fixed-volume mixing zone is allowed around the discharge point, where the water temperature can exceed the ambient temperature. The proposed model delineates the mixing zone into two regions, ambient and heated, based on water temperature (see Figure 1). Development of the model is guided by the assumption that the primary concern is with heat plumes that exceed a defined threshold temperature T_L, which is greater than the ambient water temperature T_A. Temperatures above T_L could harm the aquatic biota but for temperatures below T_L no harmful effects should occur. The ambient region is the part of the mixing zone where the water temperature is less than the threshold temperature (and above the ambient water temperature). For the model formulation, the water temperature of the ambient region is assumed to be constant and equal to T_A. The heated region is where the water temperature exceeds the threshold temperature T_L and within this region the water temperature can vary with time. The size, duration, and temperature of the heated region are the focus of the model development.

The T_L-isotherm in the mixing zone forms the boundary between the ambient and heated regions (see Figure 1). The motion of the isotherm is governed by the amount of energy that is needed to raise the water temperature from T_A to T_L and this energy is determined by the net heat flux at the boundary surface. This is similar to a Stefan condition (Crank, 1984) in heat conduction problems where the fusion/melting isotherm delineates the liquid and solid regions of a material that is undergoing a phase transition due to heating or cooling. However for the problem described herein, instead of a phase transition occurring at the boundary between the ambient and heated regions there is an instantaneous temperature jump from T_A to T_L (see Figure 2). The Stefan condition is also extended to include the heat fluxes due to advection at this boundary.

The model that is presented in this paper can be used to study the effects of time-varying discharge rates and temperatures, caused by different weather events (or series of events) and urban surface temperatures, on the size and temperature of the heated region in a water body. Results from such studies provide a means of focusing efforts on the key weather scenarios and urban surface temperatures that could adversely affect the aquatic ecosystem of a water body and that could require more detailed modelling and possible field studies to identify options for mitigating these effects.

MODEL PRELIMINARIES

Consider a region, as shown in Figure 1, which depicts a mixing zone for a stormwater discharge point that is located on the bottom shoreline. The average width of the mixing zone is B (in the transverse direction) and its length is Z (in the longitudinal direction). Let h be the depth of the zone throughout which the water is at thermal equilibrium, with no heat flux in the vertical direction. The value of h may represent the entire depth of the mixing zone or some fraction of it. For the time period under investigation, which will be hereafter referred to as the discharge transient, all dimensions of the mixing zone, and thus its volume, remain fixed. Prior to the start of the discharge transient, the ambient region, with water temperature T_A, occupies the entire mixing zone and the volumetric flow upstream of the discharge point is Q_U.

