Thermal conductivity of frozen soils
Penner, E.

Publisher's version / Version de l'éditeur:

Canadian Journal of Earth Sciences, 7, 3, pp. 982-987, 1970-05-01

NATIONAL RESEARCH COUNCIL OF CANADA
CONSEIL NATIONAL DE RECHERCHES DU CANADA

THERMAL CONDUCTIVITY OF FROZEN SOILS

BY

E. PENNER

Reprinted from
CANADIAN JOURNAL OF EARTH SCIENCES
VOL. 7, NO. 3, 1970
P. 982

RESEARCH PAPER NO. 427
OF THE
DIVISION OF BUILDING RESEARCH

OTTAWA
May 1970

PRICE 25 cents

NRCC 11277

CONDUCTIVITE THERMIQUE DES SOLS GELES

SOMMAIRE

Des essais sur la conductivité thermique de deux sols gelés à des températures variant entre 0 et —22 °C, ont été faits à l'aide d'une sonde thermique et de la méthode de transfert de chaleur en régime variable. Les résultats se comparent favorablement avec les estimés de la conductivité thermique calculée selon la méthode de DeVries. Les valeurs estimées et mesurées indiquent une tendance à une augmentation de la conductivité thermique à mesure que la température est abaissée et que la teneur en glace augmente. Cette augmentation peut être reliée à la conductivité thermique plus élevée de la glace par rapport à celle de l'eau.

Thermal conductivity of frozen soils

E. PENNER

Geotechnical Section, Division of Building Research, National Research Council of Canada, Ottawa, Canada

Received December 10, 1969

Accepted for publication March 2, 1970

Thermal conductivity measurements of two frozen soils, Leda clay and Sudbury silty clay, taken at temperatures between 0 and −22 °C by means of a thermal probe and a transient heat flow technique, compare favorably with estimates of thermal conductivity calculated by the DeVries method. Both measured and estimated values show a similar trend of increasing thermal conductivity as the temperature is lowered and the ice content grows. This increase is associated with the higher thermal conductivity of ice compared with that of water.

The thermal conductivity of soil is known to be strongly dependent on density, mineral type, grain size, and moisture content (Kersten 1949), but relatively insensitive to temperature changes above 0 °C. Below 0 °C, where soils have a temperature-dependent ice content, a constant thermal conductivity cannot be assumed in heat flow calculations because the thermal conductivity of ice is more than four times greater than that of water.

It is known that all the water in soil and other porous systems does not freeze at the same temperature; that the ice content gradually increases as the temperature is lowered (Lovell 1957, Penner 1963*a*, Williams 1964). The relation between temperature below freezing and the amount of ice and water is peculiar to each soil in much the same way as is the pF[1]—moisture content relation. The main factors thought to control the amount of water in frozen soils are specific surface area, mineral type, kind of exchangeable ions, soluble salt content of the pore water, and pore size distribution. In general and for an originally saturated system, fine textured soils contain more unfrozen water at a given temperature below freezing than do coarse textured soils.

In the present paper thermal conductivity was determined by a transient heat flow method using a line heat source developed (according to Woodside 1958) by Stalhane and Pyk (1931). This method has been used by Woodside and Cliffe (1959) and by Penner (1962) for unfrozen soils, but no previous attempt has been made to apply the method to frozen soils. For comparison with measured values the thermal conductivity of two soils is estimated using the DeVries (1952, 1963) approach.

Methods and Materials

Thermal Conductivity Measurements

A transient heat flow method using a line heat source was employed to measure thermal conductivity. The line heat source,[2] similar to that used previously by D. D'Eustachio and Schreiner (1952), Woodside (1958), and Penner (1963*b*), is a hollow metal probe with an outside diameter of 0.02 in. (0.051 cm) and 4 in. (10.2 cm) long (Fig. 1). The thin hollow metal probe contains a uniformly spaced spiral heating coil with a resistance of about 885 Ω. A constantan–chromel P thermocouple is located inside the spiral heater coil half way along the probe. The thermocouple was calibrated over the required temperature range with a platinum thermometer and a G-2 Mueller bridge capable of measuring temperatures to about 1/1000 °C.

