

NRC Publications Archive Archives des publications du CNRC

Improving document clustering in a learned concept space

Pessiot, Jean-François; Kim, Young-Min; Amini, Massih-R.; Gallinari, Patrick

This publication could be one of several versions: author's original, accepted manuscript or the publisher's version. / La version de cette publication peut être l'une des suivantes : la version prépublication de l'auteur, la version acceptée du manuscrit ou la version de l'éditeur.

For the publisher's version, please access the DOI link below./ Pour consulter la version de l'éditeur, utilisez le lien DOI ci-dessous.

Publisher's version / Version de l'éditeur:

https://doi.org/10.1016/j.ipm.2009.09.007

Information processing & management, 46, 2, pp. 180-192, 2010-03-01

NRC Publications Record / Notice d'Archives des publications de CNRC:

https://nrc-publications.canada.ca/eng/view/object/?id=11e1935d-5d95-4bb7-82fd-b4dcc0ab00fe https://publications-cnrc.canada.ca/fra/voir/objet/?id=11e1935d-5d95-4bb7-82fd-b4dcc0ab00fe

Access and use of this website and the material on it are subject to the Terms and Conditions set forth at https://nrc-publications.canada.ca/eng/copyright

READ THESE TERMS AND CONDITIONS CAREFULLY BEFORE USING THIS WEBSITE.

L'accès à ce site Web et l'utilisation de son contenu sont assujettis aux conditions présentées dans le site https://publications-cnrc.canada.ca/fra/droits

LISEZ CES CONDITIONS ATTENTIVEMENT AVANT D'UTILISER CE SITE WEB.

Questions? Contact the NRC Publications Archive team at

PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca. If you wish to email the authors directly, please see the first page of the publication for their contact information.

Vous avez des questions? Nous pouvons vous aider. Pour communiquer directement avec un auteur, consultez la première page de la revue dans laquelle son article a été publié afin de trouver ses coordonnées. Si vous n'arrivez pas à les repérer, communiquez avec nous à PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca.





FISEVIER

Contents lists available at ScienceDirect

Information Processing and Management

journal homepage: www.elsevier.com/locate/infoproman



Improving document clustering in a learned concept space

Jean-François Pessiot a,*, Young-Min Kima, Massih R. Amini a,b, Patrick Gallinari a

ARTICLE INFO

Article history: Received 11 September 2008 Received in revised form 9 September 2009 Accepted 16 September 2009 Available online 21 October 2009

Keywords: Document clustering Aspect models Concept learning

ABSTRACT

Most document clustering algorithms operate in a high dimensional bag-of-words space. The inherent presence of noise in such representation obviously degrades the performance of most of these approaches. In this paper we investigate an unsupervised dimensionality reduction technique for document clustering. This technique is based upon the assumption that terms co-occurring in the same context with the same frequencies are semantically related. On the basis of this assumption we first find term clusters using a classification version of the EM algorithm. Documents are then represented in the space of these term clusters and a multinomial mixture model (MM) is used to build document clusters. We empirically show on four document collections, Reuters-21578, Reuters RCV2-French, 20Newsgroups and WebKB, that this new text representation noticeably increases the performance of the MM model. By relating the proposed approach to the Probabilistic Latent Semantic Analysis (PLSA) model we further propose an extension of the latter in which an extra latent variable allows the model to co-cluster documents and terms simultaneously. We show on these four datasets that the proposed extended version of the PLSA model produces statistically significant improvements with respect to two clustering measures over all variants of the original PLSA and the MM models.

© 2009 Elsevier Ltd. All rights reserved.

1. Introduction

With the ever-increasing volume of on-line textual information, an efficient partitioning of documents into clusters can constitute a real saving in terms of efficiency for various information retrieval or enterprise portal applications. Document clusters can for example, help users to quickly evaluate classical search engines results, or navigate through huge document collections (Cutting, Pedersen, Karger, & Tukey, 1992). They can also be useful in distributed search (Xu & Croft, 1999) or in extractive text summarization where topically related documents facilitate the search of relevant text-spans to be extracted for the summary (Amini & Gallinari, 2002; Kummamuru, Lotlikar, Roy, Signal, & Krishnapuram, 2004).

Most existing text clustering approaches rely on the bag-of-words representation (Cutting et al., 1992). Using words as features, each document is represented in a high dimensional vocabulary space as a vector of (normalized) word frequency counts (Rijsbergen, 1979). The *sparsity* (most documents contain less than 5% of the vocabulary terms), (Dhillon & Modha, 2001) and the *noise* (text data extracted from internet pages, chat logs or e-mails may often contain spelling errors and abbreviations), (Knoblock, Lopresti, Roy, & Subramaniam, 2006) in this representation indeed affect the final clustering performance.

^a University Pierre et Marie Curie, 104, Av. du President Kennedy, 75016 Paris, France

^b National Research Council Canada, 283, Bd. Alexandre-Taché, QC, Canada J8X 3X7

^{*} Corresponding author.

E-mail address: jean-francois.pessiot@lip6.fr (J.-F. Pessiot).

These difficulties have motivated the development of dimensionality reduction techniques as a pre-processing step to determine a more compact and relevant document representation. Examples of such approaches are the singular value decomposition used in the Latent Semantic Indexing (LSI) (Deerwester, Dumais, Landauer, Furnas, & Harshman, 1990) or other matrix factorization approaches like random projections (Bingham & Mannila, 2001) or non-negative matrix factorization (Lee & Seung, 1999; Xu, Liu, & Gong, 2003). Matrix factorization approaches have successfully been applied to the clustering of text data, including web access log pages (Oyanagi, Kubota, & Nakase, 2001). Other approaches for dimensionality reduction of text data include probabilistic models and co-clustering approaches. The former include the popular Probabilistic Latent Semantic Analysis (Hofmann, 1999) and Latent Dirichlet Allocation (Blei, Ng, & Jordan, 2003). Those two models have successfully been used for the task of topic discovery (Hofmann, 1999; Griffiths & Steyvers, 2004). Co-clustering approaches aim at simultaneously clustering documents and words. Recent advances include formulations in the bipartite graph framework (Dhillon, 2001) and in the matrix factorization framework (Banerjee, Dhillon, Ghosh, Merugu, & Modha, 2007). Non-informative words can also be removed by simple heuristics based for example, on their document frequencies (Salton & McGill, 1986). These unsupervised approaches though less efficient than supervised feature selection methods allow to find less noisy representation space than the initial bag-of-words space.

