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DEGREE DAYS: THE DIFFERENT TYPES

ANALYZED

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Some energy requirements such as the amount of fuel used for heating and several natural phenomena such as the growth of plants and the thickness of lake ice are correlated more or less closely with the accumulated air temperature below or above some standard or base tempera- ture. When computed from daily mean temperatures, these temperature differences are called degree days. There are two main classes of degree days, differing in the method of calculation. This Note will define degree days, distinguish between the two classes, and comment on a few examples from each class.

In general, a degree day is defined as a measure of the departure of the mean temperature for a day from a given base temperature. The accumulation of such departures over a season or up to any day can be used as an indication of past temperature effect on some quantity. Negative departures (or positive departures, if one is primarily concerned with the negative ones) are treated in two different ways for different purposes: they may be considered as zero, that is, contributing nothing to the accumulated total, or they may be considered as having a negative value which reduces the accumulated total. Heating, cooling and growing degree-days belong to the former class; freezing and thawing to the latter.

HEATING DEGREE-DAYS

The fuel or energy required to maintain a temperature of about 21°C in a small building when outside air temperature is below 18°C is roughly proportional to the difference between 18°C and the outside temperature. The energy requirements also depend on other factors including wind speed, solar radiation, and internal heat sources. There is no convenient way of combining these effects, but for average conditions of wind, radiation, and internal sources the proportionality with temperature still holds. Heating degree-days based on temperature alone are therefore useful when more complex methods of calculating fuel requirements are not feasible.

If t is the mean outside air temperature for a particular day, then the heating degree-days h for that day are

$$h = 18 - t,$$

providing t is less than 18. For mean air temperatures greater than 18, no energy is required to keep the building sufficiently warm and therefore the heating degree-days are zero.

Heating degree-days H for a few days, a month, or a season are simply the sum of the daily values for that period

$$H = \Sigma \ (18 - t) \quad \text{for all days when } t < 18,$$
$$H = 0 \qquad\qquad \text{for all days when } t > 18.$$

In Figure 1 the 1977 daily mean temperatures at Ottawa International Airport have been plotted. The cross-hatched area represents fairly accurately the number of heating degree-days below 18°C.

Until recently, the published heating degree-days in Canada were Fahrenheit degree-days below 65°F (1). If the base temperature in Celsius had been exactly equal to 65°F, then conversion of degree days would have been simply a matter of multiplying by 5/9. The new base temperature, however, is 18°C or 64.4°F, so that the accumulated degree days are 0.6 Fahrenheit degree-days less for each day that the temperature is below 64.4°F. This adjustment is usually less than 18 Fahrenheit degree-days per month, but if accuracy is important a conversion procedure for monthly values has been prepared by Boyd (2).

Annual totals in Fahrenheit degree-days should be reduced (by a value depending on the length of the heating season) before multiplying by 5/9 to convert to Celsius degree-days. This reduction ranges from about 150 F degree-days for annual totals near 7500 to about 215 F degree-days for annual totals of 12 500 or over.

Normal (i.e. 30-year average) Celsius degree-days below 18°C for each month are available from the Atmospheric Environment Service (3). Current daily values are published in the Monthly Meteorological Summaries issued each month by over 50 weather stations in Canada.

COOLING DEGREE-DAYS

Just as the energy required for heating is roughly proportional to the temperature departure below some base temperature, so the energy required for cooling is roughly proportional to the temperature departure above some base. This required energy is also influenced by sun, wind, internal heat sources, and the humidity of the outside air. Dependence on humidity (or, more correctly, on wet-bulb temperature) makes cooling degree days somewhat less reliable than heating degree-days, but they are still close enough for some applications.

If 18°C is selected as the base temperature, then the cooling degree-days C for any period will be

$$C = \Sigma \, (t - 18) \quad \text{for all days when } t > 18,$$
$$C = 0 \qquad\qquad \text{for all days when } t < 18.$$

The dark stippled area in Figure 1 represents fairly accurately the number of cooling degree-days above 18°C.