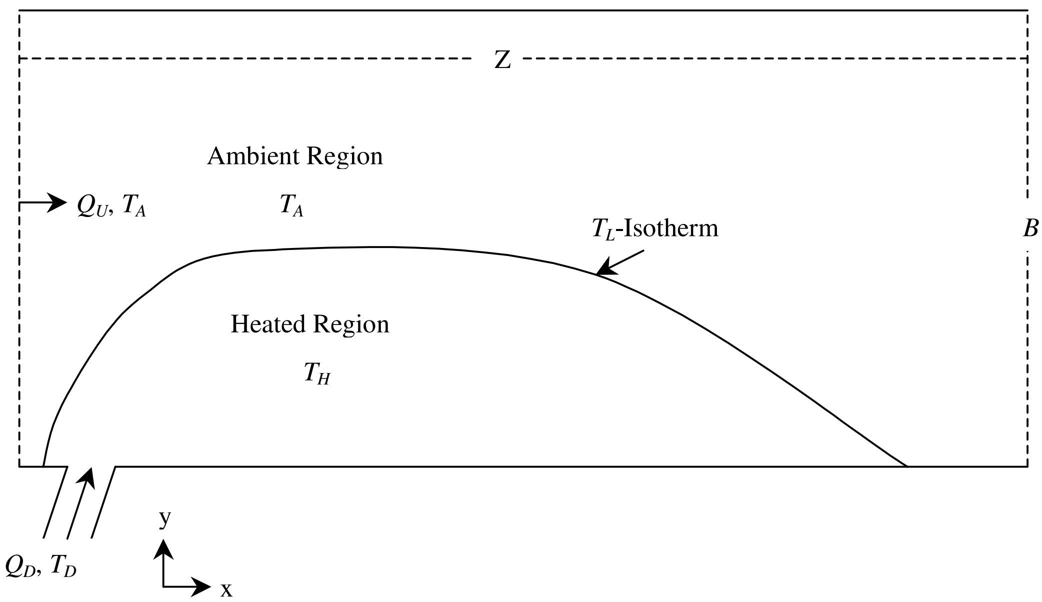

Figure 1. The mixing zone for the discharge of stormwater effluent into a receiving water body showing the division between the ambient and heated regions by the T_L-isotherm.

The transient begins when stormwater effluent starts to enter the mixing zone. The stormwater discharges into the mixing zone at a volumetric flow rate of Q_D and at a temperature T_D. Values for both parameters are expected to vary during the transient. The cross sectional area of the discharge and its orientation relative to the mixing zone are known and fixed. The stormwater effluent mixes with the water body, transferring heat to it via turbulent diffusion and advection, and creates a heat plume that grows in the transverse and longitudinal directions downstream of the discharge point. The transient ceases when there is insufficient thermal power in the effluent (i.e., mass flow multiplied by stored thermal energy) to maintain the temperature in the plume above the threshold limit.

The mixing zone, with dimensions B, Z, and h, will define the fixed control volume V_{MZ} for this system. Two constituents can occupy V_{MZ}: ambient water, which has a fixed temperature T_A, and heated water that can have a variable temperature $T_H \geq T_L$. The ambient and heated water occupy the sub-volumes V_A and V_H, respectively. These two sub-volumes can change size with time but $V_{MZ} = V_A + V_H$ must always hold. Mass and energy can be exchanged between the two sub-volumes and between V_{MZ} and the environment (these processes will be subsequently discussed). The stormwater runoff discharges into the heated region. A single, continuous T_L-isotherm forms the interface between the two constituents. Motion of the T_L-isotherm is governed by the heat flux at the interface and is described in more detail in a later section.

The discharge transient is driven by the thermal power in the stormwater effluent, which is split between raising the temperature T_H in the heated region and expanding its volume V_H through the conversion of ambient to heated water. The time rate of change of T_H is modelled by combining the mass and energy balances for the heated zone and the time rate of change of V_H is determined by modelling the movement of the T_L-isotherm. With prescribed initial and boundary conditions, the simultaneous solution of these two equations along with a momentum balance equation, to describe the velocity field in the mixing zone, can be used to simulate the discharge transient. Derivation of the local mass balance and the time rate of change of T_H will be presented. Derivation of the momentum equation is left for future research. First, attention is turned to deriving the differential equation for tracking the T_L-isotherm, which is the interface between the ambient and heated regions.

MOTION OF THE T_L-ISOTHERM

The motion of the interface between the ambient and heated regions will be treated using a Stefan condition (Crank, 1984) where the heat flux at the interface governs the conversion between ambient to heated water. Stefan conditions are typically applied to track the melting or fusion front in heat conduction problems. In such cases, the temperature profile is continuous across the interface between the material phases (as the temperature is held constant while latent energy is consumed or produced for the phase conversion). However, for this application a temperature discontinuity is assumed at the interface, as shown in Figure 2.

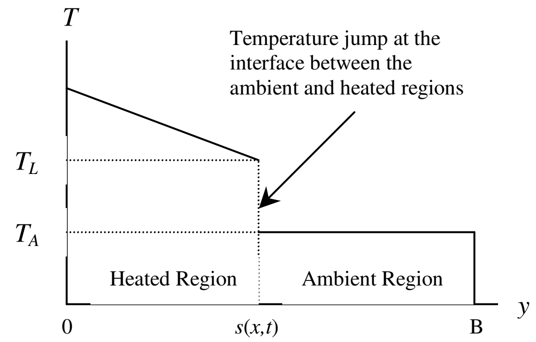

Figure 2. Temperature profile between the ambient and heated regions.