In this paper we propose two methods for unsupervised dimensionality reduction in the context of document clustering. Both approaches rely on the idea of replacing the usual BOW document representation by a condensed semantic representation in a concept space. Concepts (also referred as word topics in the following) are identified by a probabilistic model. The first technique makes the hypothesis that words occurring with the same frequencies in the same documents are semantically related. Based on this assumption words are first partitioned into word topics. Each document collection is then represented in the bag-of-concepts space by a vector where each feature corresponds to a word-topic representing the number of occurrences of words from that word-topic in the document. Documents are then clustered in this concept space. We empirically show on Reuters-21578, Reuters RCV2-French, 20Newsgroups and WebkB document collections that the clustering performance of a multinomial mixture (MM) model in the concept space is significantly better than its clustering performance on the original vocabulary space. The second approach is an extension of the PLSA model (denoted by Ext-PLSA in the following) and it jointly performs topic identification and document clustering. In this case, we use two latent variables which respectively identify the word topics and the document clusters. This allows using PLSA for both multiple topic identification and clustering in a principled framework. Empirical results indicate that this extended version of the aspect model produces statistically significant improvements with respect to two clustering measures compared to the original PLSA and the MM models operating in the original and the induced concept spaces.

The remainder of this paper is organized as follows: Section 2 presents different probabilistic models for dimensionality reduction and document clustering we propose and use in our experiments. We present experimental results in Section 3 and summarize our contribution in Section 4.

2. Models

This section presents four probabilistic frameworks for modeling the nature of documents in an unsupervised setting. Each framework defines a generative model for documents and encompasses different probabilistic assumptions for their generation. The first two models are state of the art and correspond to the multinomial mixture model (Section 2.2) and the Probabilistic Latent Semantic Analysis (Section 2.3) (Hofmann, 1999). We bridge the gap between these models by our first contribution which represents documents in a reduced word-topic space (Section 2.4) and in Section 2.5, we present our extension of the PLSA model.

2.1. Notations

We assume that the collection consists of a set of n unlabeled documents $\mathscr{D} = \{d_i\}_{i \in \{1, \dots, n\}}$ containing words from a vocabulary $\mathscr{V} = \{w_j\}_{j \in \{1, \dots, m\}}$. Each word $w \in \mathscr{V}$ is represented by a vector of its occurrences in documents of the collection $\vec{w} = \langle n(d_i, w) \rangle_{i \in \{1, \dots, n\}}$, and each document $d \in \mathscr{D}$ is represented by the vector of word frequencies $\vec{d} = \langle n(d, w_j) \rangle_{j \in \{1, \dots, m\}}$. We further assume that the collection contains K latent document clusters $A = \{\alpha_1, \dots, \alpha_K\}$ and L latent word topics (or concepts) $B = \{\beta_1, \dots, \beta_I\}$.

2.2. Multinomial mixture

In this framework each document is assumed to be generated by a mixture model:

$$p(\vec{d}|\Theta) = \sum_{k=1}^{K} p(\alpha_k|\Theta) p(\vec{d}|\alpha_k,\Theta)$$
(1)

We further assume that there is an univocal correspondence between each document cluster $\alpha \in A$ and each mixture component. A document d is therefore generated by first selecting a mixture component according to the prior cluster

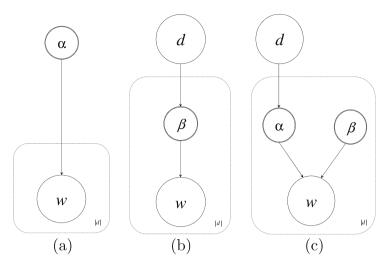


Fig. 1. Graphical model representation of the multinomial mixture model (a), the PLSA/aspect model (b) and its extended version (c). The *plates* indicate the repeated sampling of the enclosed variables.

probabilities $p(\alpha_k|\Theta)$ and then generating the document from the selected mixture component, with probability $p(\vec{d}|\alpha_k,\Theta)$ (Fig. 1a).

We further assume that the word occurrences within each document are independent. This assumption corresponds to the Naive Bayes model in the supervised case and is referred to the mixture of multinomials in unsupervised learning (Pavlov, Balasubramanyan, Dom, Kapur, & Parikh, 2004). In this case, the probability of a document d given cluster α_k can be expressed as

$$p(\vec{d}|\alpha_k, \Theta) \propto \prod_{i=1}^{|f'|} p_{jk}^{n(d,w_j)}$$
 (2)

where p_{jk} is the probability of generating word w_j in document cluster α_k . The complete set of model parameters consists of multinomial parameters for the cluster priors $p(\alpha_k)$ and word generation probabilities p_{jk} :

$$\Lambda = \{ p(\alpha_k) : \alpha_k \in A; \quad p_{ik} : w_i \in \mathcal{V}, \ \alpha_k \in A \}$$

We estimate the parameters Λ by maximizing the complete data log-likelihood using the Expectation Maximization algorithm (EM) (Dempster, Laird, & Rubin, 1977). The algorithm can be sketched out as follows: The initial set of parameters $\Lambda^{(0)}$ is obtained randomly. We then iteratively estimate the probability that each mixture component $\alpha_k \in A$ generates each document $d \in \mathscr{D}$ using the current parameters $\Lambda^{(t)}$ (E-step) and update the Multinomial mixture parameters $\Lambda^{(t+1)}$ by maximizing the complete data log-likelihood (M-step).

2.3. Probabilistic Latent Semantic Analysis

The PLSA model introduced by Hofmann (1999) is a probabilistic model which characterizes each word in a document as a sample from a mixture model, where mixture components are conditionally-independent multinomial distributions. This model associates an unobserved latent variable (called aspect or concept) $\beta \in B$ to each observation corresponding to the occurrence of a word $w \in \mathscr{V}$ within a document $d \in \mathscr{D}$. The underlying generation process of this aspect model is (Fig. 1b):

- Choose a document d with probability p(d).
- Choose a topic β according to $p(\beta|d)$.
- Generate a word w with probability $p(w|\beta)$.

The generation of a word w within a document d can then be translated by the following joint probability model:

$$p(w,d) = p(d) \sum_{\beta \in \mathbb{R}} p(\beta|d)p(w|\beta) \tag{3}$$

This model overcomes the simplifying assumption of the multinomial mixture model where all words are supposed to be generated from the same topic (Eq. (2)). In PLSA, a topic is drawn independently from $p(\beta|d)$ each time that a new word is generated in a document. This provides a much more natural way to handle unusual words or multi-topicality.

The model parameters in this case are

$$\Delta = \{ p(d), p(\beta|d), p(w|\beta) : d \in \mathcal{D}, \beta \in B, w \in \mathcal{V} \}$$

and they are obtained by maximizing the (log-)likelihood,

$$\mathcal{L}_1 = \sum_{d \in \mathcal{Q}} \sum_{w \in \mathcal{V}} n(d, w) \log p(d, w) \tag{4}$$

using the Expectation Maximization (EM) algorithm (Dempster et al., 1977). The iterative update rule consists of first computing posterior probabilities of latent variables using the current parameters $\Delta^{(t)}$ (E-step), and then to update $\Delta^{(t+1)}$ by maximizing the log-likelihood function (4) in the M-step.