The current to the probe was monitored by measuring the voltage across a calibrated 10-Ω standard resistor with a K-3 Leeds and Northrup potentiometer. This current could be adjusted so as to maintain a constant rate of heating. A dummy resistance equal to that of the probe was switched into the circuit to stabilize the DC voltage from a regulated power supply unit before each run. The circuitry was submerged in a temperature-controlled oil bath.

The output from the thermocouple was amplified 10 000 times and read on a stabilized millivolt digital voltmeter. When necessary, the zero was offset so as to place the output from the

[1] pF is the logarithm of the height (cm of water) necessary to produce the desired suction (Baver 1948, p. 215).

NRCC No. 11277

[2] The probe in Fig. 1 was obtained from Custom Scientific Instruments Inc., Whippany, New Jersey.

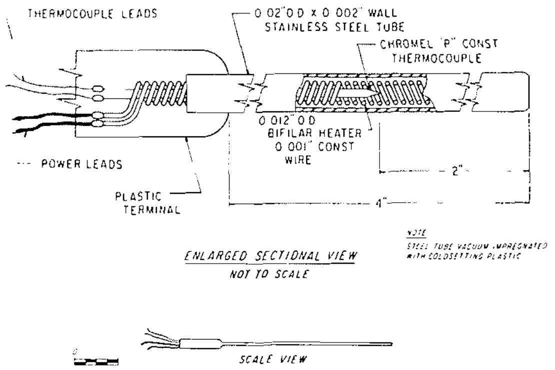

FIG. 1. Schematic diagram of thermal probe (Courtesy Custom Scientific Instruments, Inc.).

thermocouple within the range of the digital voltmeter. Thermal conductivity k was determined with the heater probe at a current level of 10 mA and gave a heat input of 2.07 mcal/s per cm of probe length.

Conductivity was calculated from the temperature rise of the probe, using the following equation

$$[1] \qquad k = \frac{Q}{4\pi\Delta T}(\ln t_2/t_1)$$

where t_2 and t_1 are the times corresponding to probe temperature change ΔT. The initial temperature rise from 0 to 2 min was not used to calculate thermal conductivity. Measurements were terminated after 10 min. For a given soil k was obtained from the least squares fit to a plot of the temperature change vs. the logarithm of the ratio of associated times.

Estimating Thermal Conductivity

Maxwell (1873) derived equations to predict the electrical conductivity of a material consisting of spherical particles immersed in a continuous medium. According to DeVries (1952), Burger (1915) extended the theory to include ellipsoidal particles and this was used by Euken (1932) for the calculation of thermal conductivity of multicomponent systems. DeVries reviewed the development of the theory and applied the following equation in various expanded forms to soil

$$[2] \qquad k = \frac{\sum_{i=0}^{N} x_i k_i F_i}{\sum_{i=0}^{N} x_i F_i}$$

where k = thermal conductivity of the system,
N = number of different kinds of particles contained in continuous medium (all particles with approximately the same shape and conductivity are considered as of one type),
x_i = volume fraction of ith kind of particles,
k_i = thermal conductivity of ith kind of particles (subscript $i = 0$ is continuous medium),
F_i = ratio of average temperature gradient in the ith kind of particles to average temperature gradient in continuous medium.

The value for F_i was estimated by DeVries (1952) from the following expression

$$[3] \qquad F_i = \frac{1}{3}\sum_j [1 + (k_i/k_0 - 1)g_j]^{-1} \; (j = a, b, c)$$

and

$$[4] \qquad \sum_j g_j = 1 \; (j = a, b, c)$$

The values for g_a, g_b, and g_c depend on relative lengths of the major and minor axes of the dispersed particles. Hence, for spherical particles $g_a = g_b = g_c = 1/3$. DeVries (1952) compared estimated and measured thermal conductivities for wet and dry soils. On a trial and error basis he was able to establish that $g_a = g_b = 0.125$ and $g_c = 0.750$, which corresponds to particles shaped like an ellipsoid of revolution. These values gave estimated thermal conductivities, using eq. [2], that compared closely with measured values. Woodside and Cliffe (1959) and Penner (1962) have also used eq. [2] to predict the conductivity of air dry and water saturated soils. Good agreement was obtained between predicted and measured thermal conductivities for these two-phase systems.