Patel (1968) produced a general expression for the motion of the solidification (or melting) front for heat conduction problems. This expression will be used to describe the motion of the interface that separates the ambient and heated regions of the mixing zone. Let T_i be the temperature along the interface of these two regions and be equal to the temperature limit T_L, i.e., $T_i(x,s(x,t),t) = T_L$. Let T_H and T_A be the temperatures in the heated and ambient regions, respectively, n be the outward normal vector at the interface (pointing into the ambient region), v_n be the velocity of the interface in the normal direction, ρ be the average water density in the two regions, ΔH be the energy required to increase the temperature of water from T_A to T_L, and q_n be the net heat flux to the interface from other sources in the normal direction. With these definitions, the energy balance at the interface can be expressed as:

$$T_i\left(x, s(x,t), t\right) = T_L$$

$$k_H \frac{\partial T_H}{\partial n} - k_A \frac{\partial T_A}{\partial n} = \rho\left(\Delta H\right)v_n - q_n$$

As mentioned above, the expression that Patel formulated was for tracking the motion of a solidification (or melting) front. Thus in the above equation ΔH was defined as the latent heat produced or consumed to change material phases but not temperature, however, for the application in this paper ΔH is the amount of energy consumed to change a unit mass of water from the ambient "phase" to the heated "phase" and this produces a temperature change from T_A to T_L in that mass of water. As there is no spatial temperature gradient in the ambient region, i.e. the water in entire region is at temperature T_A, the second term of the above equation vanishes. The equation can then be written as

$$k_H \frac{\partial T_H}{\partial n} = \rho c_p (T_L - T_A) v_n - q_n$$

where c_p is the specific heat of water. For the range of temperatures that would be considered for this type of problem (e.g. 4°C to 40°C), the difference in c_p is less than one percent (Haar, 1984; NBS/NRC, 2002) and is, therefore, assumed to be constant. This equation can be re-written to express the velocity of the interface as a balance of the heat fluxes:

$$v_n = \frac{1}{\rho c_p (T_L - T_A)} \left(k_H \frac{\partial T_H}{\partial n} + q_n \right)$$

For this two-dimensional problem, the transverse coordinate can be expressed as a function of the longitudinal coordinate, i.e., $y=s(x,t)$, and the above equation can be reduced to one-dimension using relationships derived by Patel:

$$\frac{\partial s}{\partial t} = \frac{1}{\rho c_p (T_L - T_A)} \left(\left[\left(\frac{\partial s}{\partial x} \right)^2 + 1 \right] k_H \frac{\partial T_H}{\partial y} - q_x \frac{\partial s}{\partial x} + q_y \right)$$

where q_x and q_y are x- and y-components of the advective heat flux, respectively. For heat conduction applications, k_H is the thermal diffusivity of the material, however, for the application in this paper the thermal diffusivity will be expressed as a function of the turbulent diffusion coefficient D_y. These relationships are as follows:

$$k_H = D_y \rho\, c_p$$
$$q_x = u_{x,s} \rho\, c_p (T_L - T_A) \quad \text{where } u_{x,s} = u_x(x, s(x,t), t)$$
$$q_y = u_{y,s} \rho\, c_p (T_L - T_A) \quad \text{where } u_{y,s} = u_y(x, s(x,t), t)$$

Note that in the above expressions the parameters $u_{x,s}$ and $u_{y,s}$ are respectively the x- and y-components of velocity at the interface. Substitution of these expressions into the above equation and simplification of the terms yields:

$$\frac{\partial s}{\partial t} = \frac{1}{(T_L - T_A)} \left[\left(\frac{\partial s}{\partial x} \right)^2 + 1 \right] D_y \frac{\partial T_H}{\partial y} - u_{x,s} \frac{\partial s}{\partial x} + u_{y,s} \qquad (1)$$

This expression relates the movement of the interface between the ambient and heated regions at every point along the T_L-isotherm to the heat fluxes caused by turbulent diffusion and advection. The author believes this to be the first application of Patel's interface conditions to an advection-diffusion problem. This equation coupled with the local mass balance and the time rate of change for the temperature in the heated region (which is discussed in the next section), and the appropriate initial and boundary conditions can be used to simulate the transient behaviour of a heat plume in a receiving water body that is caused by discharging stormwater effluent.

