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Optimal Scheduling of Rehabilitation and Inspection/condition assessment in large buried pipes

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ABSTRACT

A decision framework is described to assist municipal engineers and planners in optimising the scheduling of rehabilitation as well as inspection and condition assessment of large buried pipes. These may include water transmission pipes, trunk sewers or other buried pipes with high costs of failure and high costs of inspection/condition assessment.

A semi-Markov process is used to model the deterioration of a buried asset. The life of the asset is discretised into condition states, whereby the waiting times in each state are assumed to be random variables with known probability distributions. These probability distributions can be derived in two ways. Initially, when data are scarce the probability distributions can be based on expert opinion. Over time, as observed deterioration data are collected these probability distributions are continually updated to reflect the new observations.

Age-dependent transition probability matrices are compiled, using conditional survival probabilities in the various states. The expected discounted total cost associated with an asset is computed as a function of time. The time to schedule the next inspection/condition assessment is when the total expected discounted cost is minimum, while immediate intervention should be planned if the time of minimum cost is less than a threshold period (2 to 3 years) away.

The proposed framework was implemented for proof of concept in a demonstration computer application. Although usable in its current form, this paper identifies some issues that require as yet unavailable data as well as more research in order to develop the framework into a comprehensive application tool.

***Key words*:** Renewal of large buried pipes, scheduling inspection/condition assessment, scheduling intervention, semi-Markovian deterioration.

INTRODUCTION

Large buried pipes typically have low failure rates but the consequences can be severe when they fail. This low rate of failure seems to have contributed to the current situation where most municipalities lack the data necessary to model the deterioration rates of these assets and subsequently make rational decisions regarding their renewal.

There are published guidelines e.g., WRc, 1993 and 1994; Edmonton, 1996; and Zhao and McDonald, 2000, which are very useful for the mapping of distress indicators into condition states of buried pipes. However, the decision process that these guidelines provide are largely qualitative and prescriptive, and as such tend to be rather broad and general. Recommendations are provided depending on the severity of the relevant condition state and on the perceived impact of failure; economics and deterioration rates are considered only in an implicit and qualitative manner.

The literature reflects various efforts to provide quantitative decision methods for infrastructure or other components of the built environment. Examples include: The Factor Method by ISO, to estimate service life of built components (ISO/CD 16696-1, 1997). The method multiplies the reference service life of the component by factors affecting it (factors smaller than 1 reduce the service life and visa versa). The values of these factors can be determined by a Delphi process (Moser, 1999) or individual experience. The Factor Method can also be applied probabilistically (Aarseth and Hovde, 1999). Flourentzou et al. (1999) divided the life of every built element into four condition states, good, fair, poor and need replacement. They used field data to estimate the age distribution of a component in any condition state, and then conditional probabilities to estimate the time to replacement and the expected costs. Abraham and Wirahadikusumah (1999) modelled the deterioration of sanitary sewers as a Markov chain process with four life phases, where the deterioration in each phase is characterised by a stationary transition matrix. These transition matrices are compiled using expert opinion. Ariaratnam et al. (1999) proposed a multinomial logit model to model the likelihood of a sewer being in a deficient state given age category, material type, effluent transported, diameter category and depth category. The sewers were then ranked in an ascending order of likelihood, to provide a priority list for inspection.

In this paper an approach is presented to make the decision process more quantitative and explicit. The deterioration of the asset is modelled as a semi-Markov process, which means that the condition of the asset is discretised into a finite number of states. The durations of the asset in each condition state, also called state waiting times, are modelled as random variables with known probability distributions. The parameters for these distributions can initially be derived based on expert opinions, but over time these parameters will be continuously updated using actual survival data. These probability distributions are used to derive the transition probabilities from one state to the next. The transition probabilities are age-dependent, which means that the older the asset, the higher the likelihood of deterioration to the next state in a given period of time. The total expected cost associated with the asset can then be calculated as a function of time, and a decision made as to whether to rehabilitate or schedule the next inspection/condition assessment.

MODELLING PIPE DETERIORATION AS A SEMI-MARKOV PROCESS

A Markov process is defined as a stochastic process comprising a series of consecutive probabilistic trials, in which the outcome of each trial is independent of all previous trials except the outcome of the immediately preceding trial (Misra, 1992). Asset deterioration can be modelled as a discrete Markov process, whereby the transition from one deterioration state to the next corresponds to a probabilistic trial. The asset thus has a known probability of deterioration from state to state. When the probability of transition from state to state is constant the process is said to be a homogeneous (or stationary) Markov process. The stationary Markov process with n states can be represented by

[1] $$A(t) = A(t-1)P$$

Where $A(t)$ is a vector $\{a^t_1, a^t_2, \dots a^t_n\}$ representing the probability mass function (pmf) of the process in time step t, and P is a transition probability matrix with members p_{ij} $(i,j = 1,2,..n)$ representing the (constant) probability of transition from state i to state j. It is also assumed that if no intervention (renewal or rehabilitation) is implemented the process is unidirectional, i.e., if state 1 denotes good as new and state n denotes failure, then the process can move only from state i to state j where $j \geq i$.