Once the model parameters have been estimated, each vocabulary word $w \in \mathscr{V}$ can be assigned to a word topic $\beta \in B$ following the Bayes rule:

cluster
$$(w) = \underset{\beta \in B}{\operatorname{argmax}} p(\beta|w) \equiv \underset{\beta \in B}{\operatorname{argmax}} p(w|\beta) \sum_{d} p(d)p(\beta|d)$$
 (5)

In PLSA, latent word topics and document clusters are identical (Gaussier & Goutte, 2005). The probability of sampling from topic β given document d, $p(\beta|d)$, is interpreted as the posterior cluster probability for document d. Clustering is performed using Bayes decision rule: When document clustering is performed with the PLSA model, the latent word topics play the role of the document clusters (Gaussier & Goutte, 2005). In this case, the probability of observing the topic β given the document d, $p(\beta|d)$ is interpreted as the posterior probability of the document d and clustering is performed from the following Bayes decision rule

cluster:
$$(d) = \underset{\beta \in B}{\operatorname{argmax}} p(\beta|d)$$

2.4. Concept learning

In this section, we present our first dimensionality reduction model which finds latent word topics, or *concepts*, by directly grouping words of the vocabulary. The inherent assumption which leads to this partitioning is that *words occurring* with the same frequencies in the same documents are semantically related. This assumption takes into account the presence of synonym terms in the context of a discourse. Once concepts are found, documents are then represented in the deduced concept space where clustering is performed.

Formally, we assume that each word $w \in \mathscr{V}$ is generated by a mixture density:

$$p(\vec{w}|\Theta) = \sum_{l=1}^{L} \pi_l p(\vec{w}|\beta_l, \Theta_l)$$
 (6)

where as previously noted, L is the number of latent topics to be found and Θ is the set of all model parameters (mixing rates and density parameters). We further suppose that each word belongs to exactly one word topic. This assumption can be formalized using a topic indicator vector $C_i = \{C_{hi}\}_h$ for each word $w_i \in \mathscr{V}$ defined as:

$$\forall w_i \in \mathcal{V}, \quad \exists \beta_l \in B; \quad w_i \in \beta_l \iff C_{li} = 1 \text{ and } \forall h \neq l, \quad C_{hi} = 0$$

Word clustering is then performed by searching the parameters Θ maximizing the complete data log-likelihood:

$$\mathcal{L}_2(C, \Theta) = \sum_{w \in \mathcal{Y}} \sum_{l=1}^{L} C_{lj} \log p(\vec{w}_j, \beta_l, \Theta)$$
 (7)

Here, the cluster indicator vectors C are estimated together with model parameters Θ . In our experiments, we assumed that words are independently generated by the mixture density (6) where each mixture component $p(\vec{w}|\beta)$ obeys a Naive Bayes model. Hence by denoting, for each document $d_i \in \mathscr{D}$ and cluster $\beta_l \in B$, the probability of generation of d_i in β_l as q_{il} ; the complete set of model parameters consists of multinomial parameters of mixing components $\pi_l = p(\beta_l)$ and document generation probabilities q_{il} :

$$\Theta = \{ p(\beta_l) : \beta_l \in B; \quad q_{il} : d_i \in \mathcal{D}, \ \beta_l \in B \}$$

From this assumption, the probability of a word $w \in \mathcal{V}$ given a latent topic β_l can be expressed as

$$p(\vec{w}|\beta_l) \propto \prod_{i=1}^n q_{il}^{n(d_i,w)} \tag{8}$$

Algorithm 1. The CEM algorithm for word clustering

Input

- A partition $P^{(0)}$ is randomly initialized and the conditional probabilities $p(w|\beta=l, \Theta_l^{(0)})$ are estimated on the corresponding clusters.
- $t \leftarrow 0$

repeat

• \mathbb{E} -step: Estimate the posterior probabilities that each word w_j in the vocabulary belongs to each of the partitions $P_j^{(t)}$:

$$\forall w_j \in \mathscr{V}, \ \forall l \in \{1, \dots, L\}, \mathbb{E}[C_{ij}^{(t)}|w_j; P^{(t)}, \Theta^{(t)}] = \frac{\pi_l^{(t)} p(w_j|\beta_l)}{p(w, \Theta^{(t)})}$$

- C-step: Assign to each word $w_j \in \mathcal{V}$ the cluster $P_k^{(t+1)}$ with the highest posterior probability $\mathbb{E}[C|w]$. Let $P^{(t+1)}$ be the new partition.
- M-step: Update the parameter estimates $\Theta^{(t+1)}$ by maximizing Eq. (7).
- $t \leftarrow t+1$

until convergence of \mathcal{L}_2 ;

Output: Word Clusters, $P^{(t)}$

We used a classification version of the EM algorithm proposed by Celeux and Govaert (1992) to estimate the model parameters Θ . This algorithm is depicted on the right (Algorithm 1) and it may be sketched out as follows: The initial set of model parameters $\Theta^{(0)}$ are estimated on the basis of randomly obtained word partitions $P^{(0)}$. Three steps are then repeated until the convergence of the complete data log-likelihood (7). In the E-step, a conditional expectation of each word $w_j \in \mathscr{V}$ with respect to each of its associated cluster indicators C_{ij} ; $l \in \{1, \ldots, L\}$ is computed. As each C_{ij} is a binary random variable, this conditional expectation is equal to the posterior probability of the word w_j within the latent topic β_l which can be estimated by the Bayes rule and the current model parameters $\Theta^{(t)}$. In the C-step, words are assigned to each cluster with the highest posterior probability estimated previously and new model parameters $\Theta^{(t+1)}$ are estimated by maximizing the complete data log-likelihood. Lagrange multipliers are used to enforce $\sum_l \pi_l = 1$ and $\forall l$, $\sum_{i=1}^n q_{il} = 1$ constraints. In the M-step, the model parameters are updated as:

$$\pi_l^{(t+1)} = \frac{\sum_{j=1}^{|\gamma'|} C_{lj}^{(t+1)}}{|\gamma'|}, \quad q_{il}^{(t+1)} = \frac{\sum_{j=1}^{|\gamma'|} C_{lj}^{(t+1)} \times n(d_i, w_j)}{\sum_{j=1}^{|\gamma'|} \sum_{i=1}^{n} C_{li}^{(t+1)} \times n(d_i, w_j)}$$

Finally, we obtain word clusters (or topics) which serve as a new feature space representation for each document in the collection, where each feature corresponds to a word cluster and represents the total number of occurrences of its words appearing in the document.

From Eq. (8), it is clear that words occurring in the same documents with similar frequencies will have similar $p(\vec{w}|\beta_l)$ and therefore, using the Bayes decision rule, belong to the same word topic. Note also that combining Eq. (5) and the M-step equation for $p(\beta|d)$ shows that PLSA also tends to assign words with similar co-occurrence patterns to the same topic/cluster.