Cooling degree-days for locations in Canada are not readily available. They can be computed, however, from heating degree-days, provided the base temperature is the same and the mean temperature for the period concerned is known. The equations above can be rewritten for any period of Na days above and Nb days below 18°C

For Na days when t is equal to or above 18°C,

$$C = \Sigma t - 18 \, Na \qquad\qquad H = 0.$$

For the remaining Nb days when t is below 18°C

$$C = 0 \qquad\qquad -H = \Sigma t - 18 \, Nb.$$

Adding gives

$$C - H = \Sigma t - 18 \, (Na + Nb),$$
$$C = H + N \, (\overline{t} - 18),$$

where $\overline{t}$ is the mean temperature for the N days. This equation is exact, but any rounding-off error in $\overline{t}$ will be multiplied by N.

GROWING DEGREE-DAYS

The rate of growth of plants is roughly proportional to the departure of the air temperature above about 5°C, providing other conditions are satisfactory. Below 5°C there is no appreciable growth. Growing degree-days are therefore defined in the same way as cooling degree-days, except that the base temperature is 5°C. In Figure 1 the light and dark stippled areas together represent fairly accurately the number of growing degree-days above 5°C. Normal (i.e. 30-year average) Celsius degree-days above 5°C for each month have been tabulated (3).

When the air temperature is above 5°C for only a part of the day, it may be more appropriate to use degree hours or even some more complex function of temperature. The result should never be referred to as degree days, however, because, by definition, degree days are based on the daily mean temperature.

FREEZING AND THAWING DEGREE-DAYS

Depth of freezing of the ground and thickness of ice on lakes depend on many factors, but they can be estimated roughly from the degree days below and above freezing. In these cases, however, the days with mean

temperature above the base temperature of 0°C cannot be ignored as was
done for heating degree-days. On a day with mean temperature above 0°C
there will be a tendency for some lake ice to melt or ground to thaw.
The temperature departures above freezing must therefore be subtracted
from the accumulation of temperatures below freezing. Since the base
temperature is zero, the freezing degree-days are

$$F = \Sigma \; (-t)$$

for all days in the period, whether t is positive or negative.

The time required to melt lake ice or thaw ground in the spring is
sometimes estimated using thawing degree-days. As with freezing degree-
days, both positive and negative departures must be accumulated and hence
the thawing degree-days are

$$T = \Sigma t$$

for all days in the period.

The thawing index or thawing degree-days for a whole thawing season
are represented fairly accurately in Figure 1 by the area between the
graph and the zero temperature line for the season when temperature is
generally above zero, decreased by the area below zero for short periods
such as those in early April and mid-November, indicated by arrows. This
is a poor method of representing the thawing index because it requires
the visualization of the difference of areas. A much better method is
shown in Figure 2, where degree days above and below zero have been
accumulated day by day, starting with 1000 degree days in 1976 at the
first of October, to illustrate both freezing and thawing indices. The
areas above and below 0°C in Figure 1 are represented by vertical
distances in Figure 2.

The freezing index for winter is defined as the number of degree
days (below and above freezing) between the highest point in the autumn
and the lowest point the following spring on the cumulative degree-day
time curve. Similarly, the thawing index is the number of degree days
from the lowest point in the spring to the highest point the following
autumn (Figure 2). Because only degree-day differences are of concern,
the zero point on the degree-day scale is quite arbitrary. It was
started at 1000 to avoid negative values.

Any spring or autumn month that includes a seasonal minimum or
maximum of the cumulative degree-days above freezing curve will be called
a "change-over" month. In all other months the Celsius degree-days above
freezing are

$$N\overline{t} = \Sigma t$$

where $\bar{t}$ is the monthly mean temperature and each t is a daily mean temperature. These values are positive in summer and are called thawing degree-days. Days below freezing will decrease the thawing degree-day total for the month, but there will be no freezing degree-days. In winter months the values will be negative. Their absolute values are called freezing degree-days and the thawing degree-days are zero.