It has been observed by others, e.g., Jiang et al. (1989), Madanat et al. (1995, 1997), Guignier (1999), that infrastructure assets deteriorate in a non-stationary manner. It is thus assumed here that the transition probabilities p_{ij} depend on the age of the asset. This time-dependent Markov process can be represented by

[2] $$A(t) = A(t-1)P^{t-1,t}$$

It is often assumed (e.g., Madanat et al., 1995) that an infrastructure asset can deteriorate only one state at a time, that is, the asset will deteriorate from state 1 to state 2, then to state 3, and so on to failure (providing no renewal was implemented). The process therefore cannot jump from state 1 to state 3, for example, without passing through state 2. This results in a relatively simple transition probability matrix

[3] $$P^{t,t+1} = \begin{bmatrix} p_{11}^{t,t+1} & p_{12}^{t,t+1} & 0 & \dots & 0 \\ 0 & p_{22}^{t,t+1} & p_{23}^{t,t+1} & \dots & 0 \\ \dots\dots & \dots\dots & \dots\dots & \dots\dots & \dots\dots \\ 0 & \dots\dots & \dots\dots & p_{n-1,n-1}^{t,t+1} & p_{n-1,n}^{t,t+1} \\ 0 & 0 & \dots\dots & 0 & p_{nn}^{t,t+1} \end{bmatrix}$$

A semi-Markov process is defined as a Markovian process with an additional property of a sojourn time (or "waiting time") in each state. These sojourn times in the various states are random and independently distributed variables (Lawless, 1992). An asset, which is said to deteriorate in a semi-Markovian manner, will thus sojourn in each deterioration state for a random time period with an independent probability distribution.

Suppose that $T_1, T_2, \ldots, T_{n-1}$ are random variables denoting the sojourn times in states $\{1, 2, \ldots, n-1\}$, respectively. Their corresponding probability density functions (pdfs), cumulative density functions (cdfs) and survival functions (sfs) can be denoted by $f_i(t)$, $F_i(t)$, $S_i(t)$ respectively. Suppose further that $T_{i \to k}$ is a random variable denoting the sum of sojourn times in states $\{i, i+1, \ldots, k-1\}$ This can be expressed as

$$[4] \qquad T_{i\to k} = \sum_{j=i}^{k-1} T_{j,j+1} \quad ; \quad i = \{1, 2, \ldots, n-1\},\ k = \{2, 3, \ldots, n\}$$

$T_{i \to k}$ is the time it will take the process to go from state i to state k. In addition, $f_{i\to k}(T_{i\to k})$, $F_{i\to k}(T_{i\to k})$, $S_{i\to k}(T_{i\to k})$ are the pdf, cdf and sf of $T_{i\to k}$, respectively.

If the deterioration process is in state 1 at time t, the conditional probability that it will transit to the next state in the next time step Δt is given by

$$[5] \qquad \Pr[X(t+1) = 2 | X(t) = 1] = p_{1,2}(t) = \frac{f_1(T_1)\Delta t}{S_1(T_1)}$$

where $X(t)$ denotes the state of the process at time t, and $t = 0$ is the time at which the process entered into state 1 (i.e., new asset – in most cases). The formulation in equation (5) corresponds to discrete time steps that are assumed small enough to exclude a two-state deterioration. In addition, Δt is assumed to be one unit (year) and can thus be omitted.

Further, if the process is in state i at time t, the conditional probability that it will transit to the next state in the next time step Δt is given by

$$[6] \qquad \Pr[X(t+1) = i+1 | X(t) = i] = p_{i,i+1}(t) = \frac{f_{1\to i}(T_{1\to i})}{S_{1\to i}(T_{1\to i}) - S_{1\to(i-1)}(T_{1\to(i-1)})} \quad ; \quad i = \{1, 2, \ldots, n-$$

where $t = 0$ is the time when the process entered into state 1. Note that, the denominator in the rhs expresses the simultaneous condition that $T_{1\to i} < t$ and $T_{1\to(i-1)} < t$, which is equivalent to the condition $X(t) = i$. Equation (6) thus provides all the transition probabilities $p_{i,i+1}(t)$ to populate the time-dependent transition probability matrix for the semi-Markov process.

Once the transition probability matrix is established as a function of time, the deterioration process can be modelled by using the following equation to obtain the probability mass function (pmf) of the process after any number of time steps k.

$$[7] \qquad A(t+k) = A(t)P^{t,t+1}P^{t+1,t+2}\ldots P^{t+k-1,t+k}$$

where $A(t+k)$ denotes the pmf of the process k timesteps after time t

If state n is defined as failure and if at time t the asset has a pmf $A(t+k)=\{a_1, a_2,\dots, a_n\}$, the probability that the asset will fail at time $(t+k)$ is a_n.