TEMPERATURE OF THE HEATED REGION

In the previous section, an equation was derived to model the movement of the T_L-isotherm within the mixing zone, which translates into modelling the volume expansion/contraction of the heated region. Solution of this equation requires the transverse temperature gradient within the heated region at each location adjacent to the T_L-isotherm. The approach taken here is to derive a one-dimensional, differential equation for the time rate of change of T_H at each point along the longitudinal axis (x-axis) of the heated region and to define $T_H(x,t)$ to be the transverse-averaged temperature at each point x. The transverse temperature gradient is then approximated with a polynomial function (which will be discussed later in this section).

The time rate of change for T_H is modelled by combining the mass and energy balance equations for the heated region. Figure 3 shows the mass and energy processes that act upon a differential element of the heated zone. The differential element, Δx, is selected to be small enough that the curve $s(x,t)$ is approximated by a straight line in the interval $(x,x+\Delta x)$. In Figure 3, the differential element is shown attached to the south bank of the water body, which is representative of a wall-attached heated region and is a common effluent configuration (Demetracopoulos and Stefan, 1983). The derivations of the balance equations that are presented herein will assume this type of geometry for the heated region. Derivations for heated regions that are partially detached will include elements that are bounded above and below by $s(x,t)$ and will require equations of motion for the interface at both boundaries.

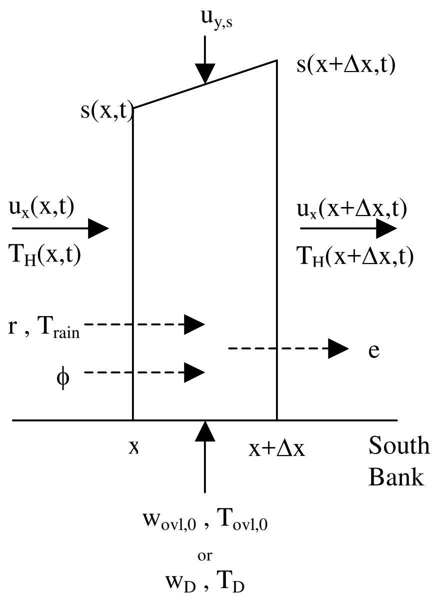

Figure 3. The mass and energy processes that act upon a fluid element in the heated region.

Mass and Energy Processes

Advection. Without loss of generality, flow entering the fluid element will be positive (for both the x- and y-components) and negative for flow exiting the element. The flow velocity through the cross sectional area $s(x,t)\times h$ will be the average velocity over this cross section and be denoted as $u_x(x,t)$. As mentioned above, the water temperature is averaged over the length $s(x,t)$ and is denoted by $T_H(x,t)$. Likewise the flow velocity and temperature exiting the element through the cross sectional area $s(x+\Delta x,t)\times h$ is $-u_x(x+\Delta x,t)$ and $T_H(x+\Delta x,t)$, respectively. The y-component velocity through the cross sectional area $\Delta x\times h$ at the interface is $u_y(x,t)$ and has a temperature of T_L. For the mass and energy balances, the discharge flow will be treated as a lateral mass flux (defined below).

Lateral Flows. Flows from the banks of the water body into the heated region will be treated as lateral flows and be defined per unit length along the longitudinal axis (with dimensions m^2/s). Lateral flows will be separated into two categories: discharge (w_D) and overland flows (w_{ovl}). The discharge flow is that resulting from stormwater effluent entering the heated region and overland flow accounts for direct runoff from the bank into the heated region.

Rainfall. As the purpose of this model is to examine the effects of heat addition from stormwater effluent on a water body, it is expected that some portion of the transient will include rainfall. Although rainfall may have a minimal effect on the mass balance in the heated region, it could influence its temperature and, therefore, it should be considered in the model. The rainfall parameter, r, is in depth per unit time (m/s).

Evaporation and Convection. Evaporative and convective losses will be treated as a single process (i.e., with application of the Bowen Ratio) and will collectively be termed as evaporation. During the rainfall phase of the transient, evaporation may be minimal but for the post-rainfall phase where stormwater continues to discharge into the water body evaporation can occur. The evaporation parameter, e, is in depth per unit time (m/s).