2.5. An extended version of PLSA

The problem of the aforementioned clustering in the concept space is that the clustering process passes through two mixture models and it may be altered from successive generative assumptions. In this section we propose an extension of the PLSA model in which document and word clustering are performed simultaneously in a single aspect model. In this case the underlying generation process is as follows:

- Pick a document d with probability p(d).
- Choose a document cluster α with probability $p(\alpha|d)$.
- Choose a word topic β with probability $p(\beta)$.
- Generate a word w with probability $p(w|\alpha, \beta)$.

Fig. 1c depicts this process. Words are in this case generated not only by latent topics (as it is the case with the PLSA model) but also by document clusters. This assumption hence enables the generative model to capture the discourse on two different semantic levels: document clusters and word topics.

In this case the generation of a word w within a document d can be expressed by the following probability:

$$p(d, w) = \sum_{\alpha \in A} \sum_{\beta \in B} p(d) p(\alpha|d) p(\beta) p(w|\alpha, \beta)$$
(9)

Following the maximum likelihood principle, the model parameters are hence,

$$\Phi = \{p(d), p(\alpha|d), p(\beta), p(w|\alpha, \beta) : (d, w, \alpha, \beta) \in \mathcal{D} \times \mathcal{V} \times A \times B\}$$

and they are estimated by maximizing the log-likelihood function (4) using an EM-type algorithm (Dempster et al., 1977). In the E-step, conditional probabilities of latent variables given documents and words are estimated from the current model parameters:

$$p^{(t+1)}(\alpha,\beta|d,w) = \frac{p^{(t)}(\alpha|d)p^{(t)}(\beta)p^{(t)}(w|\alpha,\beta)}{\sum_{\alpha'\in A}\sum_{\beta'\in B}p^{(t)}(\alpha'|d)p^{(t)}(\beta')p^{(t)}(w|\alpha',\beta')} \tag{10}$$

In the M-step, new model parameters maximizing the expectation of the log-likelihood (4) are estimated:

$$p^{(t+1)}(\alpha|d) = \frac{\sum_{w \in \mathscr{V}} \sum_{\beta \in B} n(d, w) p^{(t)}(\alpha, \beta|d, w)}{\sum_{w \in \mathscr{V}} \sum_{\alpha' \in A} \sum_{\beta \in B} n(d, w) p^{(t)}(\alpha', \beta|d, w)}$$
(11)

$$p^{(t+1)}(\beta) = \frac{\sum_{d \in \mathcal{D}} \sum_{w \in \mathcal{V}} \sum_{\alpha \in A} n(d, w) p^{(t)}(\alpha, \beta | d, w)}{\sum_{d \in \mathcal{D}} \sum_{w \in \mathcal{V}} \sum_{\alpha \in A} \sum_{\beta' \in B} n(d, w) p^{(t)}(\alpha, \beta' | d, w)}$$

$$(12)$$

$$p^{(t+1)}(\beta) = \frac{\sum_{d \in \mathscr{D}} \sum_{w \in \mathscr{Y}} \sum_{\alpha \in A} n(d, w) p^{(t)}(\alpha, \beta | d, w)}{\sum_{d \in \mathscr{D}} \sum_{w \in \mathscr{Y}} \sum_{\alpha \in A} \sum_{\beta' \in B} n(d, w) p^{(t)}(\alpha, \beta' | d, w)}$$

$$p^{(t+1)}(w | \alpha, \beta) = \frac{\sum_{d \in \mathscr{D}} n(d, w) p^{(t)}(\alpha, \beta | d, w)}{\sum_{d \in \mathscr{D}} \sum_{w' \in \mathscr{Y}} n(d, w') p^{(t)}(\alpha, \beta | d, w')}$$

$$(12)$$

Algorithm 2. Extended version of PLSA

- A document collection @
- Random initial model parameters $\Phi^{(0)}$

repeat

- Re-estimate model parameters using multiplicative update rules (10)–(13).
- $t \leftarrow t+1$

until convergence of \mathcal{L}_1 (Eq. (4));

Output: A generative classifier with parameters $\Phi^{(t)}$

This algorithm (Algorithm 2) is also an EM-like algorithm, and the iterative use of Eqs. (10)–(13) corresponds to alternating the E-step and M-step. Convergence is therefore guaranteed to a local maximum of the likelihood.

Notice that the maximum likelihood estimate of parameter p(d) has an exact analytical expression (and does not need iterative estimation):

$$p(d) = \frac{\sum_{w \in \mathscr{V}} n(d, w)}{\sum_{d' \in \mathscr{Q}} \sum_{w \in \mathscr{V}} n(d', w)}$$

Once model parameters are obtained, each document $d \in \mathcal{D}$ is assigned to a cluster using the Bayes decision rule:

$$\text{cluster } (d) = \operatorname*{argmax}_{\alpha \in A} p(\alpha|d)$$

3. Experimental setup

We conducted a number of experiments aimed at evaluating how the representation in the concept space can help to improve the clustering performance. The multinomial model was used both in its basic version (Section 2.2) and for the concept learning method (Section 2.4). PLSA was used both for direct document clustering and for finding concept spaces (word topics). In the following, these models are denoted by MM and PLSA when they perform clustering in the original vocabulary space. When clustering is performed in the concept space, we use the following notations. A first letter identifying the word topic algorithm (respectively C or P for CEM and PLSA) followed by the acronym of the clustering approach. Thus C-MM denotes document clustering with MM on the concept space induced by CEM, P-MM denotes document clustering with MM on the concept space induced by PLSA, C-PLSA is the document clustering with PLSA on the concept space of CEM and finally P-PLSA denotes two successive applications of PLSA, first for word topics identification and then for document clustering onto the induced concept space. In the light of these results we compare in the second part of our experiments the performance of the extended PLSA (Ext-PLSA) against all the previous clustering models as well as the Kmeans and LDA algorithms (Blei et al., 2003) and a NMF model obtained by minimizing the Frobenius norm. As KL-minimal NMF is essentially equivalent to PLSA (Gaussier & Goutte, 2005), we do not include it in our comparison. Latent Semantic Indexing (Deerwester et al., 1990) is another popular matrix factorization method for the analysis of text data. But as NMF outperforms LSI for document clustering (Xu et al., 2003), we do not include LSI in our comparison. In our experiments, we used standard labeled

Table 1
Class proportions in Reuters-21578. Reuters RCV2-French. 20Newsgroups and WebKB datasets.