WAITING TIMES IN THE SEMI-MARKOV PROCESS

As stated earlier, the waiting (or sojourn) times of the semi-Markov process are random variables with independent probability distribution. There are currently insufficient data to ascertain the probability distribution of waiting time. It is therefore assume that the waiting time T_i of the process in any state i could be modelled as a random variable with a two-parameter Weibull probability distribution.

$$
\begin{aligned}
&F_i(t) = \Pr[T_i \le t] = 1 - e^{-(\lambda_i t)^{\beta_i}} \\
[8] \qquad &S_i(t) = 1 - F_i(t) = e^{-(\lambda_i t)^{\beta_i}} \\
&f_i(t) = \frac{\partial F_i(t)}{\partial t} = \lambda_i \beta_i (\lambda_i t)^{\beta_i - 1} e^{-(\lambda_i t)^{\beta_i}}
\end{aligned}
$$

The procedure is not limited to any one distribution, and it is possible to use different distributions for different states in the same deterioration process.

The pdf, cdf and sf, $f_{i\to k}(T_{i\to k})$, $F_{i\to k}(T_{i\to k})$, $S_{i\to k}(T_{i\to k})$ of the sum of waiting times $T_{i\to k} = \sum_{j=i}^{k-1} T_{j,j+1}$; $i = \{1, 2,..., n-1\}$, $k = \{1, 2,..., n\}$ cannot generally be calculated analytically, therefore Monte-Carlo simulations can be used to numerically calculate these functions for sums of Weibull-distributed random variables $T_{i\to k}$.

The current lack of pertinent data is also a barrier to deriving parameters λ_i and β_i based on historical observations and condition assessments of large buried pipes. Consequently, these parameters will initially have to be derived from expert opinion and perception. The following process is suggested. An expert or a group of experts (e.g., in a Delphi process) would have to answer questions pertaining to their beliefs about the likelihood of an asset remaining in a given state for a certain period of time. For example, the following statement would have to be made: “In my opinion, the asset has a probability $x_{i,u}$ of being in state i for more than u years”. Since there are two parameters λ_i and β_i to be estimated for every state i, two such statements have to be made for every state i, i={1, 2,…, n-1}, with u years and v years, $u \neq v$, to produce two quantiles $x_{i,u}$ *and* $x_{i,v}$. Parameters λ_i and β_i could then be easily derived. Once parameters λ_i and β_i are established for every i = {1,2,…, n-1}, the transition probability matrix could be calculated by substituting equation (8) into equation (7).

EXAMPLE

Suppose the state space of a large buried pipe comprises 5 states, where state 1 is as good as new and state 5 is failure. Suppose further that a group of experts have determined that, for this type of pipe under similar conditions, if it is as good as new at age zero (i.e., it is entering state 1 at age zero) then the probabilities in Table 1 apply, resulting in the parameters β_i and λ_i calculated using [8] .

Table 1. *Example expert opinion tabulated as probabilities of survival.*

State *i*	***u*** (years)	$x_{i,u}$	***v*** (years)	$x_{i,v}$	β_i	λ_i
1	30	50%	40	10%	4.173	0.031
2	20	50%	30	10%	2.961	0.044
3	20	50%	25	10%	5.380	0.047
4	10	50%	15	10%	2.961	0.088

Note: The buried pipe is $x_{i,u}$% likely to remain in the state i for more than u years, and $x_{i,v}$% likely to remain in the state i for more than v years:

Parameters λ_i and β_i describe the probability density function (pdf), for the waiting time T_i in every state i, as is shown in Figure 1.

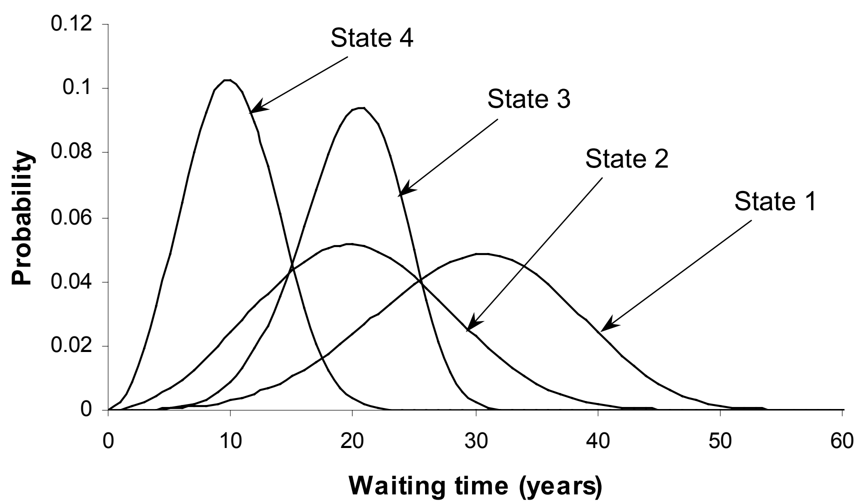

Figure 1. *Example pdfs of waiting times in all condition states.*

Next the sums of waiting times in the various states, $T_{i \to k}$, can be calculated using Monte-Carlo simulations. Their respective pdfs, cdfs and sfs can subsequently be found. Figure 2 illustrates the survival functions (sf) of the sums of waiting times.