In a change-over month both freezing and thawing degree-days have non-zero values, one occurring only before the changeover date and the other only after. It can be shown that if T is the thawing degree-days and F the freezing degree-days, then

$$T - F = N\bar{t} \quad or \quad \Sigma t$$

where T and F are never negative but one of them is zero in each month unless it is a change-over month (4). Normal monthly freezing and thawing degree-days and annual totals for over 800 weather stations have been computed from normal monthly mean temperatures as explained in (4), and published by the Atmospheric Environment Service (5). Conversion from Fahrenheit degree-days to Celsius is simply a matter of multiplying by 5/9, since the base temperatures are exactly equivalent.

As indicated, Reference (3) includes tabulations of the normal degree days below 18°C and above 5°C, calculated according to the accepted method for that class of degree day. Unfortunately, it also includes degree days above and below 0°C calculated in the same way. This means that short freezing periods during the thawing season and short thawing periods during the freezing season are ignored instead of decreasing the accumulated degree days. The degree days above and below 0°C in Reference (3) are therefore not the same as the freezing and thawing degree-days under discussion.

There seems to be a good deal of confusion and misunderstanding connected with degree days, caused partly by the different methods of calculation for different purposes, and partly by the difficulty of converting from Fahrenheit to Celsius. This discussion should lead to a better understanding of the different kinds of degree days and their relationship.

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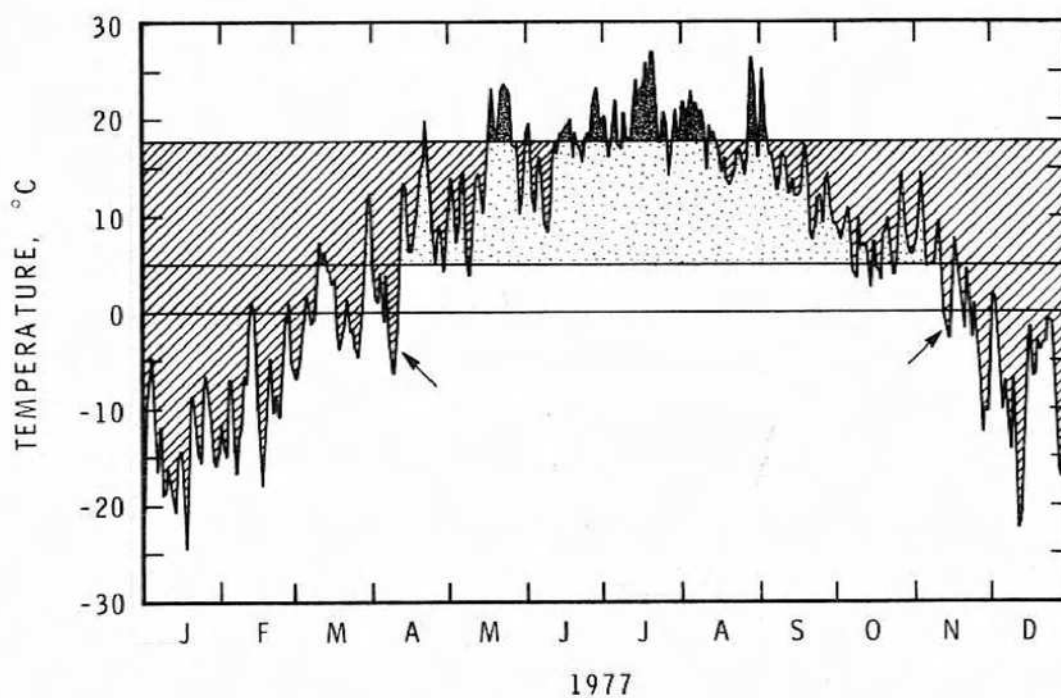   CLI 4-73, 1973.