Class	Perc. (%)
Reuters-21578	
Earn	46.2
acq	24.9
Money	8.4
Crude	7
Grain	6.5
Trade	4.8
Interest	2.4
RCV2-French	
C15	16.7
CCAT	16.7
E21	16.7
ECAT	16.7
GCAT	16.7
Mll	16.5
20Newsgroups	
Comp.	30.5
Sci.	24.7
Rec.	20.3
Talk,	19.5
Alt.	5.0
WebKB	
Student	39.1
Faculty	26.8
Course	22.1
Project	12.0

text classification corpora with the class labels representing an objective knowledge reflecting the datasets implicit structure. The following sections describe the corpora we used in our experiments as well as the accuracy measures we used to evaluate the performance of the proposed models.

3.1. Datasets

We conducted our experiments on Reuters-21578, Reuters RCV2-French, 20Newsgroups and WebKB datasets.

The Reuters-21578 1 collection contains Reuters news articles from 1987. We selected documents in the collection that are assigned to at least one class. Each document in the corpus can have multiple labels, but in practice more than 80% of articles are associated to a single topic. In addition, for multiply-labeled documents, only the first class from the $\langle TOPIC \rangle$ field was retained. We kept the documents associated with the seven most frequent classes, which resulted in 4335 documents, each with a unique label

The Reuters RCV2-French² (denoted by RCV2-French in the following) is the French part of the multilingual Reuters corpus which contains over 487,000 Reuters News stories in thirteen languages. We focused on six relatively populous classes and for each class we sampled up to 5000 documents from RCV2. Documents belonging to more than one of our six classes were assigned the label of their smallest class ending to a monolingual case.

The 20Newsgroups³ dataset is a collection of newsgroup postings from 20 different Usenet discussion forums. We deleted cross-posted and duplicate messages, and we regrouped the documents into one of the five following classes (alt., comp., sci., rec. talk.) which resulted in 16,010 documents. We ignored in this case file headers and subject lines.

The WebkB⁴ contains 8145 web pages gathered from computer science departments. The collection includes web pages of four departments divided into seven categories. In this paper, we used the four most populous ones all together containing 4196 pages. Our specific pre-processing for this collection consists in filtering the text by removing html tags.

General pre-processing steps for these four collections consist in converting all words to lowercase, mapping digits to a single *digit* token and suppressing non-alpha-numeric characters. We also used a stop-list to remove very frequent words and also filtered words occurring in less than three documents. These pre-processing lead to an initial vocabulary size of 6990, 34,272, 38,630 and 11,170 words for respectively the Reuters-21578, RCV2-French, 20Newsgroups and WebKB datasets. Table 1 summarizes the characteristics of these four collections. Classes are shown in the decreasing order of their sizes in the respective collections.

¹ http://www.daviddlewis.com/resources/testcollections/reuters21578/.

² http://trec.nist.gov/data/reuters/reuters.html.

³ http://kdd.ics.uci.edu/databases/20newsgroups/.

⁴ http://www.cs.cmu.edu/afs/cs.cmu.edu/project/theo-20/www/data/.

We finally performed experiments over 10 random cross-validation splits of each initial collection while preserving the proportions between different classes in each subset. This setting avoids bias caused by the inherent structure of each collection. It helps reduce the effects of the random initializations made in the generative models we considered. The reported performance are averaged over these 10 randomly selected subsets.

3.2. Evaluation criteria

In order to compare the performance of the algorithms, we used the micro-averaged precision and recall (Slonim & Tishby, 2002) as well as the Normalized Mutual Information (Strehl & Ghosh, 2002). To estimate these measures we first assigned documents in the clusters to the majority class present in that cluster. From these assignments we hence estimate the performance measures as follows.

3.2.1. Micro-averaged precision and recall

For each class l in the collection, we first estimate its correctly and incorrectly assigned number of documents, respectively denoted by $\kappa(l)$ and $\vartheta(l)$; as well as $\gamma(l)$, the number of documents incorrectly not assigned to l. We then compute the precision and recall of the class l from the following:

$$\operatorname{Prec}(l) = \frac{\kappa(l)}{\kappa(l) + \vartheta(l)}, \quad \operatorname{Rec}(l) = \frac{\kappa(l)}{\kappa(l) + \gamma(l)}$$

The micro-averaged precision and recall are thus defined by:

$$\label{eq:micro-averaged} \mbox{Micro-averaged precision} = \frac{\sum_{l} \kappa(l)}{\sum_{l} \kappa(l) + \vartheta(l)}$$

and,

Micro-averaged recall =
$$\frac{\sum_{l} \kappa(l)}{\sum_{l} \kappa(l) + \gamma(l)}$$

By noticing that $\sum_{l} \kappa(l) + \gamma(l)$ and $\sum_{l} \kappa(l) + \vartheta(l)$ are both equal to the total number of documents in a collection, we have in this case a perfect equality between the micro-averaged precision and recall. In our experiments we will refer to these terms by *Average precision*.

3.2.2. Normalized Mutual Information

The Normalized Mutual Information (NMI) is a measure which estimates the quality of a clustering with respect to the true classes of a dataset (Strehl & Ghosh, 2002). It corresponds to the Normalized Mutual Information between the cluster assignments of instances and their underlying class labels and is given from the following expression:

$$NMI = \frac{\sum_{h=1}^{c} \sum_{l=1}^{c} n_{h,l} \log \left(\frac{n \times n_{h,l}}{n_h n_l} \right)}{\sqrt{\left(\sum_{h=1}^{c} n_h \log \frac{n_h}{n}\right) \left(\sum_{l=1}^{c} n_l \log \frac{n_l}{n}\right)}}$$

where n is the total number of documents, c the number of classes (or clusters), n_h and n_l are the number of documents in respectively cluster h and class l, and $n_{h,l}$ is the common number of documents in both cluster h and class l. As previously the range of NMI values is within [0,1] and it tends to 1 for increasingly better cluster qualities.

3.3. Experimental results

We begin our experiments by first examining the effect of changing the document representation from the initial vocabulary space into the new learned concept space (induced by the CEM algorithm). We analyze this effect on the final clustering results of the multinomial mixture model. The two clustering schemes involved here are the multinomial mixture model operating in the vocabulary space (MM) and in the concept space (C-MM). We recall that documents are represented in the latter space by a vector where each component corresponds to a word topic and represents the total number of words from this topic occurring in the document.

Table 2 An example of term clusters found with CEM in 20Newsgroups (top) and WebKB (down) data collections.

 ${\it Cluster i:}$ wrong christian words christians truth meaning paul john bible word faith fact reason men

Cluster j: suicide suicides deaths stat selfdefense risks guns statistic accidents homicides nejm

Cluster k: course course homework project projects umd class assignment due assignments exam fall berkeley lecture

Cluster l: computer research systems cs science programming acm sciences engineering software system algorithms program

The number of document clusters is fixed to the original number of class labels of each dataset, we have then varied the number of word topics from 10 to 100. Table 3, shows the precision, recall and Average precision performance over the four Reuters-21578, RCV2-French, 20Newsgroups and WebKB databases. In these experiments, the number of word topics on each collection was fixed to the number which provided the best clustering results of the MM model in the concept space. This corresponds to |B| = 10 on the Reuters-21578, RCV2-French and 20Newsgroups datasets and |B| = 20 on the WebKB collection. The Table 2 illustrates the ability of CEM to identify word topics on the 20Newsgroups and WebKB datasets.