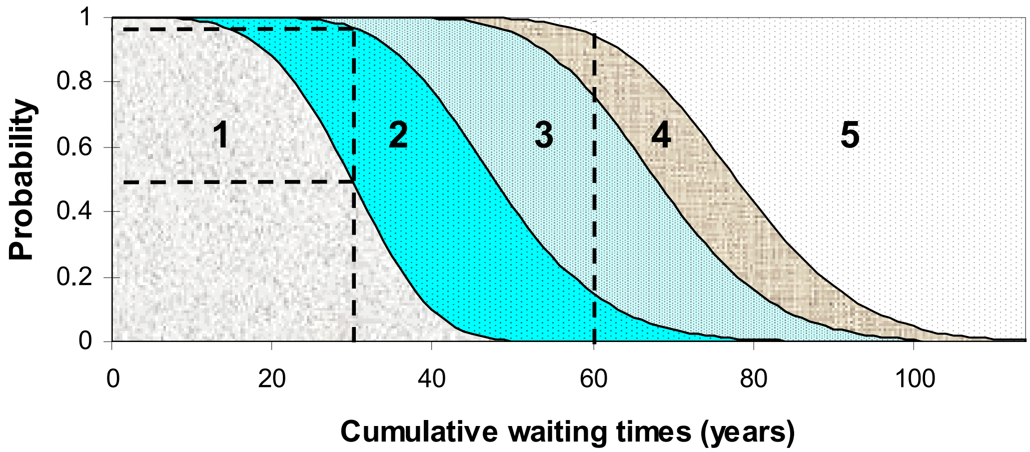

Figure 2. *Example survival functions of cumulative waiting times.*

In this example, (assuming 'as good as new at age zero') the pipe at age 30 years is about 48% likely to still be in state 1, 50% likely to be in state 2 and 2% likely to have deteriorated to state 3. At this age the probability of failure (state 5) is virtually zero. This can be expressed in terms of probability mass function (pmf) as $A(30)=\{0.48, 0.50, 0.02, 0, 0\}$. It can be seen that at age 60 for example, there already is an appreciable (about 4%) likelihood of failure with a corresponding pmf of $A(60)=\{0, 0.17, 0.62, 0.17, 0.04\}$.

Next, the age-dependent transition probabilities $p_{i,i+1}(t)$, can be generated using [6]. Once these transition probabilities are determined, the deterioration process can be modelled of any buried pipe, given only the its age and current pmf (without knowing whether it was as good as new at age zero). In our example for the 30 year old pipe above, by applying [6], the transition probability matrix can be found:

[9] $$P^{30,31} = \begin{bmatrix} 0.904 & 0.096 & 0 & 0 & 0 \\ 0 & 0.988 & 0.012 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.997 & 0.003 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

It can be seen that if an asset is in state 1 at age 30, during the next year it is about 90% likely to remain in state 1 and 10% likely to deteriorate to state 2, etc. Equation [7] can then be used to obtain the pmf of the asset at age 31,

$$A(30+1) = A(30)P^{30,31} =$$

[10]
$$= \{0.48, 0.50, 0.02, 0, 0\}\begin{bmatrix} 0.904 & 0.096 & 0 & 0 & 0 \\ 0 & 0.988 & 0.012 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.997 & 0.003 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \{0.430, 0.543, 0.027, 0, 0\}$$

In general, if the analysis is done at year $\tau = 0$ (the present) and t_0 denotes the pipe age at present, then its pmf at any time τ in the future can be found by

[11]
$$A(\tau) = A(t_0)\prod_{k=0}^{\tau-1} P_{ij}^{t_0+k,t_0+k+1}$$

Figure 3 illustrate how the pmf of the example pipe progresses over time from the present to the future.