FIGURE 1

DAILY MEAN TEMPERATURES. SHADED AREAS ILLUSTRATE:
- HEATING DEGREE DAYS BY CROSS HATCHING
- COOLING DEGREE DAYS BY DARK STIPPLING
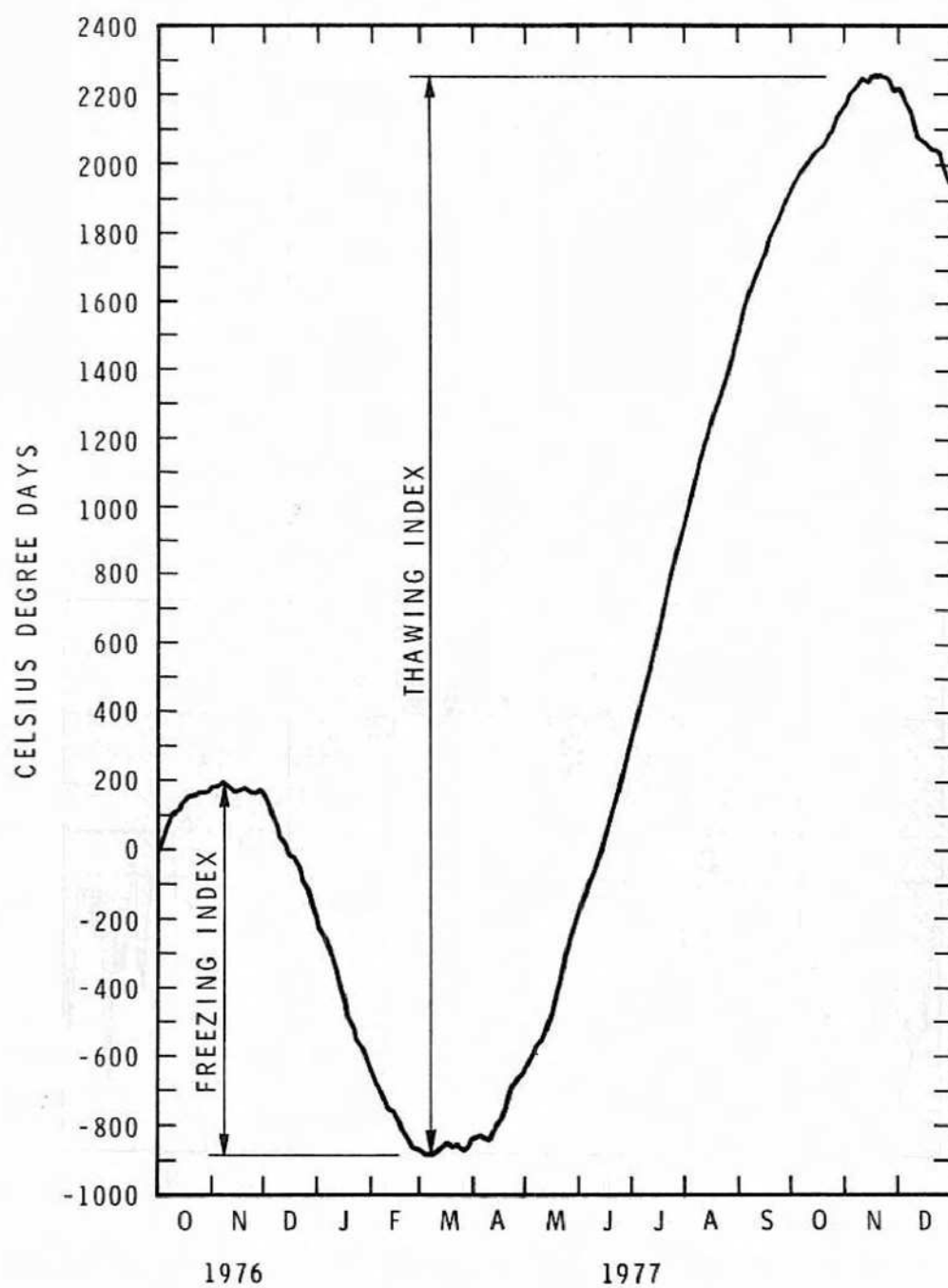- GROWING DEGREE DAYS BY BOTH STIPPLED AREAS

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FIGURE 2

CUMULATIVE DEGREE DAYS ABOVE AND BELOW
FREEZING FOR 15 MONTHS TO ILLUSTRATE
DEFINITIONS OF FREEZING AND THAWING INDICES

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