It should be noticed that the value of |B| influences the generality level of the resulting word topics. When |B| is small, the word topics are constrained to be quite general. A high value will allow more detailed and specialized words topics. Hence the optimal |B| values for Reuters-21578, RCV2-French, 20Newsgroups and WebkB correspond to rather general word topics. There are two explanations for this observed tendency for fewer word topics. First, if there are lots of specialized word topics, then two documents dealing with the same topics may appear dissimilar in the induced concept space. This phenomenon degrades the clustering quality. Second, a high number of word topics implies a higher number of parameters to be learnt. Hence the learning task is more difficult, resulting in poor clustering performance. As a consequence, the optimal number of word topics |B| tends to be small in our experiments.

The Figs. 2 and 3 show the evolution of performance of the MM and PLSA algorithms for a varying number of word topics (induced by the CEM and the PLSA models).

In the tables, the symbol \(\preceq\) indicates that performance is significantly worse than the best result, according to a Wilcoxon rank sum test used at a p-value threshold of 0.01 (Lehmann, 1975). On these four collections and for about all classes, the precision, recall, and the Average precision of C-MM are significantly better than the MM algorithm. We notice here that in the case where classes are unbalanced (especially on Reuters-21578, 20Newsgroups and WebkB), small classes are absorbed by larger ones. This phenomenon is however attenuated when clustering is performed in the concept space, the class trade in Reuters-21578 becomes even visible. These results suggest that the independence hypothesis of the MM model between document vector components is more likely to be true when documents are represented in a concept space. Indeed, it is more reasonable to assume that the discourse of a document consists of independent word topics, than assuming that vocabulary words occurring in a document be independent from one another.

Fig. 2 shows the document clustering performance, on the four datasets, of the MM algorithm operating in the bag-of-words space and in the concept spaces induced by the CEM and PLSA algorithms, for varying numbers of word concepts (L = |B|). The figure also depicts the performance of the PLSA model for document clustering. We recall that in this case,

Table 3
Precision, recall clustering performance of the Naive Bayes model learnt on the vocabulary space (MM) and on the concept space (C-MM) for the Reuters-21578, RCV2-French, 20Newsgroups and WebkB datasets.

Dataset	B = 10	Precision		Recall	
		MM	C-MM	MM	C-MM
Reuters-21578	Earn	0.77↓	0.89	0.93	0.84
	acq	0.43↓	0.65	0.60↓	0.77
	Money	0.35↓	0.41	0.26^{\downarrow}	0.48
	Crude	0.34^{\downarrow}	0.46	0.57 [↓]	0.48
	Grain	0.43^{\downarrow}	0.52	0.13↓	0.54
	Trade	0	0.37	0	0.22
	Interest	0	0	0	0
	Average	0.61 [↓]	0.70	0.61 [↓]	0.70
RCV2-French	C15	0.38^{\downarrow}	0.45	0.10↓	0.31
	CCAT	0.53^{\downarrow}	0.77	0.79^{\downarrow}	0.85
	E21	0.63↓	0.68	0.43^{\downarrow}	0.79
	ECAT	0.48	0.43^{\downarrow}	0.18↓	0.34
	GCAT	0.40^{\downarrow}	0.51	0.61 [⊥]	0.71
	Mll	0.46^{\downarrow}	0.65	0.76	0.57↓
	Average	0.48^{\downarrow}	0.60	0.48^{\downarrow}	0.60
20Newsgroups	Comp.	0.62^{\downarrow}	0.76	0.92	0.87↓
	Sci.	0.57↓	0.78	0.37 [⊥]	0.62
	Rec.	0.60^{\downarrow}	0.67	0.72^{\downarrow}	0.89
	Talk.	0.67↓	0.84	0.50↓	0.78
	Alt.	0	0	0	0
	Average	0.62^{\downarrow}	0.75	0.62^{\downarrow}	0.75
WebKB	Student	0.47^{\downarrow}	0.72	0.83	0.77↓
	Faculty	0.41↓	0.55	0.21↓	0.64
	Course	0.58↓	0.86	0.36↓	0.77
	Project	0.37↓	0.45	0.10↓	0.29
	Average	0.48 ¹	0.68	0.48 ¹	0.68

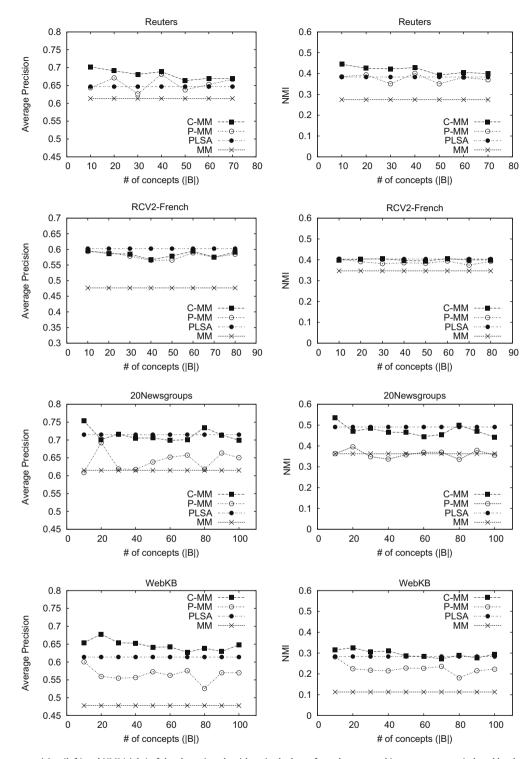


Fig. 2. Average precision (left) and NMI (right) of the clustering algorithms in the bag-of-words space and in concept spaces induced by the CEM and PLSA algorithms.

latent topics in PLSA correspond to document partitions which are hence fixed to the number of class labels on each dataset. In our experiments, the size of the concept space for Reuters-21578 is limited to 70 since we obtained empty partitions for higher values with the CEM algorithm on this collection.