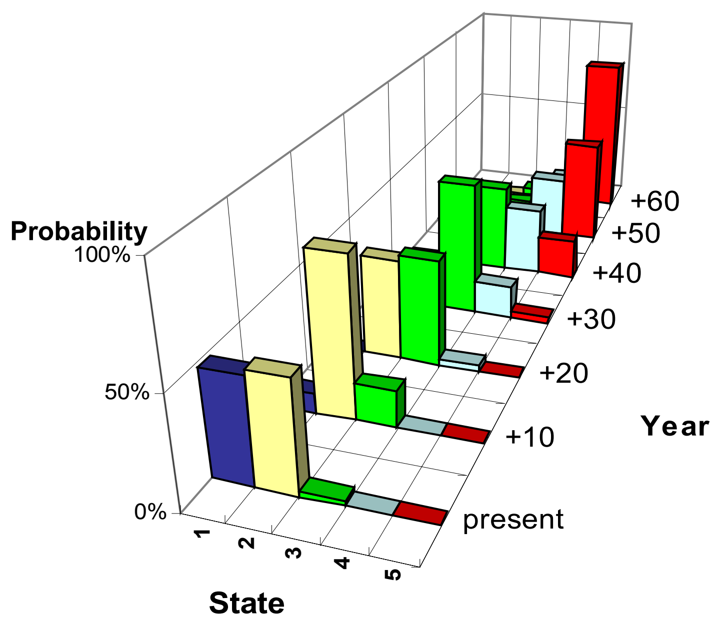

Figure 3. *Example progression of pipe pmf over time.*

CONSIDERATION OF COSTS

Failure cost. Failure is defined as an event where an unplanned emergency intervention is required to restore or to prevent imminent loss of pipe operability. The cost of failure C^F includes emergency repair, direct damages, indirect costs and social costs. While indirect and social costs are hard to quantify, an effort should be made to provide a rational approximation. The expected cost of failure at age t is the product of the cost of failure and the probability of failure at age t, i.e., $E[C^F(t)]=C^F a^t_n$.

Inspection and condition assessment cost. These may vary with the type, size, depth, accessibility and functional state of the pipe. This cost is denote by C^I and is assumed to be time-independent.

Planned intervention (rehabilitation, renovation) cost. It is assumed that the cost of planned intervention may vary with with the state of the pipe. Thus, the expected cost of intervention at age t is $E[C^R(t)] = \{c^r_1, c^r_2, ..., c^r_{n-1}\} \cdot \{a^t_1, a^t_2, ..., a^t_{n-1}\}^T$, where c^r_i is the cost of planned intervention with a pipe in state i.

The total discounted expected cost that is associated with the pipe at time τ is thus

[12] $$C^{tot}(\tau) = \left(E[C^F(t_0+\tau)] + C^I + E[C^R(t_0+\tau)]\right)e^{-r\tau}$$

where r is the (continuous) discount rate and t_0 is the age of the pipe at present ($\tau = 0$).

The expected cost of failure tends to increase with time due to the increase in the probability of failure. On the other hand, the expected cost of intervention as well as inspection and condition assessment tends to decrease over time due to discounting. The total cost thus typically forms a convex curve over time as illustrated in Figure 4.

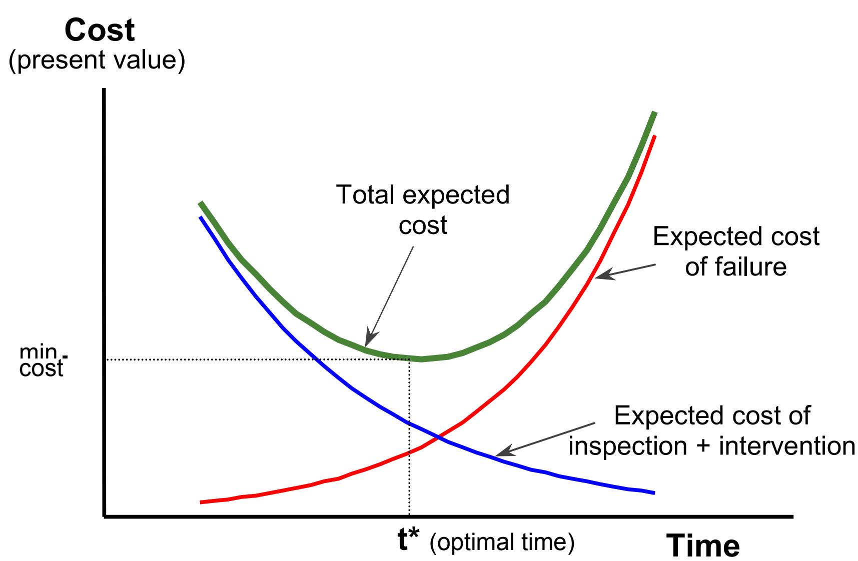

Figure 4. *Expected costs variation over the pipe life-time.*

THE DECISION PROCESS

The decision process comprises the following fundamental assumptions:

- An optimal decision strategy will minimise the total expected costs that are associated with the buried asset throughout its life.
- Upon inspection and condition assessment, the decision alternatives are:
    1. no immediate intervention is required, therefore the next inspection/condition assessment must be scheduled, or,
    2. an immediate intervention is required. (note that "immediate" can mean a threshold period of one to three years - in the realm of large buried pipes, planning, designing bidding and executing rehabilitation projects require this threshold period).
- A decision is always preceded by an inspection/condition assessment. It is unlikely that an intervention will be planned more than two to three years (the threshold period) in advance.