The present paper is concerned with frozen soil, which is assumed to be a three-phase system. Equation [2] then becomes

$$[5] \qquad k = \frac{x_w k_w F_w + x_i k_i F_i + x_s k_s F_s}{x_w F_w + x_i F_i + x_s F_s}$$

where the subscripts w, i and s refer to water, ice and soil components. Because F is the ratio of the average temperature gradient in the ith particles to the average temperature gradient in the continuous medium, F_w is taken as *1*. The continuity of the water medium in frozen clay soils has been shown by Hoekstra (1966), based on moisture flow studies down to −10 °C. The unfrozen water in clay soils exists to much lower temperatures, to be shown also in the present study, and such overlapping films are believed to constitute the continuous water medium.

Ice Content Determinations

Ice content was determined by a calorimetric technique similar to that used by Lovell (1957). A sufficient number of samples to allow two for each of five equilibrium temperatures were suspended in a 2-gallon air well submerged inside a temperature-controlled tank of antifreeze solution. The temperature of the tank was lowered in successive steps and the soil specimens were allowed to establish thermal equilibrium at each temperature before being removed for ice content determinations. The specimen temperatures were measured with calibrated copper-constantan thermocouples embedded inside each sample. The reliability of the temperature measurements was considered to be within ±0.02 °C.

The amount of ice in each sample at a given temperature was determined calorimetrically in a 1-quart (1136-ml) thermoflask equipped with a motorized glass stirrer. The water equivalent of the calorimeter was evaluated by melting ice cubes of known weight with initial temperatures in the same temperature range as that used for the frozen soils. The temperature change in the calorimeter was evaluated according to the method described by Lovell (1957). The temperature in the calorimeter was recorded continuously during thawing.

Values for the specific heat of water and ice and for the latent heat of fusion of ice were taken from handbooks. Those for the specific heat of the soil solids were taken from Kersten's (1949) work for similar soils in the same temperature range.

Soils and Soil Sample Preparation for Ice Content and Thermal Conductivity Measurements

The particle size data for the two soils studied, Leda clay and Sudbury silty clay, are given in Table I. Leda clay is a postglacial marine clay found extensively as a surficial deposit along the St. Lawrence and Ottawa Rivers and their tributaries (Crawford 1968). The sample of Leda clay was from an undisturbed soil core taken with a 12.7-cm diameter Osterberg sampler at a depth of 20 to 25 ft (6.1 to 7.6 m) on the site of the National Research Council near the eastern city limits of Ottawa. For thermal conductivity measurements the sample was cut to the length of the thermal probe (10.2 cm), which was inserted down the centerline of the 12.7 cm diameter cylindrical sample. The sample was then waxed and suspended in the air well of the freezing unit. When equilibrium temperature was reached, conductivity measurements were made without removing it from the air well. Measurements were made at temperatures to −22 °C.

TABLE I
Particle size distribution of soil

	% passing by weight	
Particle size (mm)	Leda clay	Sudbury silty clay
0.001	70	29
0.002	81	40
0.005	94	62
0.01	100	93
0.02	—	99
0.05	—	100

Samples for calorimetric ice content determinations were cut from the undisturbed soil core directly in contact with that used for conductivity measurements. The samples were trimmed to 3.58 cm in diameter and 8 cm in length. They were then covered with a rubber membrane and suspended in the same freezing unit. Ice contents were established from averages of duplicate samples at each temperature.

The silty clay sample was an alluvial soil from a shallow excavation within the city limits of Sudbury, Ontario. As the natural structure was destroyed in sampling, the soil was air dried, lightly crushed, and mixed. Sufficient water was added to give a viscous slurry, which was placed in a 12.7-cm diameter compaction mould and consolidated under a pressure of 2.54 kg/cm^2, the maximum for the consolidation apparatus used. Two samples were prepared, one for thermal conductivity measurements and one for ice content determinations. The specimens were kept saturated, and the technique of preventing water loss during the experiments, placement of the probe, and the method of ice content determination were the same for both soils.

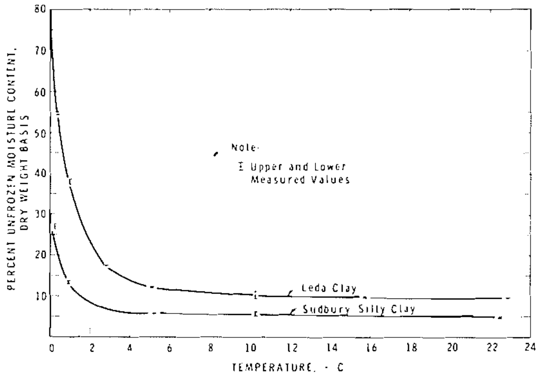

FIG. 2. Percentage of unfrozen moisture content for partially frozen soils.