Absorbed Solar Radiation. As with evaporation, during the rainfall phase of the transient the effects of this process on the time rate of change of T_H may be minimal but for the post-rainfall phase of the transient it may be important. The solar radiation parameter, ϕ, is expressed as a net heat flux with units of W/m^2.

Mass Balance Equation
The time rate of change of mass per unit of longitudinal length (kg/(m·s)) in the heated region can be written as:

$$\frac{dM_H(x,t)}{dt} = -\rho hs\frac{\partial u_x}{\partial x} + \rho s(r-e) + \rho w_{dis,0} + \rho w_{ovl,0} - \rho hu_{y,s} + \rho h\frac{\partial s(x,t)}{\partial t} \quad (2)$$

where $M_H(x,t)$ is the mass of water in the volume $h \times dx \times s(x,t)$ in the heated region at time t. The first five terms on the R.H.S. of Equation 2 are the continuity terms for the element and the last term accounts for the expansion/contraction due to the movement of the T_L-isotherm (see previous section). Assuming water to be incompressible over the temperatures and pressures for this type of problem, the sum of the continuity terms is equal to zero. Therefore, the change of mass for the heated region is only caused by expansion/contraction of its volume, i.e., the last term. However, the continuity terms are kept as they help simplify the equation for the time rate of change of T_H (as will be shown later). For the range of temperatures that would be considered for this type of problem (e.g. 4°C to 40°C), the difference in the density of water, ρ, is less than one percent (Haar, 1984; NBS/NRC, 2002) and is, therefore, assumed to be constant.

Energy Balance Equation
The time rate of change in energy per unit of longitudinal length (W/m) in the heated region is:

$$\frac{\partial E_H(x,t)}{\partial t} = -\rho c_p hs \frac{\partial (u_x T_H(x,t))}{\partial x} + \rho c_p s (T_{rain} r - T_H(x,t)\, e) + \rho c_p T_{dis} w_{dis} + \rho c_p T_{ovl,0} w_{ovl,0}$$
$$- \rho c_p h T_L u_{y,s} + \rho c_p h T_L \frac{\partial s(x,t)}{\partial t} + \phi \cdot s(x,t) \qquad (3)$$

where c_p is the specific heat of water and is assumed constant for the range of temperatures considered in this problem type (see discussion in previous section), T_{rain} is the temperature of the rainwater, T_D is the temperature of the stormwater effluent at the discharge point, T_{ovl} is the temperature of the overland flow entering the mixing zone at location x on the south bank, and ϕ is the net heat flux due to solar radiation.

Time Rate of Change for the Temperature of the Heated Region
This change in energy can also be expressed in terms of the volume expansion of the element and the change in water temperature in the element:

$$\frac{\partial E_H(x,t)}{\partial t} = \frac{\partial (c_p M_H(x,t) T_H(x,t))}{\partial t} = c_p T_H(x,t) \frac{\partial M_H(x,t)}{\partial t} + c_p M_H(x,t) \frac{\partial T_H(x,t)}{\partial t}$$

By substituting Equation 2 and the above equation into Equation 3, the following expression is obtained (note that the (x,t) indices are omitted):

$$c_p T_H \left[-\rho c_p hs \frac{\partial u_x}{\partial x} + \rho s (r - e) + \rho w_{dis} + \rho w_{ovl,0} - \rho h u_{y,s} + \rho h \frac{\partial s}{\partial t} \right] + c_p M_H \frac{\partial T_H}{\partial t} =$$
$$- \rho c_p hs \frac{\partial (u_x T_H)}{\partial x} + \rho c_p s (T_{rain} r - T_H e) + \rho c_p T_D w_D + \rho c_p T_{ovl} w_{ovl,0} - \rho c_p h T_L u_{y,s} + \rho c_p h T_L \frac{\partial s}{\partial t} + \phi \cdot s$$

Dividing through by ρc_p and collecting terms, the above equation yields the time rate of change for temperature in the heated region:

$$\frac{\partial T_H}{\partial t} = -u_x \frac{\partial T_H}{\partial x} + \frac{r}{h}(T_{rain} - T_H) + \frac{w_D}{hs}(T_D - T_H) + \frac{w_{ovl,0}}{hs}(T_{ovl,0} - T_H) - \frac{u_{y,s}}{s}(T_L - T_H)$$
$$+ \frac{1}{s}\frac{\partial s}{\partial t}(T_L - T_H) + \frac{\phi}{\rho c_p h} \qquad (4)$$