Ideally, intervention should be implemented just before failure, thus benefiting from the deepest possible discount on the cost of intervention, while avoiding high failure costs. In reality the probability of failure can only be evaluated at any given time, therefore the objective is to defer intervention as much as possible without taking too high a risk of failure. Thus the optimal age for intervention is when the marginal benefits of postponing intervention becomes smaller than the marginal increase in the expected cost of failure. This age corresponds to point t^* in Figure 4. If $t^* = t_0 + \tau^*$ denotes the optimal age, and t_0 denotes the pipe age at the time of analysis, then the time τ^* is the optimal time for intervention.

Recall that one of the fundamental assumptions made was that any intervention must be preceded by an inspection/condition assessment, which implies that the asset must be inspected at time τ^* before commencing rehabilitation. After assessing the condition of the asset at time τ^*, its true probability mass function (pmf) can be evaluated and compared to the predicted pmf. If the true pmf is approximately equal to or worse than the predicted pmf, then indeed τ^* is the optimal time for intervention. If, on the other hand, the true pmf is better (less deteriorated) than the predicted pmf, then it may be too early to intervene and the deterioration model should be re-applied with the new pmf in order to find a later optimal time τ^{**}.

It should be noted that if the observed pmf is significantly better or worse than predicted, it may be necessary to update some or all the parameters λ_i and β_i in light of the newly obtained data.

The decision process is illustrated using the example presented in the previous sections, where the sfs of the cumulative waiting times in the various states are shown in Figure 2. The following costs are assumed: Cost of failure, C^F = \$200,000. Cost of inspection and condition assessment, C^I = \$5000. Discount rate, $r = 3\%$.

Cost of intervention at various states, C^R =

State	1	2	3	4
Cost ($)	10,000	10,000	15,000	20,000

A few scenarios are examined:

Scenario #1: Suppose the pipe is as good as new (entering state 1) at age zero. The discounted costs associated with it as a function of age [12] are depicted in Figure 5. The total discounted costs associated with the pipe appear to be minimum at $t^* = 52$ years. That means that if post-installation inspection/condition assessment determine that the pipe is in perfect condition, then the next inspection and condition assessment should be scheduled about 52 years after installation, based on expert opinion (as expressed in the parameter derivation procedure). The pmf of the pipe at age 52 is predicted to be $A(52) = \{0, 0.404, 0.553, 0.040, 0.003\}$.

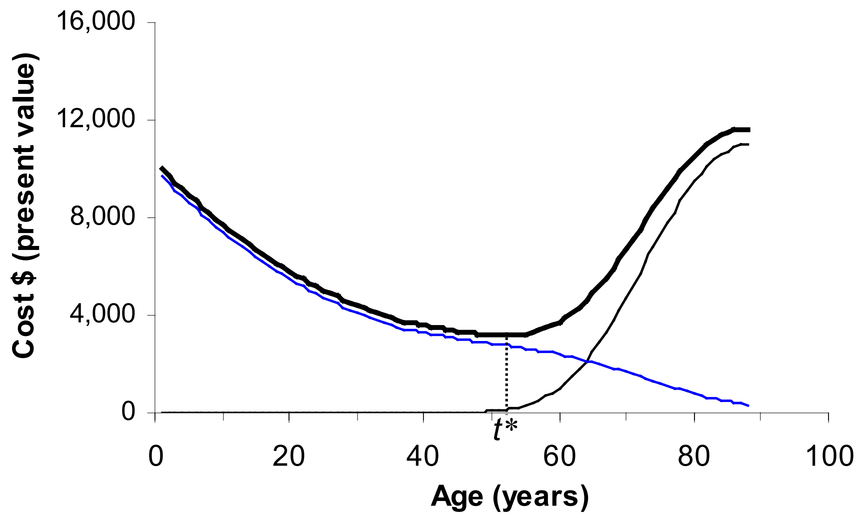

Figure 5. *Example: Total cost curve as a function of age – scenario #1 .*

Scenario #2: Supposes the pipe was not inspected after installation and thus it is not known whether it was in perfect condition at age zero. Suppose further that an inspection and condition assessment were arbitrarily implemented 30 years after installation. According to the original expert opinion one would anticipate at age 30 (Figure 2) a pmf of roughly $A(30) = \{0.5, 0.5, 0, 0, 0\}$. If the observed condition assessment reveals that the pipe is either in state 1 or in state 2, that would indicate that the initial expert opinion was adequate. However, if the pipe is observed to be in state 3 or 4, that would indicate that the initial expert opinion was overly optimistic and it should be modified to reflect shorter waiting times in the earlier states.

Scenario #3: Suppose that following scenario #1 an inspection was carried out at age 50. The anticipated pdf (Figure 2) is roughly $A(50) = \{0, 0.46, 0.51, 0.03, 0\}$. If the condition assessment reveals that the pipe is in state 2 or 3 that would mean that the initial expert opinion was adequate. Further, re-applying equation [12] to find the optimal time for the next inspection τ^{**}, yields the cost curve depicted in Figure 6, which implies that intervention should be planned immediately.