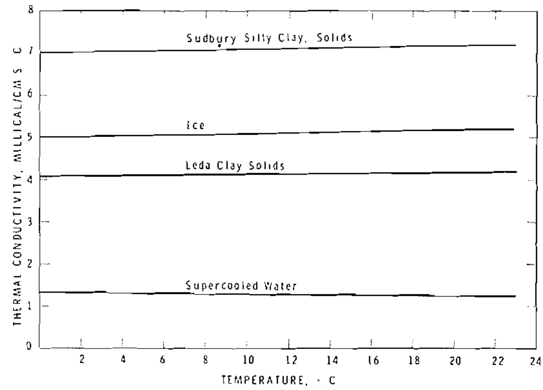

FIG. 3. Thermal conductivity of the soil constituents as a function of temperature.

Results and Discussion

Ice Contents

The temperature dependence of the unfrozen water content of the two soils is given in Fig. 2. The average saturated moisture content of the undisturbed Leda clay samples was 80%; that for the prepared samples of Sudbury silty clay was 29%.

The ice content calculations involve the use of a value for the latent heat of fusion of water. The handbook value used in this paper was 79.7 calories, which applies only at 0 °C. Antoniou (1964) used a value adjusted for phase changes occurring at temperatures other than 0 °C. The quantity required in the present calculations is the latent heat of fusion of adsorbed water. No method of determining this value for any adsorbate on any adsorbent has yet been devised. The possibility exists that too high a value was used in this study and that the ice content calculated would therefore be too low, particularly at lower temperatures. According to the calorimetric determinations, Leda clay samples contain about 10% liquid water (by weight) at −22 °C. The coarser textured Sudbury silty clay contains about 5% at the same temperature.

Measured and Estimated Conductivities

The thermal conductivity of a soil system depends on the conductivity of the individual constituents, in this case, soil solids, water and ice. As the thermal conductivity of these constituents has a small but significant temperature dependence, Fig. 3 was prepared as described below.

The conductivity of the soil solids in Leda clay was determined earlier by an extrapolation method (Penner 1962). The temperature dependence of thermal conductivity for the individual minerals was obtained from handbook values (Clark 1966) between 0 and 100 °C to give an estimated temperature dependence in the region 0 to −22 °C. On this basis the thermal conductivity of both soils was estimated to increase by 0.124% for each degree below 0 °C. Values for the thermal conductivity of water below 0 °C were obtained by extrapolation of above-zero values (Dorsey 1968) to the temperature range required. Values for ice are given by Dorsey. The conductivity value used for Sudbury silty clay solids was taken from the study by DeVries (1952), and the variation with temperature was assumed to be the same as that calculated for Leda clay.

The volume fractions of the various constituents calculated from the ice content, specific gravity of the soil solids and water content are shown in Figs. 4 and 5. Allowance was made for the 9% water to ice volume change.

Implicit in the development of the DeVries (1952) method for estimating thermal conductivity is the influence of shape and the geometric arrangement of the various constituents of the system: ice, water and soil solids. It was assumed for the present case that the arrangement consists of ellipsoids of soil solids and ice dispersed in a continuous medium of water. It is believed the assumption that water constitutes the continuous medium is justified, based on the experiment of Hoekstra (1966), unless severe ice lensing occurs. The assumption regarding the values assigned to g_a, g_b, and g_c (eq. [3]), which depend on the length of the major and minor axes of the dispersed par-

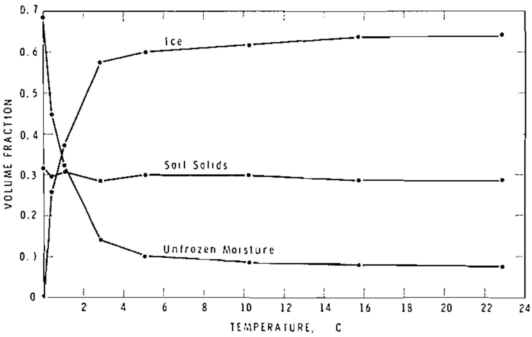

FIG. 4. Phase composition as a function of temperature for undisturbed Leda clay samples.