The first observation is that in the original bag-of-words space, PLSA outperforms the MM model over all four datasets. The gap in performance between these two models are the largest on the WebkB collection reaching 17% and 14% for respectively

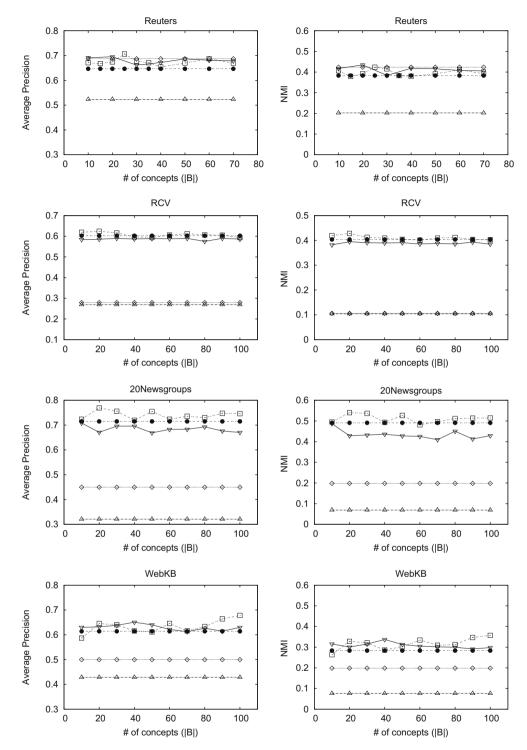


Fig. 3. Average precision (left) and NMI (right) performance of the PLSA (\bullet) and its proposed extension version, Ext-PLSA (\square) compared to the performance of Kmeans (\triangle), NMF (\diamondsuit) and C-PLSA (∇).

the NMI and average precision measures. These results confirm the effectiveness of the document generation assumption made by the PLSA model and discussed in Section 2.3. The MM model becomes however, more competitive in different concept spaces induced by the CEM and the PLSA algorithms (i.e. C-MM and P-MM). The latter two have significantly better results than the MM model performing in the initial bag-of-words space, for concept spaces with various sizes. These observations confirm our findings made previously, that the independence assumption made by the MM model is more accurate on a space

where each direction corresponds to a group of similar words. We also notice that both C-MM and P-MM models achieve results comparable to PLSA in terms of NMI and Average Precision. Moreover, the C-MM model performs uniformly better than the P-MM model on the four collections, in both NMI and Average precision.

As the underlying assumption of the MM model is that each dimension axes are mutually independent, from the clustering results of C-MM and P-MM it becomes apparent that the CEM algorithm groups dependent terms more efficiently than PLSA. These results suggest that the simple assumption of CEM (Section 2.4) is in better agreement with the assumption used by both PLSA and CEM to assign words to topics which, we recall, is: words co-occurring with the same frequency in the same documents are similar.

The performance curves of the PLSA model operating in the original bag-of-words space and in the concept space induced by the CEM algorithm (C-PLSA) as well as its extension Ext-PLSA introduced in Section 2.5 are shown in Fig. 3.

On the Reuters-21578 and 20Newsgroups datasets, Ext-PLSA outperforms PLSA on average precision for any number of concepts. The best performances of Ext-PLSA are obtained for 25 concepts on Reuters-21578 and 20 concepts on 20Newsgroups, and correspond to improvements of respectively 6% and 8% in average precision over PLSA. The NMI curves of Ext-PLSA on Reuters-21578, RCV2-French, 20Newsgroups and WebkB are very similar to the corresponding average precision curves. On the WebkB collection, the performance of the Ext-PLSA model is better than that of the PLSA model on both measures (except when the number of concepts is equal to 10).

We further notice that C-PLSA shows performances similar to Ext-PLSA on Reuters-21578 and WebKB. On the 20Newsgroups dataset, C-PLSA is consistently outperformed by Ext-PLSA for any number of concepts. On the other hand, the Ext-PLSA model captures the cluster structure of the document collection and the topics or concepts. The joint clustering of Ext-PLSA reduces the bias that might result from the two successive clusterings performed in the first model described in Section 2.4.

Finally we notice that there is a big gap between the performance of PLSA and NMF using the Frobenius norm especially on RCV2-French, 20Newsgroups and WebKB where initial dimension sizes are higher than the one of the Reuters-21578 collection. This difference might be due to the use of the quadratic norm which is not relevant in high dimensional spaces. This is consistent with the results of the Kmeans algorithm in which we used the Euclidean norm.

In Table 5, we present a comparison of all the different algorithms operating in the initial bag-of-words space and in the different concept spaces. The number of concepts was fixed to the one for which each respective model obtained its best results in terms of Average precision. We compared clustering performance on the 10 subsets of the four data collections. The best Ext-PLSA model here is significantly better than almost all other models except the C-MM model which behaves

Table 4Computational and complexity comparisons on different datasets. *M* represents the number of (term, document) pairs observed in the corpus. Recall that *n* is the total number of documents, *m* the vocabulary size, *K* the number of document clusters and *L* the number of topics. For the execution time, the latter is fixed to 10.

Algorithm	Complexity	Execution time				
		Reuters	RCV2	News	WebKB	
Kmeans	$O(K \times M)$	1 s	2 s	1 s	1 s	
NMF	$O(K \times n \times m)$	40 s	28 min	5 min	40 s	
MM	$O(K \times n \times m)$	5 s	10 min	3 min	20 s	
C-MM	$O(L \times m \times n)$	10 s	20 min	7 min	1 min	
P-MM	$O(L \times M)$	10 s	5 min	2 min	25 s	
PLSA	$O(K \times M)$	5 s	3 min	1 min	10 s	
Ext-PLSA	$O(K \times M \times L)$	1 min	30 min	11 min	1 min	
LDA	$O(K \times n \times m)$	1 min	30 min	15 min	2 min	

Best average precision and the corresponding average NMI of different clustering algorithms on the Reuters-21578, RCV2-French, 20Newsgroups and WebkB datasets. The symbols \$\psi\$ indicate the cases of algorithms significantly worse than that of the best algorithm which performance is shown in bold.

Algorithm	Reuters-21578		RCV2-Fre	RCV2-French		20Newsgroups		WebKB	
	AP	NMI	AP	NMI	AP	NMI	AP	NMI	
Kmeans	0.52↓	0.20↓	0.27↓	0.10↓	0.32↓	0.07↓	0.43↓	0.08↓	
NMF	0.69	0.42	0.28^{\downarrow}	0.11 [↓]	0.45↓	0.20^{\downarrow}	0.50↓	0.20^{\downarrow}	
MM	0.61 [⊥]	0.27↓	0.48^{\downarrow}	0.35↓	0.62^{\downarrow}	0.36 [⊥]	0.48↓	0.11 [↓]	
P-MM	0.68	0.40^{\downarrow}	0.59^{\downarrow}	0.40^{\downarrow}	0.69↓	0.40^{\downarrow}	0.60↓	0.28↓	
C-MM	0.70	0.44	0.60	0.40^{\downarrow}	0.75	0.53	0.68	0.32^{\downarrow}	
PLSA	0.64^{\downarrow}	0.38↓	0.60	0.40^{\downarrow}	0.71 [⊥]	0.49^{\downarrow}	0.61 [⊥]	0.28^{\downarrow}	
P-PLSA	0.68	0.39↓	0.60	0.39^{\downarrow}	0.69 [↓]	0.40^{\downarrow}	0.63↓	0.28 [↓]	
C-PLSA	0.69	0.42	0.59^{\downarrow}	0.39^{\downarrow}	0.71 [↓]	0.49^{\downarrow}	0.65↓	0.34^{\downarrow}	
Ext-PLSA	0.71	0.42	0.62	0.43	0.77	0.54	0.68	0.36	
LDA	0.69	0.38↓	0.55↓	0.32^{\downarrow}	0.67↓	0.41 [↓]	0.59^{\downarrow}	0.25^{\downarrow}	

nearly the same. Note also that the clustering performance of MM and PLSA in the concept space induced by the CEM algorithm is considerably increased. This is especially true on the NMI measure, where, on the WebkB corpus, the performance of C-MM is three times higher than for MM.