The above equation compactly describes the processes that affect the time rate of change of a fluid element in the heated region. Simultaneous solution of Equations 1, 2 and 4 yield solutions for u_x, $s(x,t)$, and $T_H(x,t)$ where the parameters r, T_{rain}, w_D, T_D, $w_{ovl,0}$, $T_{ovl,0}$, and ϕ are assumed to be known time-varying boundary conditions. Closure relations are needed for the velocities at the T_L-isotherm ($u_{x,s}$, and $u_{y,s}$) and for the lateral temperature gradients ($\partial T_H / \partial y$). The velocities $u_{x,s}$ and $u_{y,s}$ require either the solution of the momentum balance for the entire mixing zone or some method to approximate them. This is the focus of future research efforts.

Approximation of the lateral temperature gradient
Solution of Equation 1, which describes the motion of the T_L-isotherm, requires the lateral temperature gradient ($\partial T_H / \partial y$) in the heated region at the interface (recall that there is no temperature gradient in the ambient region). One option for obtaining the temperature

gradient is to formulate the problem in two spatial dimensions and directly solve for $T_H(x,y,t)$ and from the solution obtain the necessary gradients. The main drawback to this option is that a lot of computational overhead is created to obtain gradients that are only needed near the interface. Also, the calculated gradients could be sensitive to the mesh spacing. Another option is to approximate the temperature gradient with an assumed profile that can be matched to values already calculated. For example, a linear temperature profile can be simply derived knowing that the temperature at the interface is T_L and the average temperature of the x-element is $T_H(x,t)$. The constant lateral gradient resulting from this profile is:

$$\frac{\partial T_H}{\partial y}(x,t) = \frac{2\left(T_L - T_H(x,t)\right)}{s(x,t)}$$

By adding the assumption that there is no lateral heat transfer at y=0 (i.e., between the water and the south bank of the water body in Figure 1), a quadratic temperature profile can be expressed in the form:

$$T_H(x,y,t) = \frac{1}{2}\left(3T_H(x,t) - T_L\right) + \frac{3}{2\,s(x,t)^2}\left(T_L - T_H(x,t)\right)y^2$$

The lateral gradient, at the interface, that results from this profile is:

$$\left.\frac{\partial T_H}{\partial y}(x,t)\right|_{y=s(x,t)} = \frac{3\left(T_L - T_H(x,t)\right)}{s(x,t)}$$

This gradient is larger than the one from the linear profile and will thus induce a faster volume expansion. Comparisons to experimental data and to two-dimensional numerical solutions will be needed to determine if either approximation is suitable for this application.

SUMMARY

A set of equations was derived to model the dynamics of the thermal advection/diffusion processes that are caused by the discharge of stormwater effluent into a mixing zone of a water body. The main features of this model are:

- Definition of a mixing zone that is divided into two regions, ambient and heated, which are separated by an isotherm that is the threshold temperature limit for the water body. By defining the mixing zone in this manner, one can model the duration, size, and temperature of the region that exceeds a temperature threshold given the time-varying conditions of the stormwater discharge into the water body.
- Formulation of an equation to describe the motion of the threshold temperature isotherm (T_L-isotherm). This equation permits one to model the size of the heated region as a function of time. The motion of the isotherm is governed by the net heat flux at the boundary surface, which is similar to a Stefan condition in heat conduction problems but is extended to include the heat fluxes at the boundary due to advection.
- An equation to describe the time rate of change of the water temperature in the heated region. This equation allows one to model the magnitude that the temperature exceeds the threshold in the heated region, as a function of time.

Future research efforts will focus on the development of the momentum balance equation for the mixing zone so that the velocities at the T_L-isotherm (i.e., $u_{x,s}$ and $u_{y,s}$) can be calculated. Work will also proceed on a numerical solution method for the equations, which will likely draw upon previous work on the Isotherm Migration Method (Crank and Gupta, 1975; Crank, 1984).

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