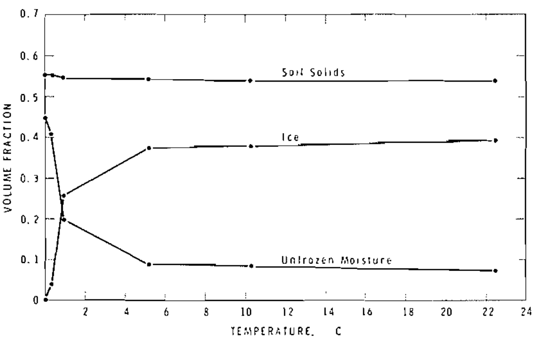

FIG. 5. Phase composition as a function of temperature for laboratory prepared Sudbury silty clay.

ticles, is much more questionable. These values were established by trial and error (DeVries 1963) so that the general approach for estimating thermal conductivity is, as best, semi-empirical.

Despite such criticism, estimations have been reasonably satisfactory in the past and appear to give good agreement with measured values for unfrozen saturated soil systems. Similarly, in the present study with partially frozen soils the estimated values reflect the influence of the increasing ice phase as the temperature is lowered (Fig. 6).

Thermal conductivity measurements with the thermal probe were repeated three or four times at each equilibrium temperature. The standard deviation at each temperature ranged from ± 0.1 to ± 0.3 mcal/s per cm °C for the Sudbury silty clay; and from ± 0.01 to ± 0.07 mcal/s per cm °C for Leda clay. In the unfrozen state the estimated values for both soils were a little lower than the

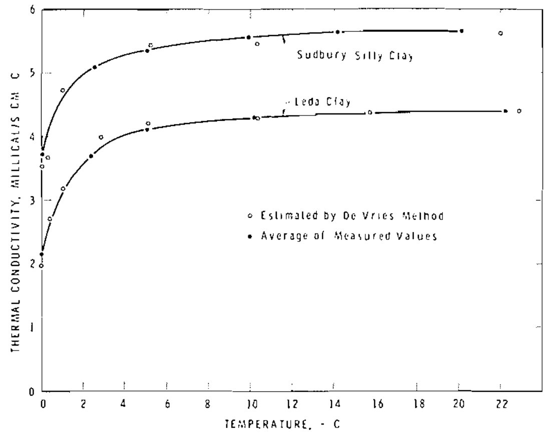

FIG. 6. Estimated and measured thermal conductivity for partially frozen soil.

measured values. In the frozen state agreement was good except for some small scatter.

Of greater significance is the fact that by both measurement and estimation the same trend of increasing thermal conductivity is shown with decreasing temperature, i.e., as the ice content increases. Examination of the specimens following removal from the air well at -22 °C showed that no macroscopic ice lensing had taken place in the Sudbury soil. Some evidence of small lenses in the Leda clay could be seen without optical aids. On thawing this soil had a crumb structure of about $\frac{1}{4}$-in. (0.6-cm). As some lensing did occur, the geometric arrangement assumed for the DeVries model was not obeyed completely.

There is a basic objection to using the transient heat flow method for determining thermal conductivity of frozen soil because of the temperature dependence of the amount of unfrozen water. Lachenbruch (1957) states that "care must be taken that the temperature does not rise to the point at which an appreciable amount of latent heat is liberated by the melting of interstitial ice." It is obvious from the results that the ice/water content ratios are highly sensitive to small temperature changes close to 0 °C and that because of this the heat input from the probe probably interfered with the phase composition in the region between 0 and -2 °C. With present measuring equipment it was not possible to reduce the heat input much below 2.07 mcal/s per cm of probe length and still obtain reliable temperature changes throughout the 10-min measuring period. For specimen temperatures below −2 °C the results were reproducible, as indicated earlier.

An alternative method for determining thermal conductivities of frozen soil is the steady-state method using a linear temperature gradient between two fixed plates. The temperature dependence of the unfrozen water content would also affect this method, particularly near 0 °C. It is thought that, because of the larger temperature differences required for the steady state method, the average conductivity measured by this method would not be that associated with the average temperature of the specimen.

Conclusions

Thermal conductivity of two frozen soils has been measured over a temperature range from 0 °C to −22 °C, using a transient heat flow method. Reasonably reliable measurements were possible below −2 °C, but in the region between 0 and −2 °C this method was not successful.