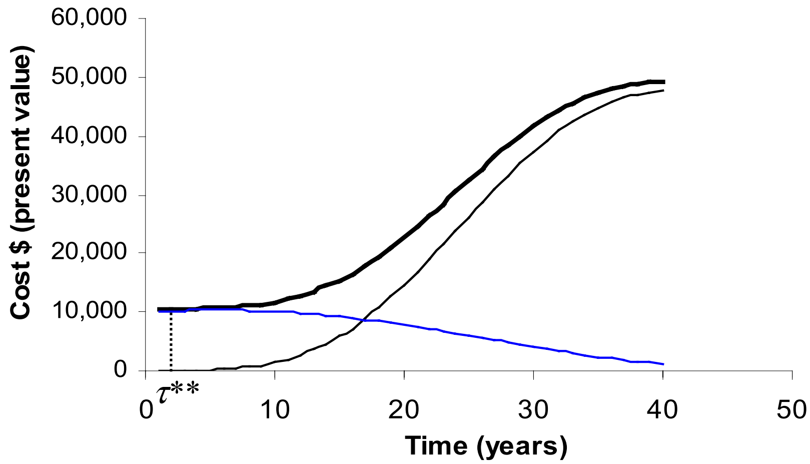

Figure 6. *Example: Total cost curve as a function of age – scenario #3 .*

If, however, condition assessment reveals that the pipe is in state 1 at age 50, that would imply that the original expert opinion was overly pessimistic and the waiting times (at least in the first state) should be modified to reflect this new observation.

Summary and conclusions

Figure 7 provides a flow diagram that summarises the decision framework. The key elements are:

- The deterioration of large buried assets is modelled as a semi-Markov process, in which the waiting times in each state are assumed to be random variables with known probability distributions. These probability distributions are initially based on expert opinion, and then continually updated as observed deterioration data are collected over time. The updating can be done using a statistical process such as Bayesian updating.
- The distributions of the cumulative waiting times in states 1+2, 1+2+3, and 1+2+3+4 are calculated using Monte-Carlo simulations and subsequently age-dependent transition probability matrices are compiled, using conditional survival probabilities.
- The expected discounted total cost associated with an asset (including cost of intervention, inspection and failure) is computed as a function of time, and the time to schedule the next inspection/condition assessment is when the total expected discounted cost is minimum.
- Immediate intervention should be planned if the time of minimum cost is less than a threshold period (2 to 3 years) away.

This framework was implemented in a computer program for proof of concept and demonstration. The framework is suitable for a computer application, however more research is required in the following areas, to develop a practical and comprehensive tool:

- As more deterioration data are collected over time, statistical procedures have to be developed for updating ‘waiting time’ parameters. The procedures will be used to shift gradually from relying on expert opinion to using deterioration data. Since assets may deteriorate at different rates under various conditions, assets and data will have to be partitioned into groups comprising relatively homogeneous characteristics.
- The asset is assumed to begin a new deterioration mode after it has undergone intervention (rehabilitation/renewal). This new deterioration mode may have a unique starting pmf as well as state waiting times. Different intervention alternatives can have different level of effectiveness at different costs. For example, alternative A can cost $10,000 to bring the deteriorated asset back to state 1, while alternative B costs $5,000 to bring the asset to state 2, or 3. Furthermore, these transitions to lower states are not deterministic but rather stochastic, with their own transition probabilities.

  Data are required to determine transition probabilities from a deteriorated state to a renewed state, given various rehabilitation techniques (e.g., if an asset is in state 4, what is the probability that it would be in state 1 or 2 after it was lined with cement mortar). Once these transition probabilities and state waiting times are determined, the decision framework could include the selection of the most efficient rehabilitation/renewal alternative for a given buried asset in a given state.
- Economies of scale in buried assets rehabilitation costs can be an important factor. Their consideration in a decision optimisation procedure, however, is very challenging from a mathematical viewpoint. This issue warrants further research.