Table 4 shows the complexity and the average execution time of different algorithms on 3.16 GHz Intel Core 2 Duo processor with 4G RAM. Comparatively, the proposed Ext-PLSA approach takes the same time in execution than LDA and NMF but it is more efficient than the two latter on all datasets.

4. Conclusion

In this paper, we have studied the problem of document clustering in a reduced concept space. Our first contribution sought to find this space by partitioning vocabulary words according to the hypothesis that words co-occurring in the same context with the same frequency are topically related. Experiments on four datasets have shown that the performance of a baseline MM model is significantly improved when it operates in the induced concept space. We further proposed an extended version of the PLSA model which learns jointly the word topics and the document clusters. Experiments conducted on the Reuters-21578, RCV2-French, 20Newsgroups and WebKB datasets have shown that the proposed Ext-PLSA performs significantly better than the original PLSA model. Compared to the C-MM which partitions documents in the concept space induced by CEM, the join clustering of documents and words performed by Ext-PLSA presents the advantage to reduce the bias of the successive clustering steps done previously.

Acknowledgements

This work was supported in part by the IST Program of the European Community, under the PASCAL Network of Excellence IST-2002-506778. This publication only reflects the authors view.

References

Amini, M.-R., & Gallinari, P. (2002). The use of unlabeled data to improve supervised learning for text summarization. In *Proceedings of the 25th annual international ACM SIGIR* (pp. 105–112).

Banerjee, A., Dhillon, I., Ghosh, J., Merugu, S., & Modha, D. S. (2007). A generalized maximum entropy approach to Bregman co-clustering and matrix approximation. *Journal of Machine Learning Research*, 8, 1919–1986.

Bingham, E., & Mannila, H. (2001). Random projection in dimensionality reduction: Applications to image and text data. In *Proceedings of the 7th ACM SIGKDD* (pp. 245–250).

Blei, D. M., Ng, A. Y., & Jordan, M. I. (2003). Latent Dirichlet allocation. Journal of Machine Learning Research, 3, 993-1022.

Celeux, G., & Govaert, G. (1992). A classification em algorithm for clustering and two stochastic versions. *Computational Statistics and Data Analysis*, 14(3), 315–332.

Cutting, D. R., Pedersen, J. O., Karger, D., & Tukey, J. W. (1992). Scatter/gather: A cluster-based approach to browsing large document collections. In *Proceedings of the 15th annual international ACM SIGIR* (pp. 318–329).

Deerwester, S. C., Dumais, S. T., Landauer, T. K., Furnas, G. W., & Harshman, R. A. (1990). Indexing by latent semantic analysis. *Journal of the American Society of Information Science*, 41(6), 391–407.

Dempster, A. P., Laird, N. M., & Rubin, D. B. (1977). Maximum likelihood from incomplete data via the em algorithm. Journal of the Royal Statistical Society. Series B (Methodological), 39(1), 1–38.

Dhillon, I. S. (2001). Co-clustering documents and words using bipartite spectral graph partitioning. In *Proceedings of the 7th ACM SIGKDD international* conference on knowledge discovery and data mining (pp. 269–274).

Dhillon, I. S., & Modha, D. S. (2001). Concept decompositions for large sparse text data using clustering. Machine Learning, 42(1/2), 143-175.

Gaussier, E., & Goutte, C. (2005). Relation between plsa and nmf and implications. In *Proceedings of the 28th annual international ACM SIGIR* (pp. 601–602). Griffiths, T. L., & Steyvers, M. (2004). Finding scientific topics. *Proceedings of the National Academy of Sciences*, 101, 5228–5235.

Hofmann, T. (1999). Probabilistic latent semantic indexing. In Proceedings of the 22nd Annual International ACM SIGIR (pp. 50-57).

Knoblock, C., Lopresti, D., Roy, S., & Subramaniam, L. V. (2006). Special issue on noisy text analytics. *International Journal on Document Analysis and Recognition*, 11, 147–155.

Kummamuru, K., Lotlikar, R., Roy, A., Signal, K., & Krishnapuram, R. (2004). A hierarchical monothetic document clustering algorithm for summarization and browsing search results. In *Proceedings of the 13th international conference on world wide web* (pp. 658–665).

Lee, D. D., & Seung, H. (1999). Learning the parts of objects by non-negative matrix factorization. Nature, 401(6755), 788-791.

Lehmann, E. (1975). Nonparametric statistical methods based on ranks. New York: McGraw-Hill.

Oyanagi, S., Kubota, K., & Nakase, A. (2001). Application of matrix clustering to web log analysis and access prediction. In WEBKDD 2001 mining web log data across all customers touch points, third international workshop (pp. 13–21).

Pavlov, D., Balasubramanyan, R., Dom, B., Kapur, S., & Parikh, J. (2004). Document preprocessing for naive Bayes classification and clustering with mixture of multinomials. In *Proceedings of the tenth ACM SIGKDD international conference on knowledge discovery and data mining* (pp. 829–834).

Rijsbergen, K. V. (1979). Information retrieval. London: Butterworth.

Salton, G., & McGill, M. J. (1986). Introduction to modern information retrieval. New York, NY, USA: McGraw-Hill.

Slonim, N., & Tishby, N. (2002). Unsupervised document classification using sequential information maximization. In *Proceedings of the 25th annual international ACM SIGIR* (pp. 129–136).

Strehl, A., & Ghosh, J. (2002). Cluster ensembles – A knowledge reuse framework for combining multiple partitions. *Journal of Machine Learning Research*, 3, 583–617.

Xu, J., & Croft, W. (1999). Cluster-based language models for distributed retrieval. In *Proceedings of the 22nd annual international ACM SIGIR* (pp. 254–261). Xu, W., Liu, X., & Gong, Y. (2003). Document clustering based on non-negative matrix factorization. In *Proceedings of the 26th annual international ACM SIGIR* (pp. 267–273).