Lowering the freezing temperature of a saturated soil increases ice content and reduces water content. Because ice has a thermal conductivity more than four times that of water, this increases the overall conductivity of the soil system. Measured values of thermal conductivity show a strong dependence on phase composition. The DeVries method of estimating thermal conductivities, not used previously for frozen soils, also predicts this dependence on phase composition. The author believes this thermal conductivity behaviour pattern should be recognized in heat flow calculations for frozen soil systems in the field.

As no meaningful thermal conductivity measurements were possible from 0 to −2 °C, the need for further study is clearly indicated for frozen soils near 0 °C.

Acknowledgments

The author is grateful to his colleagues for their interest and assistance, particularly to D. Eldred for construction of the necessary apparatus and for taking the measurements in the experiments. This paper is a contribution from the Division of Building Research, National Research Council of Canada, and is published with the approval of the Director of the Division.

ANTONIOU, A. A. 1964. Phase transformations of water in porous glass, J. Phys. Chem. **68**, No. 4, pp. 2754–2764.

BAVER, L. D. 1948. Soil physics. John Wiley and Sons Inc., New York. 398 pp.

BURGER, H. C. 1915. Das Lertvermogen verdunnter Mischkristallfreier Lonsungen. Phys. Zs **20**, pp. 73–76.

CLARK, S. P. 1966. Handbook of physical constants. Geol. Soc. Amer. Inc., 587 pp.

CRAWFORD, C. B. 1968. Quick clays of Eastern Canada. Engineering Geology, **2**, No. 4, pp. 239–265.

D'EUSTACHIO, D. and SCHREINER, R. E. 1952. A study of a transient heat method for measuring thermal conductivity. Amer. Soc. Heating Ventilating Engr. Journal Section, **24**, No. 6, pp. 1–5.

DEVRIES, D. A. 1952. The thermal conductivity of soil. Mededelingen van der Landbouwhogeschool te Wageningen **52**, pp. 1–73.

——— 1963. Thermal properties of soils. *In* Physics of plant environment (*Editor*, W. R. Wijk). North-Holland Pub. Co., Amsterdam.

DORSEY, N. E. 1968. Properties of ordinary water-substance. p. 273, 482, Hafner Pub. Co., New York. 673 pp.

EUKEN, A. 1932. VDI Forschungsheft No. 353.

HOEKSTRA, P. 1966. Moisture movement in soils under temperature gradients with the cold-side temperature below freezing. Water Resour. Res. **2**, No. 2, pp. 241–249.

KERSTEN, M. S. 1949. Thermal properties of soils. University of Minnesota, Eng. Exp. Station, Bull. 28, 227 pp.

LACHENBRUCH, A. H. 1957. A probe for measurement of thermal conductivity of frozen soils in place. Trans. Amer. Geophys. Union **38**, No. 5, pp. 691–697.

LOVELL, C. W. 1957. Temperature effects on phase composition and strength of partially frozen soils. Highway Research Board, Bull. 168, pp. 74–95.

MAXWELL, C. 1873. Treatise on electricity and magnetism, 1. Oxford. 365 pp.

PENNER, E. 1962. Thermal conductivity of saturated Leda clay. Geotechnique, **12**, No. 2, pp. 168–175.

——— 1963*a*. Frost heaving in soils. Int. Conf. on Permafrost Proc., pp. 197–202 (NRC 9271).

——— 1963*b*. Anisotropic thermal conduction in clay sediments. Int. Clay Conference, Stockholm Proceed., **1**, pp. 365–376.

STALHANE, B. and PYK, S. 1931. New method for determining the coefficients of thermal conductivity. Tek. Tidskrift, 61 (28) pp. 389–393.

WILLIAMS, P. J. 1964. Unfrozen water content of frozen soils and soil moisture suction. Geotechnique, **14**, No. 3, pp. 231–246.

WOODSIDE, W. 1958. Probe for thermal conductivity measurement of dry and moist materials. Amer. Soc. Heating Air-Conditioning Engrs. Journal, Piping and Air Conditioning, **30**, No. 9, pp. 163–170.

WOODSIDE, W. and CLIFFE, J. B. 1959. Heat and moisture transfers—Closed systems. Soil Science, **87**, No. 2, pp. 75–82.