ACKNOWLEDGEMENT

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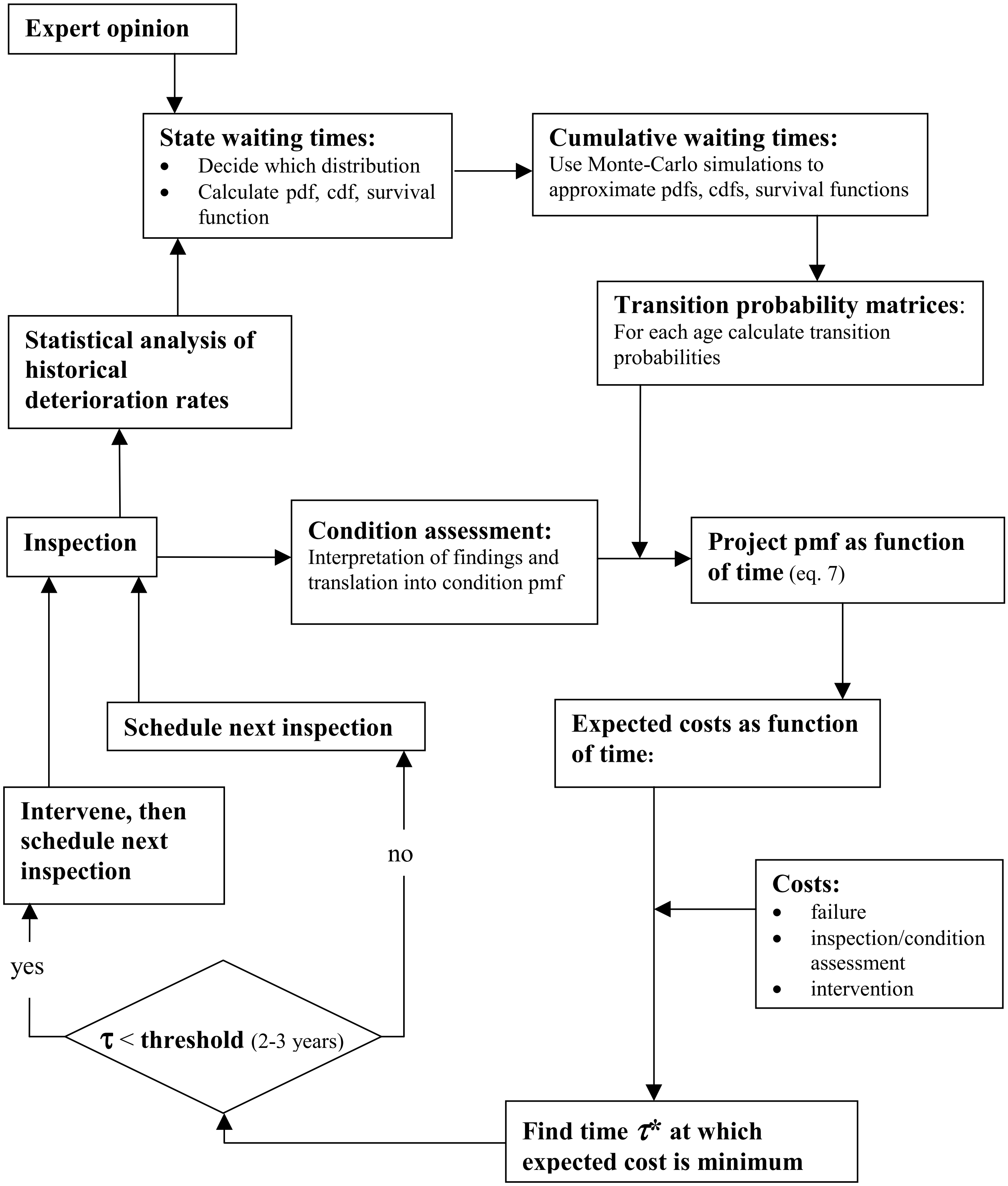

Figure 7. *Decision process flow diagram*

Notation

$p_{ij}^{t,t+1}$	=	single (time) step transition probability from state i to state j.
$P^{t,t+1}$	=	transition probability matrix with members $p_{ij}^{t,t+1}$ (for a stationary process indices t, t+1 can be omitted).
$A(t)$	=	vector with members a_i^t , denoting the probability mass function (pmf) of the Markov process at time t.
T_{ij}	=	a random variable denoting, the sojourn time in state i given that the process goes next to state j, in a semi-Markov process. i+1,
T_i	=	T_{ij} in the deterioration model (under the assumption that the process always moves from state i to state i+1, index j can be omitted) denoting waiting time in state i.
$f_i(t)$	=	probability density function (pdf) of T_i .
$F_i(t)$	=	cumulative density function (cdf) of T_i .
$S_i(t)$	=	survival function (sf) of T_i .
$T_{i\to k}$	=	a random variable denoting the sum of sojourn times in states $i, i+1,\ldots, k-1$
$f_{i\to k}(T_{i\to k})$	=	pdf of $T_{i\to k}$
$F_{i\to k}(T_{i\to k})$	=	cdf of $T_{i\to k}$
$S_{i\to k}(T_{i\to k})$	=	sf of $T_{i\to k}$
$X(t)$	=	random variable representing the state of a Markov process at timestep t.
λ_i , β_i	=	parameters for the Weibull distribution of the waiting time in state i, T_I
$x_{i,u}$, $x_{i,v}$.	=	quantiles that reflect the expert's belief that, for example, there is $x_{i,u}$ % chance that an asset will stay in state i more that u years.
τ	=	variable denoting the time elapsed from the present and on.
t^*, τ^*, τ^{**}	=	optimal age for action, optimal time for action, next optimal time for action
C^F	=	cost of failure.
C^I	=	cost of inspection and condition assessment.
C^R	=	cost of intervention (rehabilitation, renewal, repair).
c_i^r	=	cost of planned intervention with a buried pipe in state i.
r	=	discount rate.

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